

# A New Mathematical Model for Mapping Indoor Environment

Yunusa Ali S., Abdul Rahman Ramli, Siti A Ahmad  
and Arash Toudeshki  
Dept. Electrical and Electronic Engineering,  
Faculty of Engineering, Universiti Putra Malaysia,  
43400 UPM Serdang, Selangor, Malaysia

Ata Jahangir Moshayedi  
Department of Electronic Science,  
University of Pune, Pune  
411007, Maharashtra, India

**Abstract**— This paper presents a mathematical model as a new approach to object mapping, the system is proscribed to indoor and applied to approach a landmark. The contribution of this paper is to propose a new mathematical model for object mapping, the landmark is captured at varying distant points, the Scale invariant Feature Transform (SIFT) to extract object options, at the side of their uncertainty, from camera sensors. The (SIFT) features are invariant to image scaling, translation, and rotation, and partially invariant to illumination changes and affine or 3D projection, which is suitable for our application. As image options do not seem to be noise-free, the error analysis of the landmark positions and a preprocessing to obtained information which is incorporated into a model, using curve fitting techniques. Predictions created by our model square measure well and correlate with experimental knowledge. This has eliminated correspondence Problem also known as a data association problem.

**Keywords**—mapping, feature extraction, fitting, threshold, clusters, landmark, SLAM

## I. Introduction

Simultaneous Localisation and Mapping (SLAM) is one with all the most difficult issues that has to be solved before we can have autonomous navigation. Typical approaches involve the utilization of grid representations, landmark representations or topological representations. Despite the representation used, the systems created so far are probabilistic to a point [1]. Many of today's models and approaches for object class recognition are based on local features. Local features are typically extracted from images and subsequently grouped into appearance clusters [2].

This research work focuses on the study of the landmark for localization from a strictly empiric position, we tend to explore the connection between the edge and have a cluster of each image by using K-means algorithm, as the most popular method, recognition performance is better for k-means clustering, and data structure was achieved for efficient volume search in high dimensional space [3] and additionally image distance and feature cluster [4]. Coincidental localization and map building (SLAM) is that the Golem explores the atmosphere and builds a map concerning it, whereas localizing itself relative to the present map simultaneously [5-7]. Over the past 20 years, several

approaches are projected to resolve this drawback [6,8-10]. One among the known approaches is the extended Kalman filter (EKF) based mostly SLAM. Dissanayake et al. [3] Projected 3 theorems to prove that the quality EKF-SLAM rule is confluent within the case of linear motion and observation model.

The novelty of this work is the mathematical model to localize the Golem using a landmark with a low cost camera as a sensor to have a cheap map. With this a stable and accurate map for a static indoor environment can be achieved. The sampled employed in our model are compact and adaptable. The curve produce acceptable datum which make the model valid and it gave a lot of correct results.

## II. Methodology

We utilize a simple and straight forward approach. In this approach landmarks are captured, and features are extracted from the captured image after preprocessing, the features are clustered based on selected threshold, a model is selected to produce best fit of data points from curve fitting. Curve Fitting uses the linear least-squares method [12] to fit a linear model to data. A linear model is defined as an equation that is linear in the coefficients.

## III. EXPERIMENTAL SETUP

In this section we present and discuss estimated results. The experimental setup was carried out by analyzing the data, thresholding and clustering and finally processing the information for the object mapping process to be achieved, .

### A. Data

The experimental setup employed in this article requires the Golem to actively move its gaze direction to the landmark and integrate information over time in order to accomplish the task. We propose that the evolved Golem can detect the landmark within the range (150cm).

In this section we present and discuss the validated results, we used an ordinary camera (FUJIFILM Digital Camera, FinePix J38 to capture the Landmark). To normalise the effect

of illumination during the experiment the flash light was set off and Lens was at normal zoom, as you can see in the pictorial view Figure 1. Below, the traffic signal sign is used as the landmark. The landmark image was captured at various distances of 10cm interval to 150cm. And undergo an image processing to remove noise. Our landmark consists of 78 invariant features provided by Lowe, with SIFT descriptors [13]. This is shown in Figure 2.

