

Comparative Study of Bio-inspired algorithms for Unconstrained Optimization Problems

Adil Hashmi, Divya Gupta, Nishant Goel, Shruti Goel

Abstract—Nature inspired meta-heuristic algorithms are iterative search processes which find near optimal solutions by efficiently performing exploration and exploitation of the solution space. Considering the solution space in a specified region, this work compares performances of Bat, Cuckoo search and Firefly algorithms for unconstrained optimization problems. Global optima are found using various test functions of different characteristics.

Keywords— Firefly Algorithm, Bat Algorithm, Cuckoo Search Algorithm, Unconstrained Optimization, Benchmark Functions, Nature-Inspired Algorithms.

Introduction

Optimization has always been an active research area because of the fact that almost all real-world optimization problems belong to a class of hard problems. In all optimization problems the goal is to find the minimum or the maximum of the D-dimensional objective function where D is the number of variables to be optimized [1]. Population based optimization algorithms have proved to be very efficient in solving unconstrained optimization problems through motivation from nature [2][3]. Evolutionary algorithms [4] and swarm intelligence [5] based algorithms are two important classes of population based optimization algorithms. The popular example of one such evolutionary algorithm is Genetic Algorithm (GA) [6]. Swarm intelligence based algorithms have gained interest of many researchers and scientists tremendously. Since our prime focus is on solving the unconstrained problem [7] through swarm based algorithms, this research paper consists of only swarm-based algorithms which are popularly known for their efficient optimization task.

A swarm is any collection of interacting agents. Amongst them, the most popular ones are Bees Algorithm [8], Particle Swarm Optimization (PSO) [9], Firefly Algorithm (FA) [10], Cuckoo Search Algorithm (CS) [11] and Bat Algorithm (BA) [12]. We have implemented the latter three. Firefly and Bat Algorithm were developed by Xin She Yang at Cambridge University in 2007 and 2010 respectively [13][14] whereas the

Cuckoo Search algorithm was developed by Young and Deb in 2009. All the three algorithms are based on different techniques where FA is inspired by the flashing behaviour of fireflies; BA is based on the echolocation behaviour of bats and CS mimics the brooding behaviour of some cuckoo species. Our purpose is to compare the performances of these algorithms on the basis of convergence speed and precision. Benchmark functions have been used to find optimal solutions for single objective unconstrained optimization problems [15].

In this work, a comprehensive comparative study on the performances of recently introduced swarm-based algorithms for optimizing a set of numerical functions is presented. The rest of the paper is organized as follows. Section I summarizes swarm-based algorithms- Firefly, Cuckoo Search and Bat Algorithm. Section II briefly describes the test functions used along with their simulation results. Section III reports experimental settings of the algorithm and experimental analysis on the three algorithms. Finally, Section IV concludes the work done.

I. Swarm Based Optimization Algorithms

Firefly, Cuckoo Search and Bat Algorithms are well known for their efficiency in the field of optimization. These swarm inspired algorithms are well researched in providing good results for unconstrained optimization problems.

A. Firefly Algorithm (FA)

This algorithm is inspired by the flashing behavior of fireflies. Fireflies or lightning bugs belong to family of insects that are capable to produce natural light to attract a mate or a prey. If a firefly is hungry or looks for a mate its light glows brighter in order to make the attraction of insects or mates more effective. The brightness of the bioluminescent light depends on the available quantity of a pigment called *luciferin*, and more pigment means more light.

The FA uses three main basic rules:

1. A firefly will be attracted by other fireflies regardless of their sex.
2. Attractiveness is proportional to their brightness and decreases as the distance among them increases.
3. The landscape of the objective function determines the brightness of a firefly [10].

Adil Hashmi (Author)
Assistant Professor,
Maharaja Surajmal Institute of Technology, GGSIPU
New Delhi, India

Divya Gupta, Nishant Goel, Shruti Goel (Authors)
Maharaja Surajmal Institute of Technology, GGSIPU
New Delhi, India

Pseudo code for FA:

1. Objective function of $f(x)$, where $x=(x_1, \dots, x_d)$
2. Generate initial population of fireflies;
3. Formulate light intensity I ;
4. Define absorption coefficient γ ;
5. While ($t < \text{MaxGeneration}$)
6. For $i = 1$ to n (all n fireflies);
7. For $j=1$ to n (all n fireflies)
8. If ($I_j > I_i$), move firefly i towards j ;
9. end if
10. Evaluate new solutions and update light intensity;
11. End for j ;
12. End for i ;
13. Rank the fireflies and find the current best;
14. End while;
15. Post process results and visualization;
16. End procedure;

B. Cuckoo Search Algorithm (CS)

Cuckoo search algorithm implements the breeding behaviour of various cuckoo species in order to apply it to various optimization problems. Cuckoo breeding behaviour is just similar to the phenomenon of brood parasitism. The female cuckoo often called as parasite chooses a nest to lay eggs where the host bird just laid its own eggs. When the first cuckoo egg is hatched, the cuckoo eggs being hatched slightly earlier than the respective host eggs; it evicts the host eggs by propelling the eggs out of the nest. This results in increasing its feeding opportunity as the food stored by the respective host bird is consumed only by the first cuckoo chick hatched.

