

# Discrete Phase Approach for Nanofluids Flow in Pipe

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**Abstract**— Nanofluid is known as a new generation of fluid and it has been introduced almost several decades ago. But its effectiveness in practical thermal engineering applications has started to diminish with time due to the several factors such as physical instability, complex procedure for production of nanofluids and its cost, instability of suspension of nanoparticles into a base fluid, choice of thermophysical properties and reliability of nanofluids. To overcome these problems, two different phases such as a base fluid (water) and nanoparticles can be considered instead of a typical nanofluid which actually acts like a fluid-solid mixture. However, the interaction between the fluid and particles needs to be investigated to assess its performance. In the present work, Eulerian-Lagrangian discrete phase model has been used with temperature dependent thermophysical properties of the base fluid (water) and nanoparticles to study the thermal performance behaviour of  $Al_2O_3$  and  $TiO_2$  nanoparticles inside a horizontal pipe within the transition to turbulent flow regimes. SST  $\kappa - \omega$  and Realizable  $\kappa - \epsilon$  models are considered for the modelling of transition and turbulent flow fields respectively with an enhanced near wall treatment. Results reveal that the different phases for water and nanoparticles can be used instead of a nanofluid and no thermophysical properties of nanofluid are needed to explain such behaviour. Also, it is found that the enhancement of heat transfer rate is feasible and such enhancement is fully dependent of the thermal conductivity of nanoparticles as well as nanoparticles size diameters and volume concentrations.

**Keywords**— Nanofluid, Thermophoretic force, Saffman's Lift force, heat transfer, thermal performance factor.

## I. Introduction

In the area of thermal science and engineering, various techniques to enhance the heat transfer rate have been applied by researchers and recently nanotechnology also brings new and advanced solutions to this. In many engineering applications, the major limitation of enhancement of heat transfer rate is found to be the low thermal conductivity of conventional fluids such as water, air, engine oil and ethylene glycol. Thus, to improve the heat transfer solid particles of micro/mili-size into base fluids were added by Wang et al. [1], Lenert et al.[2]. But in real life applications, these micro/mili-size particles cause several problems such as clogging, erosion, quick settlement into the base fluid, and rapid decrease of pressure drops. In order to achieve heat transfer enhancement, a new generation of fluid called nanofluid is introduced and various applications of this fluid are highlighted in the recent study of Saha and Paul [3].

Also, several experimental and numerical investigations have been carried out on laminar to turbulent forced convection nanofluid flow in circular pipe using both single phase and multi-phase models as reported in Saha and Paul [3-5].

Very recently, He *et al.* [6] investigated experimentally and numerically the heat transfer behaviour of laminar  $TiO_2$ - $H_2O$  nanofluid flow in a horizontal circular pipe using Eulerian-Lagrangian discrete phase model. A similar investigation was carried out by Bianco *et al.* [7] and Moraveji and Esmaeili [8] on laminar forced convection  $Al_2O_3$ - $H_2O$  nanofluid flow using single phase and Eulerian-Lagrangian discrete phase models with constant and temperature dependent properties. A comparison was made between the results obtained by the two models and found that the maximum deviation of average heat transfer coefficient is only 10 to 11% for the volume concentration of 4%. Earlier, single phase and Eulerian-Eulerian mixture modelling approaches have been discussed in Saha and Paul [3-5]. They investigated the effect of nanoparticles size diameter and Brownian motion of nanoparticles of  $Al_2O_3$ - $H_2O$  and  $TiO_2$ - $H_2O$  nanofluids in a circular pipe within the transition and turbulent flow regimes.

Review of the above literature indicates that the different types of thermophysical properties of nanofluids commonly used in single-phase and Eulerian-Eulerian mixture models play an important role to examine their hydrodynamic and thermal performances. Eulerian-Lagrangian discrete phase model (DPM), on the other hand, is fully independent to the thermophysical properties of nanofluids and two separate phases such as a continuum fluid phase (water) and a discrete nanoparticle phase are used in this model. DPM has shown a success in laminar nanofluid flow through a pipe [6-8], but no attention has been made to date to investigate its performance in transition to turbulent flow regimes. The aim of the present work is therefore to investigate this for  $Al_2O_3$  and  $TiO_2$  nanoparticles.