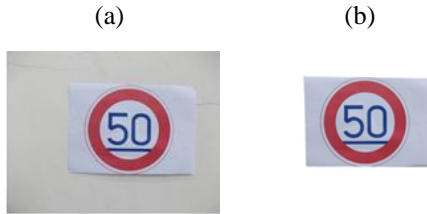


Figure 1. (a) Image captured at different distance point; (b) the de-noised images.

### B. Threshold and Clustering

K-means clustering is a partitioning method. The kmeans partitions data into k mutually exclusive clusters, and returns the index of the cluster to which it has assigned each observation. K-means clustering operates on actual observations (rather than the larger set of dissimilarity measures), and creates a single level of clusters. It is found that the k-means clustering is often more suitable for large amounts of data.



Figure 2. (a) Image of landmark; (b) Sift features found with scale and orientation

### C. Processing

The projected technique has been developed to fit data points into model kind. The format of the new knowledge points regenerated by the projected technique is appropriate for the necessities for fitting into a brand new curve with a decent form. The whole procedure of this technique involves filtering, regressing, and steps. As it was proposed in Ref. [11], the tactic is enforced and used for a utilization in landmark localization in this work.

Based on the data, the value threshold are selected to fit (1) for the relationship of threshold and cluster was proposed and optimized to achieve the best fit to the raw data points.

$$C(T) = \alpha \cdot T^\beta + \gamma . \quad (1)$$

Where C is representing the clusters, T is the value of threshold,  $\alpha$ ,  $\beta$  and  $\gamma$  are the coefficients that their values are dependent on the distance points.

In order to justify the relationship between the coefficient  $\alpha$  and the distance, (2) is suggested.

$$\alpha(d) = \psi_1 \cdot d^{\psi_2} + \psi_3 . \quad (2)$$

Where d is the distance of the camera's lens to the object and coefficients  $\psi_1$  is 2.68E+10,  $\psi_2$  is -5.113 and  $\psi_3$  is 167.7 which are obtained by using the curve fit optimization technique with R-square of 0.9107.

In order to justify the relationship between the coefficient  $\beta$  and the distance, (3) is suggested.

$$\beta(d) = \sum_{i=0}^3 \rho_i \cdot d^i . \quad (2)$$

Where d is the distance of the camera's lens to the object and coefficients  $\rho_0$  is -1.786,  $\rho_1$  is 0.02904,  $\rho_2$  is -0.0002376 and  $\rho_3$  is 4.39E-07 which are obtained by using the curve fit optimization technique with R-square of 0.6389.

In order to justify the relationship between the coefficient  $\gamma$  and the distance, (4) is suggested with R-square of 0.9993.

$$\gamma(d) = \sum_{i=1}^4 m_i \cdot \sin(\omega_i \cdot d + \phi_i) . \quad (4)$$

Where d is the distance of the camera's lens to the object and coefficients  $m_i$ ,  $\omega_i$  and  $\phi_i$  which are obtained by using a curve fit optimization technique, are shown in Table 1.

TABLE I. COEFFICIENTS

i	m	$\omega$	$\phi$
1	4.84	0.02437	3.003
2	1.441	0.1864	3.136
3	1.263	0.2665	-3.407
4	0.8779	0.09802	3.932

As a result, by replacing the coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  from (2), (3) and (4) in (1), we present the generalized and simplified (5) that can give the almost exact behavior of clusters as a function of threshold and distance.

$$\begin{aligned}
 C(T, d) = & 1.441 \sin(0.1864d + 3.136) \\
 & + 4.84 \sin(0.02437d + 3.003) \\
 & + 0.8779 \sin(0.0980d + 3.9320) \\
 & + 1.23 \sin(0.25d - 3.407) \\
 & + \left( \frac{2.8 \times 10^{10}}{d^{5.113}} + 168 \right) T^{(4.39 \times 10^{-8} d^3 - 2.37 \times 10^{-4} d^2 + 2.9 \times 10^{-2} d - 1.786)}
 \end{aligned} \quad (5)$$

Mathematical calculation of distance from  $C(T, d)$  (5) is very complicated. In order to simplify the process, we propose that another fitting process must be repeated from the beginning. As a result, the new optimal equation that fit data point shows the relationship between distance, cluster and threshold. In this new equation distance is explained as a function of cluster and threshold as the process shown in (6), (7), (8) and (9).