Some of the cuckoos also have the ability to imitate the colours and patterns of the eggs of the chosen host species. This reduces the probability of the eggs being abandoned and thus increases the productivity of the cuckoo eggs. Also, a situation may occur where the host bird finds an alien egg from the intruding cuckoos. In this case, the host bird will throw the cuckoo egg away or will simply build the new nest elsewhere.

Each egg in a nest represents a solution, and a cuckoo egg represents a new solution with the aim to employ the new and potentially better solutions (cuckoos) and to replace not-so-good solutions in the nests. Each nest has one egg.

The algorithm can be extended to more complicated cases in which each nest has multiple eggs.

The CS is based on three idealized rules:

1. Each cuckoo lays one egg at a time, and dumps it in a randomly chosen nest;
2. The best nests with high quality of eggs (solutions) will carry over to the next generations;
3. The number of available host nests is fixed, and a host can discover an alien egg with probability $p_a \in [0,1]$. In this case, the host bird can either throw the

egg away or abandon the nest to build a completely new nest in a new location. [11]

Pseudo code for CS Algorithm:

1. Initialize the popularity of the host nests, N and related parameters;
2. While ($t < \text{MaxGeneration}$) or (stop criterion)
3. Get a cuckoo randomly (say, i) and replace its solution by performing Lévy flights;
4. Evaluate its quality/fitness;
5. Choose a nest among n (say, j) randomly;
6. if ($F_i > F_j$),
7. Replace j by the new solution;
8. end if;
9. A fraction (p_a) of the worse nests are abandoned and new ones are built;
10. Keep the best solutions/nests;
11. Rank the solutions/nests and find the current best;
12. Pass the current best solutions to the next generation;
13. end while;

C. Bat Algorithm (BA)

Bats can find their prey even in complete darkness based on a property called echolocation. Bats use echolocation to a certain degree depending on their type. In echolocation, bats emit a loud sound pulse and listen for the echo that bounces back from the surrounding objects. Micro-bats use the time delay from the emission and detection of echo for echolocation. Hence it can detect the type of prey, distance and orientation of the target, and even their moving speed. Bat algorithm is implemented in order to associate it with the objective function to be optimized to give rise to new optimization algorithms.

The idealized rules for Bat Algorithm are as follows.

1. All bats use echolocation to sense distance, and they also 'know' the difference between food/prey and background barriers in some magical way.
2. Bats fly randomly with velocity v_i at position x_i with a fixed frequency f_{\min} , varying wavelength λ and loudness A_0 to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission r in the range of $[0, 1]$, depending on the proximity of their target.
3. Although the loudness can vary in many ways, we assume that the loudness varies from a large (positive) A_0 to a minimum constant value A_{\min} [12].

Pseudo code for Bat Algorithm:

1. Objective function $f(x)$, $x = (x_1, \dots, x_d)$;
2. Initialize the bat population x_i ($i = 1, 2, \dots, n$) and v_i ;
3. Define pulse frequency f_i at x_i ;
4. Initialize pulse rates r_i and the loudness A_i ;
5. while ($t < \text{Max number of iterations}$)
6. Generate new solutions by adjusting frequency, and updating velocities and locations/solutions;
7. if ($\text{rand} > r_i$)
8. Select a solution among the best solutions;
9. Generate a local solution around the selected best solution;
10. end if;
11. Generate a new solution by flying randomly;
12. if ($\text{rand} < A_i \& f(x_i) < f(x^*)$)
13. Accept the new solutions;
14. Increase r_i and reduce A_i ;
15. end if;
16. Rank the bats and find the current best x^* ;
17. end while

II. Benchmark Functions

To test the performance of the two algorithms, five well known benchmark functions are used. These functions are useful to evaluate characteristics of optimization algorithms, such as convergence speed, precision, robustness and general performance [15]. Some of the benchmark functions are unimodal, while others are multimodal. A function is called unimodal if it has only one optimum position. The multimodal functions have two or more local optima [1]. The five functions used in our research work are listed in Table I.

