

Design of an Electromechanical Positioning System with Sinusoidal Change of the Jerk

Mihaylo Y. Stoychitch

Abstract—In this paper we consider design of an electromechanical positioning system with a controlled jerk. A system that is formed from a load and an actuating device is used as an object. For the proposed sinusoidal change of the jerk, the appropriate changes of the acceleration, velocity and displacement were found. The algorithm which ensures the motion of the object with prespecified jerk so that the requirements which are related to the maximum values of the acceleration, velocity and displacement are satisfied, is also proposed. The feedforward and feedback controllers that are realized this algorithm are designed. Furthermore the simulation of that system is performed, which is confirmed the proposed theory.

Keywords—positioning system, trajectory planning, feedback and feedforward design, controlled jerk

I. Introduction

Positioning systems are often used in the industry as electromechanical drive or to drive of robots. The task of this system is to achieve adequate movement between two arbitrary points, from a point A to a point B, where the system at the initial and the final points is at the idle state. In doing so, we assume that the acceleration and deceleration times are the same, or equivalently, the trajectory is symmetric with respect to the velocity. In order to design such a system it is necessary to solve several problems, such as:

- trajectory planning: determining allowable trajectory and all parameters of motion (jerk, acceleration, velocity, ...) for all degrees of freedom and for each actuating device, separately
- controller design: designing feedback and/or feedforward controller that ensures realization of the desired trajectory for each actuating device, even in circumstances when disturbances (internal and/or external) act on the object, and there exists an unmodeled dynamic of the object, and
- other problems, such as: diagnostic, internal checks, communications, etc.

Solutions of to above problems are generally reduced to one actuator unit, on one axis or a degree of freedom.

Motion of the object (plant) between two points can typically be divided into three phases: acceleration, motion with constant speed, and deceleration. Traditionally, a trapezoidal (or triangular) speed profile has been mainly used. This means that acceleration of the object (for a time t_a) is constant until it reaches the maximum speed, then it keeps this speed (for a

time t_v), and it decelerates by a constant deceleration (also, for a time t_a) so that the total time of the movement τ is $\tau = 2t_a + t_v$.

One of the main problems with the trapezoidal speed profile is large changes of the jerk, and consequently large inertial forces. Furthermore, that can induce large vibrations of mechanical parts of the system, which leads to a large (often unacceptable) the stationary error and too long settling time.

There exist several approaches to improve the performance of these systems, which can be roughly split as:

1. Trajectory smoothing or shaping: The result can be very good, but it can lead to a significant increase in execution time of the trajectory. This approach is considered in [2,3,7,8].
2. Feedforward control based on plant inversion: This approach gives good results only if the plant model is well known, but with important disadvantage with respect to robustness of the system. Different examples of this approach can be found in papers [2,3,4].
3. Feedback control optimization: Since the feedback controller is an integral part of almost all positional systems, then its optimization leads to decrease of the stationary errors and settling time, but at the same time may increase the overshoot and reduce the stability of the closed system. This approach is considered in [1,8,9].

In this paper we use an approach that in some way includes all of the above approaches. In [10] such an approach is proposed, but for linear change of the jerk.

II. Trajectory Planning

A. Mathematical model of the object

We consider an electromechanical positioning system, Fig. 1, where mass m includes masses of all the moving parts (load and actuator). In the initial time $t_0 = 0$, we assume that the values of the jerk j , acceleration a , velocity v and displacement r are zero, i.e. $j(0) = a(0) = v(0) = r(0) = 0$. Force $F(t)$ generated by the actuator must overcome the force

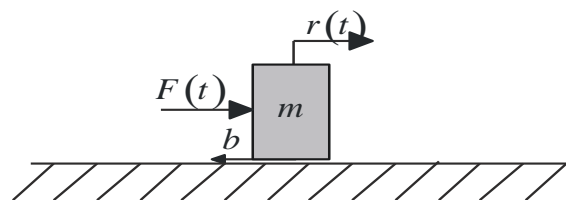


Figure 1. Electromechanical positioning system

of inertia $f_i = ma$ (of all the moving masses) and viscous friction force $f_v = bv$. The other frictions (for example Columb frictions) are neglected and their effect is modeled through disturbance $w(t)$, so behavior of the system is described by the equation:

$$F = m\ddot{r} + b\dot{r} = ma + bv. \quad (1)$$

After Laplace transformation (all the initial conditions are zero) we obtain the transfer function of the plant (object) $G_o(s)$ as

$$F(s) = (ms^2 + bs)R(s) \Rightarrow G_o(s) = \frac{1}{ms^2 + bs} = \frac{R(s)}{F(s)}. \quad (2)$$

B. Smoothing the Trajectory

In order to get little changes of acceleration (trajectory smoothing) we assume that the jerk is changing as:

$$j = \begin{cases} J \sin \frac{2\pi}{T} t, & t \in [0, T] \\ 0, & t \in [T, t_1], \\ -J \sin \frac{2\pi}{T} (t - t_1), & t \in [t_1, \tau] \end{cases} \quad (3)$$

where: $T = t_a$, $t_1 = T + t_v$ and J are the acceleration time, the time of the start deceleration and the maximum value of the jerk respectively .