$$d(C, T) = \beta(T) \sqrt{\frac{C - \gamma(T)}{\alpha(T)}} \quad (6)$$

Where:

$$\begin{aligned}
 \beta(T) = & 10.98 \sin(0.0793T + 5.701) \\
 & + 27.81 \sin(0.04181T + 1.375) \\
 & + 33.36 \sin(0.05708T - 2.536)
 \end{aligned} \quad (7)$$

$$\gamma(T) = \frac{819.3}{e^{(0.6038T - 19.8188)^2}} \quad (8)$$

$$\alpha(T) = \frac{1.235 \times 10^{15}}{e^{(0.644T - 62.863)^2}} \quad (9)$$

#### IV. Validation

The proposed data fitting techniques were implemented, once distance and the threshold are selected. Cluster are obtained, which correspond to the raw data recorded. This is good validation.

In curve fitting technique, in order to find the optimal coefficient, needed as shown in (9) derived under section 3. This technique is flexible and simply can provide a convenient interface for data fitting optimization. However, since, the data fitting is a computational technique and discretize the data, base on it design iteration, during the computing process, error arise due to uncertainty at its final result. In order to minimize

this (as much as possible) it must define correct initial input value, as it was mentioned in section 3.

Plot of reference model showing goodness of fit which are require for defining the right seeking bound and start point optimal coefficient (Figure 3, 4, 5 and 6).

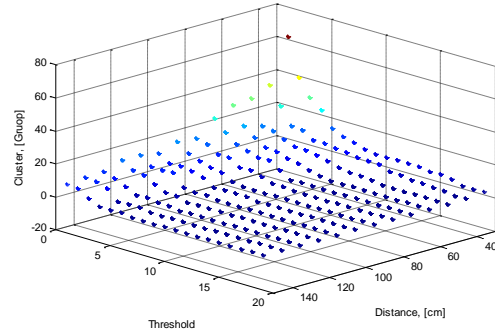


Figure 3: Showing 3D plot of Data Points as the reference model

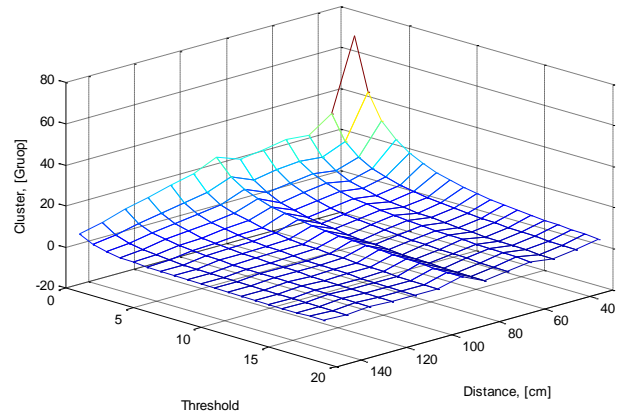


Figure 4: 3D plot showing goodness of fit, generated from the modeled equation

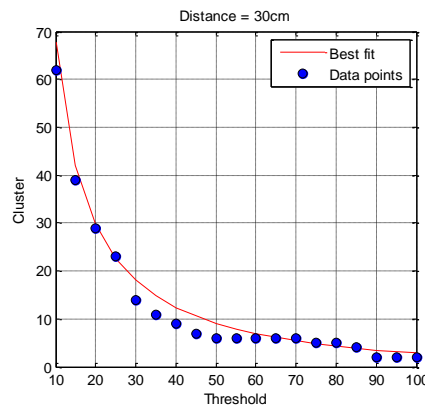


Figure 5: Validation of the fit to the data point for a distance of 30cm with R-square of 0.9908