TABLE I Test Functions: (D-dimension of the function)

Function	Range	Formulation	F(x)
Step	[-10,10]	$f(x) = \sum_{i=1}^D (x_i + 0.5)$	0
Sphere	[-2,2]	$f(x) = \sum_{i=1}^D x_i^2$	0
Sum square	[-10,10]	$f(x) = \sum_{i=1}^D 1x_i^2$	0
Trid10	[-100,100]	$f(x) = \sum_{i=1}^D (x_i - 1)^2 - \sum_{i=1}^D x_i x_{i-1}$	-210
Zakharov	[-0.5,10]	$f(x) = \sum_{i=1}^D x_i^2 + (\sum_{i=1}^D 0.5ix_i)^2 + (\sum_{i=1}^D 0.5ix_i)^4$	0

III. Experimental Analysis

The experimental environment is implemented in MATLAB programs and executed on a DELL Studio15 computer with the configuration of Intel Core I3 CPU M370 at 2.40GHz and 4GB RAM. In each experimental run, for each population size the five standard functions are processed ten times to measure the elapsed time taken and to find the average of the best objective function values.

The parameter settings for benchmark functions for all the three algorithms- Bat, Cuckoo Search and Firefly are given in Table 2 whereas the experimental results for objective function values and processing time are shown in Table 3 and Table 4 respectively. The number of variables i.e. the dimension of the algorithms has been kept constant equal to 10.

TABLE II Parameters set for Bat and Firefly algorithms

BAT ALGORITHM	FIREFLY ALGORITHM	CUCKOO SEARCH ALGORITHM
A (loudness): 0.5 R (pulse rate): 0.5 Q _{min} (frequency min) : 0 Q _{max} (frequency max): 2 Population Size : 10,20,40	α (randomness): 0.2 γ (absorption): 1.0 β : 0 β_0 : 0 Population Size : 10,20,40	P_a (discovery rate) :0.25 Tol (tolerance) : 1.0e ⁻⁵ β (Levy) : 3/2 Population Size : 10,20,40

Table 3 gives the best, mean and worst values of the optima obtained by all the three algorithms for each benchmark functions. As we can see, the Firefly algorithm has emerged out to be a winner in terms of objective function values. Except the Zakharov function, the Firefly algorithm has outperformed other two algorithms for almost every benchmark function. For unimodal-separable functions (Step, Sphere and Sum-Square), Firefly algorithm values for almost all test functions are better by factor of 10⁻⁴ for Bat algorithm and by a factor of 10⁻¹ for Cuckoo Search algorithm. For unimodal-nonseparable functions, we found different results. For Trid10 function, only the Firefly algorithm was able to achieve the global optima unlike the Bat and Cuckoo Search algorithm, and for Zakharov function, the Cuckoo Search algorithm showed better results than the firefly algorithm by a factor of 10⁻⁶.

Population size has also played a vital role in the comparison as we could clearly see that as we increase the population size all the three the algorithms reach a more optimum value.

Table 4 gives the best, mean and worst values of the processing time in seconds. The elapsed time is measured as the time taken to run the algorithms from start to finish. Here we see that the Cuckoo Search algorithm converges to an optimization problem much faster than the other two algorithms. The difference in the best, mean and the worst times clearly indicates the winner.

TABLE III Comparison of algorithms with respect to objective function values. (N-population size)