Acceleration $a(t) = \int jdt + C_a$ obtains from (3), as:

$$a = \begin{cases} \frac{A}{2}(1 - \cos \frac{2\pi}{T} t), & t \in [0, T] \\ 0, & t \in [T, t_1], \\ -\frac{A}{2}(1 - \cos \frac{2\pi}{T} (t - t_1)), & t \in [t_1, \tau] \end{cases} \quad (4)$$

where $A = JT/\pi$ is the maximum value of acceleration. The constant of integration C_a is determined from the initial conditions for every time interval.

In a similar way we can get velocity $v = \int a dt + C_v$ as:

$$v = \begin{cases} \frac{A}{2} t - \frac{AT}{4\pi} \sin \frac{2\pi}{T} t, & t \in [0, T] \\ \frac{AT}{2}, & t \in [T, t_1], \\ \frac{A}{2} (\tau - t) + \frac{AT}{4\pi} \sin \frac{2\pi}{T} (t - t_1), & t \in [t_1, \tau] \end{cases} \quad (5)$$

and displacement $r = \int v dt + C_r$ as:

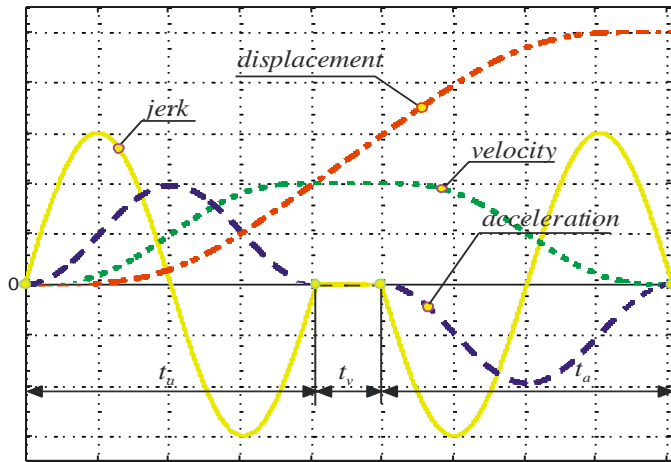


Figure 2. Changing the jerk, acceleration, velocity and displacement at the time interval $[0, \tau]$

$$r = \begin{cases} \frac{A}{4} t^2 - \frac{AT^2}{8\pi^2} (1 - \cos \frac{2\pi}{T} t), & t \in [0, T] \\ \frac{AT}{4} (2t - T), & t \in [T, t_1] \\ \frac{AT}{4} (2t - T) - \frac{A}{4} (t - t_1)^2 + \frac{AT^2}{8\pi^2} [1 - \cos \frac{2\pi}{T} (t - t_1)], & t \in [t_1, \tau] \end{cases} \quad (6)$$

The changes of the jerk, acceleration, velocity and displacement that are given in (3), (4), (5) and (6) respectively, are shown in Fig. 2.

From (4), (5), (6) it is easy to determine the maximum values of acceleration, velocity and displacement at the time interval $[0, T]$, as:

$$a\left(\frac{T}{2}\right) = \frac{JT}{\pi}, \quad v(T) = \frac{JT^2}{2\pi}, \quad r(T) = \frac{JT^3}{4\pi}. \quad (7)$$

C. Trajectory planning -algorithm

From a practical viewpoint, it is the best to give the maximum values of jerk J , acceleration A , velocity V and the total displacement R at the beginning of the trajectory planning. These values depend on the possibility of the actuator (force or torque), the application of the positioning system as well as the possibility of the control system. Then it is necessary to determine the shortest time $T = t_a$ and t_v so that given limitations are not exceeded. In this sense, we give the following algorithm:

1. The shortest time within which motion can be performed is calculated from (7) as:

$$R = 2 \times r(T) = \frac{JT^3}{2\pi} \Rightarrow T = \sqrt[3]{\frac{2\pi R}{J}}. \quad (8)$$

2. Using this time, given jerk J and (7) we can calculate the maximum value of the acceleration a_{\max} as $a_{\max} = JT/\pi$. Now we can test whether the acceleration bound A exceeds this value a_{\max} . If $a_{\max} \leq A$ we continue, but if $a_{\max} > A$ we recalculate T as: $T = \pi A / J$.
3. In the similar way we test whether the velocity bound is satisfied. The maximal velocity, from (7), is $v_{\max} = JT^2 / 2\pi$. Test $v_{\max} \leq V$. If this is true we can continue, but if it is false we recalculate T as $T = \sqrt{2\pi V / J}$.