An axi-symmetric model is considered to describe the characteristics of nanoparticles flowing through a straight circular pipe under a constant heat flux boundary condition. It consists of a pipe with length  $L$  of 1.0 m and a circular section with diameter,  $D_h$  of 0.019 m as shown in Figure 1. The flow and thermal fields are assumed to be axisymmetric with respect to the horizontal plane parallel to the  $x$ -axis.

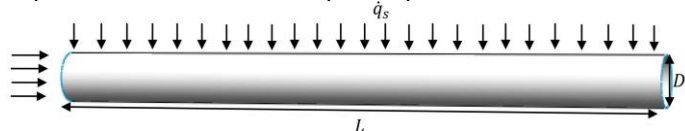


Fig. 1: Schematic diagram of the geometry under consideration

## A. Governing Equations

Dimensional steady-state governing equations for the Eulerian-Lagrangian discrete phase model are considered. It is assumed that flow is incompressible and Newtonian. Also, the Boussinesq approximation in the momentum equation, the compression work and the viscous dissipation term in the energy equation are neglected. The governing equations are expressed as follows [9]:

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$$\nabla \cdot (\rho \vec{V}) = 0 \quad (1)$$

$$\nabla \cdot (\rho \vec{V} \vec{V}) = -\nabla P + \nabla \cdot (\mu \nabla \vec{V}) + S_m \quad (2)$$

$$\nabla \cdot (\rho \vec{V} C_p T) = \nabla \cdot (\lambda \nabla T) + S_e \quad (3)$$

$$\frac{d\vec{V}_p}{dt} = F_D (\vec{V} - \vec{V}_p) \quad (4)$$

$$\rho C_p \frac{dT_p}{dt} = \frac{6h}{D_h} (T - T_p) \quad (5)$$

where Eq. (4) and (5) represent the Lagrangian form of particle momentum and energy equation. Also  $\vec{V}_p$  and  $T_p$  are the nanoparticle velocity and temperature respectively and  $\rho_p$  is the density of the nanoparticles.

The drag coefficient  $F_D$  is defined as [9]:

$$F_D = \frac{18 \mu_f}{\rho_p d_p^2 C_c} \quad (6)$$

where the factor  $C_c$  is known as the Cunningham correction which can be defined as [9]:

$$C_c = 1 + \frac{2\lambda_{mp}}{d_p} (1.257 + 0.4 e^{-(1.1 d_p/2\lambda_{mp})}) \quad (7)$$

where  $\lambda_{mp}$  is the nanoparticle mean free path and  $\mu_f$  is the dynamic viscosity of base fluid.

The heat transfer coefficient  $h$  is developed using the Ranz and Marshall correlation which is written as [9]:

$$Nu = \frac{hd_p}{\lambda_f} = 2.0 + 0.6 Re_d^{0.5} Pr^{1/3} \quad (8)$$

where  $d_p$  is the nanoparticle diameter (m),  $\lambda_f$  is the thermal conductivity of base fluid (W/m-K),  $Re_d$  is the nanoparticle Reynolds number and  $Pr$  is the Prandtl number of the base fluid.

Also, the source term  $S_m$  and  $S_e$  are defined as [9]:

$$S_m = \sum_{np} \frac{m_p}{\delta V} \frac{d\vec{V}_p}{dt} \quad (9)$$

$$S_e = \sum_{np} \frac{m_p C_p}{\delta V} \frac{dT_p}{dt} \quad (10)$$

## B. Transition Modeling

SST  $\kappa - \omega$  transitional model [10] is used and a detailed investigation was carried out in Saha and Paul [5] to assess its suitability in transitional nanofluid simulations. The equations for the kinetic energy ( $\kappa$ ) and specific dissipation rate of kinetic energy ( $\omega$ ) used in the SST  $\kappa - \omega$  transitional model are given by

$$\text{div}(\rho \kappa \vec{V}) = \text{div} \left\{ \left( \mu + \frac{\mu_t}{\sigma_\kappa} \right) \text{grad } \kappa \right\} \quad (11)$$

$$- \rho \kappa \omega \beta_1 + G_\kappa$$

$$\text{div}(\rho \omega \vec{V}) = \text{div} \left\{ \left( \mu + \frac{\mu_t}{\sigma_\omega} \right) \text{grad } \omega \right\} \quad (12)$$

$$+ \frac{2(1 - F_1) \rho \sigma_{\omega,2} \text{grad } \omega \text{ grad } \kappa}{\omega} - \rho \omega^2 \beta_2 + G_\omega$$