Test functions	N	Bat Algorithm			Firefly Algorithm			Cuckoo Search Algorithm		
		Best	Worst	Mean	Best	Worst	Mean	Best	Worst	Mean
Sphere function	10	1.12e ⁻⁶	0.0325	0.0050	2.45e ⁻⁸	6.76e ⁻⁸	4.39e ⁻⁸	9.47e ⁻⁶	9.86e ⁻⁶	9.77e ⁻⁶
	20	6.13e ⁻⁷	1.180e ⁻⁶	9.0460	1.17e ⁻⁸	4.45e ⁻⁸	4.17e ⁻⁸	8.48e ⁻⁶	9.77e ⁻⁶	9.07e ⁻⁶
	40	5.55e ⁻⁷	1.187e ⁻⁶	6.2461	1.78e ⁻⁸	7.57e ⁻⁸	2.09e ⁻⁸	6.60e ⁻⁶	8.29e ⁻⁶	8.18e ⁻⁶
Sum Square Function	10	8.01e ⁻⁶	1.695e ⁻⁵	9.219e ⁻⁵	3.24e ⁻⁶	1.54e ⁻⁵	9.23e ⁻⁷	9.23e ⁻⁶	9.99e ⁻⁶	9.72e ⁻⁶
	20	7.42e ⁻⁶	1.246e ⁻⁵	25.67e ⁻⁵	5.01e ⁻⁶	1.48e ⁻⁵	1.40e ⁻⁵	8.64e ⁻⁶	9.88e ⁻⁶	8.57e ⁻⁶
	40	0.10e ⁻⁶	9.844e ⁻⁶	5.458e ⁻⁶	3.23e ⁻⁶	1.26e ⁻⁵	0.87e ⁻⁵	5.87e ⁻⁶	9.61e ⁻⁶	9.55e ⁻⁶
Step Function	10	8.9720	125.42	45.4720	4.26e ⁻⁷	1.31e ⁻⁶	9.72e ⁻⁷	6.89e ⁻⁶	9.77e ⁻⁶	9.075e ⁻⁶
	20	0.7910	36.676	20.1534	4.52e ⁻⁷	1.95e ⁻⁶	9.37e ⁻⁷	6.52e ⁻⁶	9.75e ⁻⁶	9.17e ⁻⁶
	40	1.07e ⁻⁶	0.0104	0.00105	4.22e ⁻⁷	1.71e ⁻⁶	9.26e ⁻⁷	4.09e ⁻⁶	9.07e ⁻⁶	8.07e ⁻⁶
Trid10 Function	10	967.38	12915.7	5825.42	-209.9	-209.9	-209.9	-5.566	-45.64	-17.73
	20	657.56	5120.91	2815.86	-209.9	-209.9	-209.9	-2.765	-24.32	-23.66
	40	511.19	2674.69	2031.53	-209.9	-209.9	-209.9	-2.867	-54.28	-16.76
Zakharov Function	10	96.340	1689.21	633.858	-0.090	-0.090	-0.090	9.44e ⁻⁶	9.74e ⁻⁶	9.51e ⁻⁶
	20	71.440	1864.08	546.703	-0.090	-0.090	-0.090	8.68e ⁻⁶	9.61e ⁻⁶	8.92e ⁻⁶
	40	24.430	629.610	230.117	-0.090	-0.090	-0.090	6.26e ⁻⁶	9.45e ⁻⁶	7.63e ⁻⁶

TABLE IV Comparison of algorithms with respect to processing time in seconds (N-population size)

Test functions	N	Bat Algorithm			Firefly Algorithm			Cuckoo Search Algorithm		
		Best	Worst	Mean	Best	Worst	Mean	Best	Worst	Mean
Sphere function	10	0.304	0.362	0.326	0.456	0.655	0.534	0.195	0.226	0.203
	20	0.553	0.677	0.606	1.495	1.784	1.677	0.297	0.328	0.312
	40	1.129	1.238	1.186	5.587	6.253	5.900	0.671	0.733	0.702
Sum Square Function	10	0.463	0.568	0.498	0.359	0.487	0.438	0.343	0.593	0.421
	20	0.874	0.963	0.919	0.971	1.140	1.068	0.593	0.889	0.624
	40	1.771	1.943	1.827	3.071	4.187	3.825	1.344	1.656	1.405
Step Function	10	0.263	0.406	0.312	0.421	0.749	0.529	0.242	0.273	0.250
	20	0.158	0.590	0.546	1.490	1.817	1.482	0.359	0.489	0.421
	40	1.012	1.193	1.107	5.663	6.606	6.058	0.842	0.889	0.874
Trid10 Function	10	0.272	0.371	0.314	0.452	0.648	0.525	0.031	0.078	0.070
	20	0.534	0.654	0.597	1.560	2.122	1.736	0.109	0.764	0.198
	40	1.074	1.240	1.162	5.819	5.990	6.489	0.218	0.273	0.258
Zakharov Function	10	0.359	0.458	0.392	0.562	0.765	0.649	0.982	1.217	1.036
	20	0.942	1.078	1.014	1.763	1.997	1.910	1.076	1.287	1.137
	40	1.802	1.957	1.867	4.946	6.817	6.370	2.636	2.979	2.792

IV. Conclusions

In this paper, three swarm based algorithms were compared on accuracy and convergence speed parameters. Although there are improved versions of all the three algorithms, we have used standard version only with set of control parameters. It was observed that firefly algorithm gave better results to most of the test functions than the other two algorithms. But the Cuckoo Search algorithm had a faster convergence speed than Firefly and Bat algorithm. This conclusion is based on the five benchmark functions used in our experiment and the results may vary for some other set of benchmark functions.

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