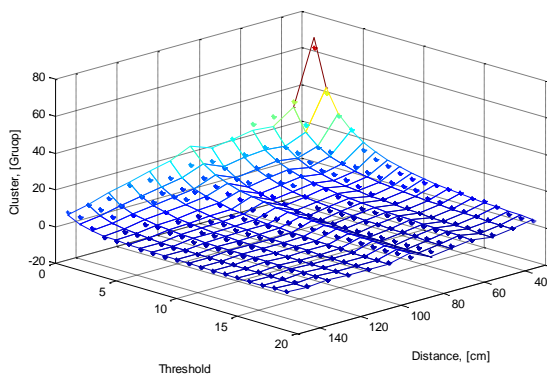


Figure 6: Validation of all the generated data showing goodness of fit

## v. Discussion and Conclusion

Based on data which are presented in this graph, the proposed equation can predict the cluster with a very small error, in the range from 30 to 70 cm distances and also from 100 to 150 cm. However, the error for distance between 70 to 100 cm is a bit higher, but since this amount of error can be neglected. The tolerance of the error's median for all distances is about  $\pm 3\%$ . Experiments were allotted to produce curves through some random nominal Points. Each open and closed curve are generated with success and a few of the representative results are conferred during this process. The gap contained expressly correspond to the metric distance within the world, that is a sign of the power to travel in an exceedingly path. The new mathematical model can be used as a new approach to object mapping and recognition for mobile Golem. This is often a giant step into the robotic world. Our model has been valid and also the results compared with our information shows ninety fifth matching. The landmarks can be artificial or natural. For future work we are going to evaluate the propose mathematical model in a real robot system.

## References

- [1] M. Milford, Wyeth, G., & Prasser, D., "Simultaneous localisation and mapping from natural landmarks using RatSLAM," in *Australasian Conference on Robotics and Automation*. Australian Robotics and Automation Association Inc, 2004.
- [2] B. Leibe, Mikolajczyk, K., & Schiele, B. , "Efficient clustering and matching for object class recognition," in *Proceedings of BMVC*, 2006, pp. 789-798.
- [3] B. Leibe, K. Mikolajczyk, and B. Schiele, "Efficient clustering and matching for object class recognition," in *Proceeding of BMVC*, 2006, pp. 789-798.
- [4] T. Bailey and W. H. Durrant, "Simultaneous localization and mapping (SLAM): Part II," *Robotics*

- & *Automation Magazine, IEEE*, vol. 13, pp. 108-117, 2006.
- [5] M. W. M. G. Dissanayake, P. Newman, S. Clark, H. F. Durrant-Whyte, and M. Csorba, "A solution to the simultaneous localization and map building (SLAM) problem," *Robotics and Automation, IEEE Transactions on*, vol. 17, pp. 229-241, 2001.
- [6] H. Durrant-Whyte and T. Bailey, "Simultaneous localization and mapping: part I," *Robotics & Automation Magazine, IEEE*, vol. 13, pp. 99-110, 2006.
- [7] A. Eliazar and R. Parr, "DP-SLAM: Fast, robust simultaneous localization and mapping without predetermined landmarks," pp. 1135-1142.
- [8] E. Flaccomio, F. Damiani, G. Micela, S. Sciortino, F. R. Harnden Jr, S. S. Murray, and S. J. Wolk, "Chandra X-Ray Observation of the Orion Nebula Cluster. I. Detection, Identification, and Determination of X-Ray Luminosities," *The Astrophysical Journal*, vol. 582, p. 382, 2008.
- [9] G. Grisetti, C. Stachniss, and W. Burgard, "Improved Techniques for Grid Mapping With Rao-Blackwellized Particle Filters," *Robotics, IEEE Transactions on*, vol. 23, pp. 34-46, 2007.
- [10] D. Hahnel, W. Burgard, D. Fox, and S. Thrun, "An efficient FastSLAM algorithm for generating maps of large-scale cyclic environments from raw laser range measurements," 2003, pp. 206-211.
- [11] M. C. Huang and C. C. Tai, "The pre-processing of data points for curve fitting in reverse engineering," *The International Journal of Advanced Manufacturing Technology*, vol. 16, pp. 635-642, 2000.
- [12] eFunda. (2011). *linear least-squares method*
- [13] D. L. a. J. L. Stephen Se, " Mobile robot localization and mapping with uncertainty using scale-invariant visual landmarks," *The International Journal of Robotics Research*, vol. VOL 21 N0. 8, pp. 735-758, Aug 1, 2002 2002.