In these equations,  $G_\kappa$  represents the generation of turbulence kinetic energy due to the mean velocity gradients,  $G_\omega$  represents the production of  $\omega$ ,  $\sigma_\kappa$  and  $\sigma_\omega$  are the effective Prandtl numbers for the kinetic energy and specific rate of

dissipation, respectively; and the turbulent viscosity  $\mu_t$  is modelled as

$$\mu_t = \frac{\rho \kappa}{\omega} \frac{1}{\max \left( \frac{1}{\alpha^*}, \frac{S F_2}{\alpha_1 \omega} \right)} \quad (13)$$

where  $F_1$  and  $F_2$  are the blending functions,  $S$  is the strain rate magnitude and  $\alpha^*$  is a model constant. Also, the model constants used are  $\beta_1 = 0.075$ ,  $\beta_2 = 0.0828$ ,  $\alpha_1 = 0.31$ ,  $\sigma_\kappa = 1.0$  and  $\sigma_\omega = 1.168$ . Further information is available in Fluent [9] for transitional modelling.

## C. Turbulent Modeling

Realizable  $\kappa - \epsilon$  turbulence model of Shih *et al.* [11] is used and a detailed investigation was also carried out in Saha and Paul [3] to assess its suitability in turbulent nanofluid simulations. The equations for the turbulent kinetic energy ( $\kappa$ ) and dissipation rate of turbulent kinetic energy ( $\epsilon$ ) used in the realizable  $\kappa - \epsilon$  turbulence model are given by:

$$\text{div}(\rho \vec{V} \kappa) = \text{div} \left\{ \left( \mu + \frac{\mu_t}{\sigma_\kappa} \right) \text{grad } \kappa \right\} \quad (14)$$

$$+ G_\kappa - \rho \epsilon$$

$$\text{div}(\rho \vec{V} \epsilon) = \text{div} \left\{ \left( \mu + \frac{\mu_t}{\sigma_\epsilon} \right) \text{grad } \epsilon \right\} \quad (15)$$

$$- \rho C_2 \frac{\epsilon^2}{\kappa + \sqrt{\nu \epsilon}} + \rho C_1 S$$

where

$$C_1 = \max \left[ 0.43, \frac{\eta}{\eta + 5} \right], \eta = S \frac{\kappa}{\epsilon} \text{ and } S = \sqrt{2 S_{ij} S_{ij}} \quad (16)$$

In these equations,  $G_\kappa$  represents the generation of turbulence kinetic energy due to the mean velocity gradients, determined from  $\mu_t S^2$  where,  $S$  is the modulus of the mean rate-of-strain tensor,  $\sigma_\kappa$  and  $\sigma_\epsilon$  are the effective Prandtl numbers for turbulent kinetic energy and rate of dissipation, respectively; and  $\mu_t$  is modelled as

$$\mu_t = \frac{\rho \kappa^2}{\epsilon} \left( A_0 + A_s \frac{\kappa U^*}{\epsilon} \right)^{-1} \quad (17)$$

where  $A_0$  and  $A_s$  are the model constants. Other model constants are  $C_2 = 1.9$ ,  $\sigma_\kappa = 1.0$  and  $\sigma_\epsilon = 1.2$  as reported in [9].