4. And finally we determine the time t_v as:

$$t_v = \frac{R - \bar{v}T}{\bar{v}}, \quad (9)$$

where velocity \bar{v} is determined as $\bar{v} = \min(v_{\max}, V)$.

III. Controller Design

Configuration of the overall system as in Fig. 3 is proposed, where r , y and w are denoted as reference, output and an immeasurable disturbance, respectively. The controller of the system includes both, the feedback controller $C(s)$ and feed-forward controller $C_f(s)$. The output of the overall system $y(s)$ and the output error $e(s)$ (see Fig. 3) are obtained as:

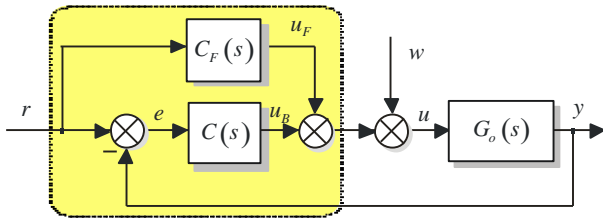


Figure 3. Overall system with feedback $C(s)$ and feedforward $C_F(s)$ controller, object $G_o(s)$ and disturbance w

$$\begin{aligned} e &= r - y = r - (Ce + C_F r + w)G_o \Rightarrow \\ y &= \frac{(C + C_F)G_o}{1 + CG_o} r + \frac{G_o}{1 + CG_o} w \text{ and} \\ e &= \frac{1 - G_o C_F}{1 + CG_o} r - \frac{G_o}{1 + CG_o} w. \end{aligned} \quad (10)$$

A. Design of Feedforward Controller

We can see from (10) that the output error contains two components, one due to the reference r (error $e_r = (1 - G_o C_F) / (1 + G_o C)$) and the other due to the disturbance w (error $e_w = G_o / (1 + G_o C)$). In order for object $G_o(s)$ to track the reference r asymptotically, the condition $e_r = 0$ must be satisfied. This leads that the transfer function of feedforward controller $C_F(s)$ is given as

$$C_F(s) = G_o^{-1}(s) = ms^2 + bs = \frac{u_F(s)}{r(s)}. \quad (11)$$

We got non-proper transfer functions, so the feedforward control $u_F(t)$ can be realized using differentiation, see [2,3], as

$$\begin{aligned} u_F &= C_F(s)r = G_o^{-1}(s) \frac{1}{s^2} \ddot{r}(t) \\ &= \frac{ms^2 + bs}{s^2} \ddot{r}(t) = \left(m + \frac{b}{s}\right) \ddot{r}(t) \Rightarrow \\ u_F(t) &= ma + bv. \end{aligned} \quad (12)$$

This controller is shown in Fig. 4.

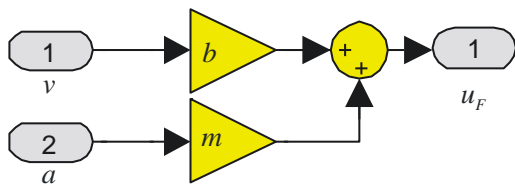


Figure 4. Feedforward controller $C_F(s)$

B. Design of feedback controller

The main task of the feedback controller is to compensate some unknown disturbances and unmodeled behavior of the object. We request that the controller $C(s)$ rejects all the step

disturbances, so that the output error tends to zero without oscillations. There are several different approaches for its design, [8,9]. In this paper we use a standard PID controller, so that the task now becomes to determine its parameters that ensure desired behavior of the overall system. The transfer function $C(s)$ of the PID controller is

$$C(s) = k_p \left(1 + \frac{1}{T_i s} + T_d s \right) = k_p \frac{T_i T_d s^2 + T_i s + 1}{T_i s}, \quad (13)$$

where the parameters k_p, T_i and T_d are proportional gain, integral and derivative time respectively. This transfer function, together with transfer function of the object (2) and using (10), gives the characteristic polynomial of the overall system as,

$$\Delta(s) = s^3 + \frac{k_p T_d + b}{m} s^2 + \frac{k_p}{m} s + \frac{k_p}{T_i m}, \quad (14)$$

that can be written as

$$\Delta(s) = (s + \alpha)(s^2 + 2\xi\omega_n s + \omega_n^2). \quad (15)$$

The overall system has three poles, which we can get as the solutions to the equation $\Delta(s) = 0$, so the poles are given as:

$s_1 = -\alpha$, $s_{2,3} = -\xi\omega_n \pm j\omega_n \sqrt{1 - \xi^2}$. In order for the output error to tend to the zero value without oscillations, both conditions $\xi \geq 1$ and the pole s_1 being much further in the left half s-plane than the poles $s_{2,3}$ must be satisfied. We adopt $\xi = 1$ (the error $e(t)$ tends to zero fastest), so the poles $s_{2,3}$ are $s_{2,3} = -\omega_n$.

Using these values, (14) and (15), we finally get the parameters of the PID controller as:

$$k_p = m\omega_n(2\alpha + \omega_n), T_i = \frac{2\alpha + \omega_n}{\alpha\omega_n}, T_d = \frac{m(2\omega_n + \alpha) - b}{k_p}. \quad (16)$$

It is easy to prove that the system (see Fig. 3) with controller (13) and object (2) rejects all step disturbances, $w(s) = \frac{w_o}{s}$.

Really, from the (10) follows,

$$e_w(s) = \frac{G_o}{1 + CG_o} w(s) = \frac{1}{m} \frac{s}{\Delta(s)} w(s), \quad (17)$$

where $\Delta(s)$ is given by (15). From the above equation we can calculate the stationary error $e_w(\infty)$, due to the step disturbance with amplitude w_o , as

$$\begin{aligned} e_w(\infty) &= \lim_{t \rightarrow \infty} e_w(t) = \lim_{s \rightarrow 0} s e_w(s) \\ &= \lim_{s \rightarrow 0} s \frac{1}{m} \frac{s}{\Delta(s)} \frac{w_o}{s} = 0, \end{aligned} \quad (18)$$

which confirms the above claim.

iv. Simulation Results

Simulation of the systems with the controllers (12) and (13), the object (2) and in the configuration as in Fig. 3 is presented in Fig. 5. To carry out the simulation we use MATLAB/SIMULINK program which SIMULINK model is

shown in Fig.6. Values $s_{2,3} = -\omega_n = -3$ are used as the poles $s_{2,3}$, and for value of the pole s_1 value $s_1 = -\alpha = -7\omega_n = -21$ is selected.

References

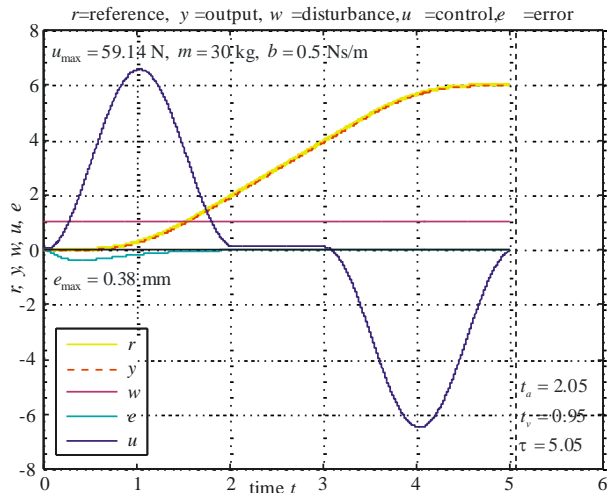


Figure 5. Simulation results for: $\xi = 1$, $\omega_n = 3$, $\alpha = -21$ and parameters from (16).

v. Conclusions

In this paper a system is designed that asymptotically follows a given reference and rejects all step disturbances. The reference is obtained from the condition that the jerk is changed in a predefined manner. As a controller the feedback and feedforward controllers are designed. The feedback is a standard PID controller, and a controller that is obtained by plant inversion is used as feedforward controller. An algorithm of planning trajectory which ensures that all bound values of the jerk, acceleration, velocity and movement, is also given. The results of the simulation confirm the given theoretical considerations.

- [1] Anton Cervin, Johan Eker, Bo Bernhardsson, Karl-Eric Arzen, "Feedback Feedforward Scheduling of Control Tasks," Real Time Systems (Special Issue on Control Theoretical Approaches to Real Time Computing), To appear in 2002
- [2] Paul Lambrechts, "Trajectory planning and feedforward design for electromechanical motion systems," Report nr. DCT 2003-08, University of Technology Eindhoven
- [3] Paul Lambrechts, Matthijs Boerlage, Maarten Steinbuch, "Trajectory planning and feedforward design for electromechanical motion systems," Control Engineering Practice 13 (2005) 145-157, ScienceDirect
- [4] Vito Cerone, Mario Milanese, Diego Regruto, "Robust feedforward design for a two-degrees of freedom controller," System & Control Letters, Volume 56, Issues 11-12, Dec. 2007, pp 736-741
- [5] H. Z. Li, Z. M. Gong, W. Lin, and T. Lippa, "Motion profile planning for reduced jerk and vibration residuals," SIMTech technical reports, Volume 8, No 1, 2007, pp 32-37
- [6] Kim Doang Nguyen, Teck-Chew Ng and I-Ming Chen, "On Algorithms for Planning S-curve Motion Profiles," International Journal of Advanced Robotic Systems, Vol. 5, No 1 (2008), ISSN 1729-8806, pp. 99-106
- [7] G.M. Clayton, S. Tien, A.J. Fleming, S.O.R. Moheimani, S. Devasia, "Inverse feedforward of charge controlled piezopositioners," Mechatronics 18 (2008) 273-281, ScienceDirect
- [8] Mihajlo J. Stojčić, Bojan Knežević, "The controller design for tracking trajectory with controlled jerk," DEMI 2011 Proceedings, Banja Luka, (2011), pp. 905-911
- [9] Mihajlo J. Stojčić, Bojan Knežević, "Design of digital controller that ensures robust tracking trajectory with controlled jerk," (In Serbian), JUPITER 37 Proceedings, Belgrade 2011, pp. 4-11-4.16.
- [10] Mihajlo J. Stojčić, "Design of Electromechanical Positioning System with Controlled Jerk," Heavy Machinery – HM2011, Proceedings, Kraljevo 2011, pp. 13-17



Mihajlo Y. Stoychitch (in Serbian language *Михајло Ј. Стојчић*) received B.Sc., M.Sc. and Ph.D. degrees, all in mechanical engineering, in 1978, 1982 and 2005 at the Faculty of Mechanical Engineering B.Luka, Zagreb and B.Luka respectively.

Now he is a head of the Department of Mechatronics within the Faculty of Mechanical Engineering, University of Banja Luka. His research interests include practical tracking, practical stability, system identification, design of control systems, intelligent systems and computer and microcontroller programming.

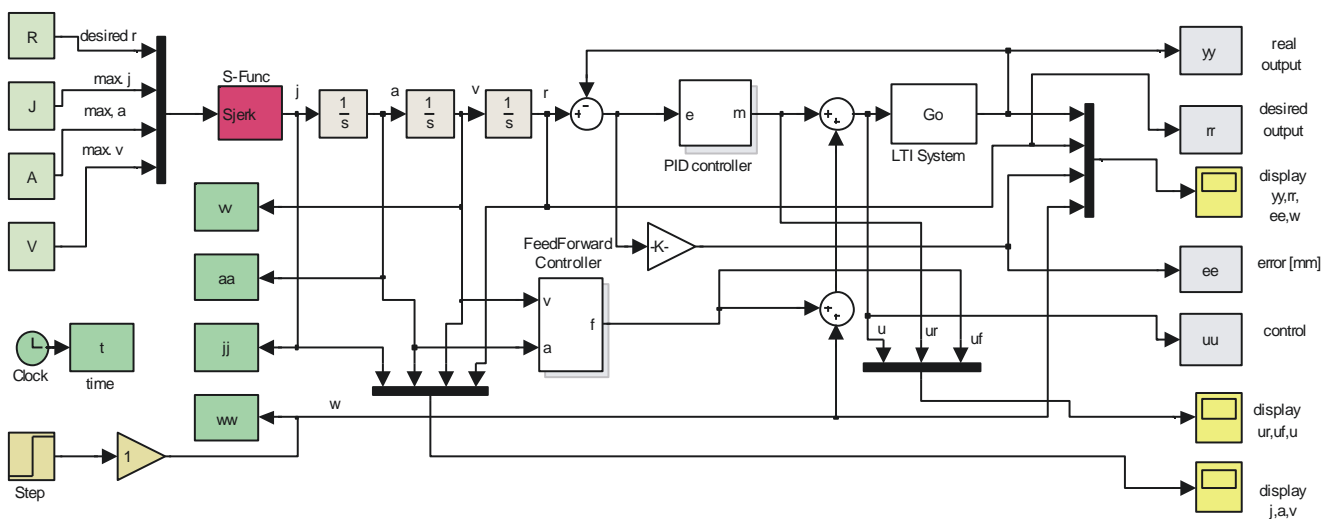


Figure 6. SIMULINK simulation model