## D. Boundary Conditions

At the pipe inlet, a uniform profile of velocity and temperature,  $T_{in} = 293$ , is used. Turbulent intensity,  $I$ , for the transitional cases remains constant while it is determined by  $I = 0.16 Re^{-1/8}$  for the turbulent cases. A hydraulic diameter,  $D_h = 0.019 \text{ m}$ , is also used. At the pipe outlet, a static gauge pressure,  $p_{gauge} = 0$  is specified. On the pipe wall, a no-slip boundary condition is introduced with a uniform heat flux condition. Further details about the boundary conditions of the Eulerian-Lagrangian discrete phase model are given in Fluent [9]. Also, the details of the temperature dependent physical properties of the base fluid (water) and  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$  nanoparticles are given in Saha and Paul [3].

## E. Numerical Procedure

The governing equations for the continuity, momentum, energy and other scalars are non-linear and coupled. These non-linear equations along with the suitable boundary conditions are discretised and then solved by using Finite volume method. Further information about the numerical scheme is given in [9]. The convergence criterion of solutions is set to be  $10^{-8}$ . Moreover, several numerical simulations have been performed to justify the consistency of the present solutions and appropriate arrangement of grid points relevant to resolve the flow and thermal field in the horizontal circular pipe is found. The details of the grid sensitivity test are also given in Saha and Paul [3, 5]. In order to validate the present numerical results for the base fluid (water), first the local Nusselt number for the fully developed laminar flow under the constant heat flux boundary condition is compared with the correlation of Shah and London [12] and experimental result of Kim *et al.* [13] as shown in Fig. 2. Then the fully developed radial velocity and turbulent kinetic energy profile for  $Re = 21800$  and  $Pr = 7.04$  have been validated against the experimental data as well as correlations. Finally, additional validation has been performed using the numerical results of Darcy friction factor and average Nusselt number against the existing correlations for different  $Re$  from 2300 to  $100 \times 10^3$  and  $Pr = 7.04$ .

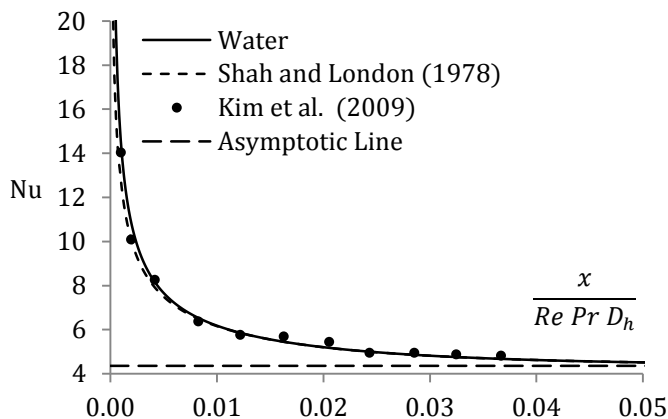


Figure 2: Comparison between the present result with Shah and London [12] and Kim *et al.* [13] of local Nusselt number for water under fully developed laminar flow regime

A good agreement was observed and thus this helps us to continue the further investigations. The details of these results are given in Saha and Paul [3, 5].

## II. Results and Discussion

Numerical investigations are carried out using the  $Al_2O_3$ - $H_2O$  and  $TiO_2$ - $H_2O$  nanofluids, with the following parameters: Reynolds number from  $Re = 250$  to 1200 (Laminar flow regime),  $Re = 2300$  to  $10 \times 10^3$  (Transition flow regime) and  $Re = 10 \times 10^3$  to  $100 \times 10^3$  (Turbulent flow regime), Prandtl number from 7 to 10, nanoparticles volume concentration of 1 to 6%, and diameter of nanoparticles of 100 nm. In the following, the performance of  $Al_2O_3$  and

$TiO_2$  nanoparticles using the Eulerian-Lagrangian discrete phase model have been presented and discussed.

### A. Average shear stress coefficient ratio

Figure 3 shows the variation of average shear stress coefficient ratio with Reynolds number for  $Al_2O_3$  and  $TiO_2$  nanoparticles. Results reveal that the average shear stress coefficient ratio has enhanced with an increase in the nanoparticles volume concentration. This is due to the effects of increase of nanofluid dynamic viscosity or pressure drop in the nanofluids. It is found that the Reynolds number has an insignificant effect on the enhancement of average shear stress coefficient ratio for any value of  $\chi$ . It is also seen from the figure that the average shear stress coefficient ratio of the  $Al_2O_3$  nanoparticles has a value, 1.10, 1.80, 2.80 and 1.12, 1.85, 2.89 for  $\chi = 1\%$ , 4% and 6% in the transition and turbulent flow regimes respectively. However, a lower average shear stress coefficient ratio is observed for the  $TiO_2$  nanoparticles. In particular, for the  $Al_2O_3$  nanoparticles in turbulent flow regime, results of the average shear stress coefficient ratio are compared with the works of Maiga *et al.* [14] and Bianco *et al.* [15]. Bianco *et al.* [15] carried out numerical investigation of turbulent flow using Eulerian-Eulerian mixture model whereas Maiga *et al.* [14] carried out similar investigation using single phase model. From the comparison point of view, it is seen that Eulerian-Lagrangian discrete phase model predicts lower average shear stress coefficient ratio than that obtained by the Eulerian-Eulerian mixture and single phase models. This is reasonable in a sense that the mixture of fluid and nanoparticles behave more like a nanofluid in both the Eulerian-Eulerian mixture and single phase models than in the Eulerian-Lagrangian discrete phase model.

### B. Average heat transfer performance

Figure 4 shows the comparison of the present result with the results of Bianco *et al.* [7] and Moraveji and Esmaili [8] for different  $Al_2O_3$ -nanoparticles volume concentrations within the laminar flow regime. It is to be note that Bianco *et al.* [7] and Moraveji and Esmaili [8] both used Eulerian-Lagrangian discrete phase model. From Fig. 4, it is seen that the results of average Nusselt number for different Reynolds number are in good agreement with the results of Bianco *et al.* [7] and Moraveji and Esmaili [8] for temperature dependent properties. Also maximum deviations of 5.12% and 3.80% for  $\chi = 1\%$ ,  $Re = 750$  and 12.88% and 9.09% for  $\chi = 4\%$ ,  $Re = 750$  are observed compared with the results of Bianco *et al.* [7] and Moraveji and Esmaili [8]. However results of Bianco *et al.* [7] for  $\chi = 4\%$  seems to be inconsistent for  $Re = 500$  and  $Re = 750$  respectively, while similar behaviour is also observed by Moraveji and Esmaili [8] for  $Re = 750$ . Furthermore, Figs. 5 and 6 show the variation of average Nusselt number with Reynolds number and nanoparticles volume concentrations using  $Al_2O_3$  and  $TiO_2$  nanoparticles under transition and turbulent flow regimes. Results reveal that average Nusselt number increases with the increase of



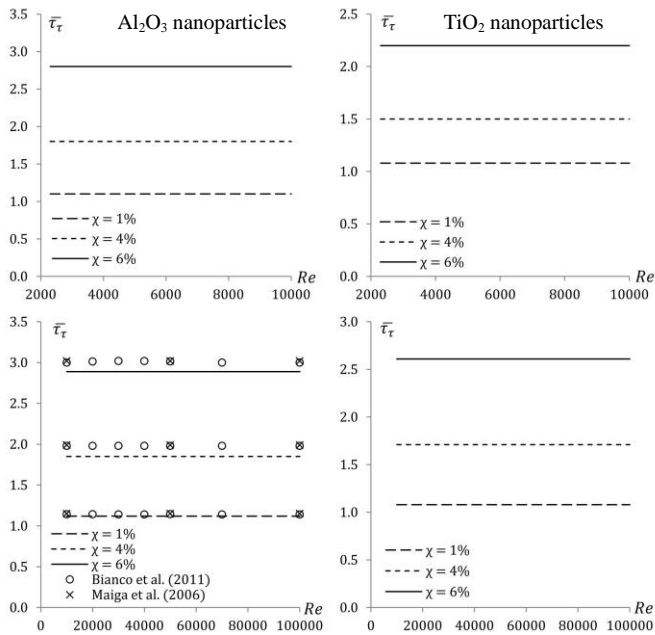


Figure 3: Variation of average shear stress coefficient ratio with different Reynolds number for different nanoparticles volume concentration and nanofluids

Reynolds number and such enhancement becomes more pronounced as the nanoparticles volume concentration increases.

It is found that average Nusselt number remains higher than of base fluid (water) at any Reynolds number. This is due to the enhancement of thermal conductivity of nanofluids compared to the low thermal conductivity of the base fluid (water). This is also due to the nanoparticle size and shapes, decrease in boundary layer thickness and delay in boundary layer growth as reported in Saha and Paul [3]. In particular, for  $\text{Al}_2\text{O}_3$  nanoparticles and volume concentration,  $\chi = 1\%$ ,  $4\%$  and  $6\%$  respectively, the maximum enhancement is approximately  $1.84\%$ ,  $13.57\%$  and  $25.40\%$  respectively under transition flow regime whereas  $4.80\%$ ,  $18.07\%$  and  $31.48\%$  respectively under turbulent flow regime. However, for the  $\text{TiO}_2$  nanoparticles and volume concentration,  $\chi = 1\%$ ,  $4\%$  and  $6\%$  respectively, the maximum enhancement is approximately  $1.55\%$ ,  $8.27\%$  and  $18.70\%$  respectively under transition flow regime whereas  $3.73\%$ ,  $13.95\%$  and  $24.61\%$  respectively under turbulent flow regime. It is also observed that  $\text{Al}_2\text{O}_3$  nanoparticles gives higher average Nusselt number than  $\text{TiO}_2$  nanoparticles. This is realistic because  $\text{Al}_2\text{O}_3$  nanoparticles have higher thermal conductivity than that of  $\text{TiO}_2$  nanoparticles.

Besides, in Fig. 6, a comparison has been made with the proposed correlations suggested by Pak and Cho [16] and Maiga *et al.* [14] for  $\text{Al}_2\text{O}_3$  nanoparticles under turbulent flow regime. For  $\chi = 1\%$  and  $10 \times 10^3 \leq Re \leq 70 \times 10^3$ , it is observed that the variation between the present results and Pak and Cho [16] correlation is insignificant. But when  $Re > 70 \times 10^3$ , the values of average Nusselt number tends to differ from

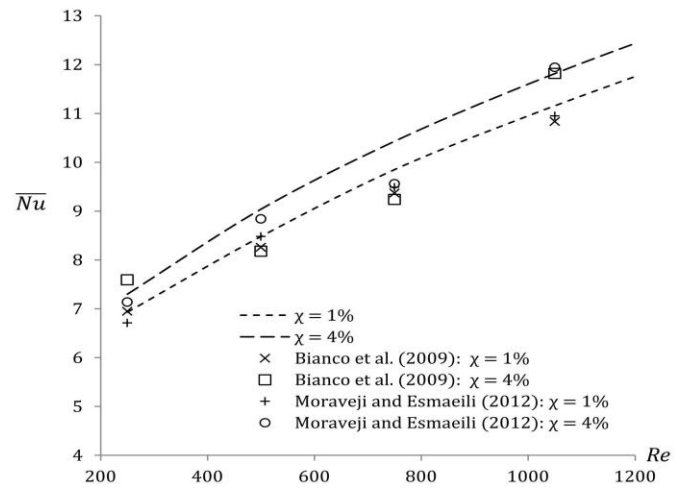


Figure 4: Variation of average Nusselt number with different Reynolds number for different nanoparticles volume concentration and  $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  nanofluid under laminar flow regime

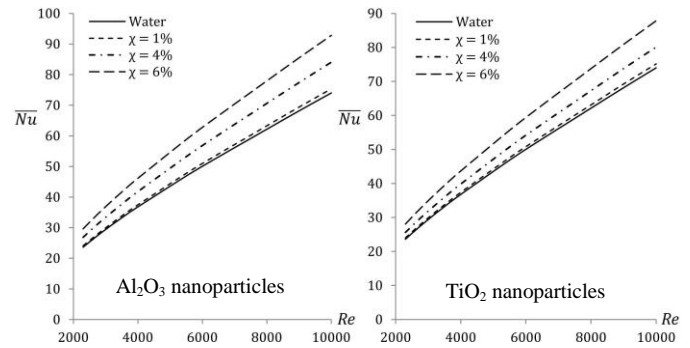


Figure 5: Variation of average Nusselt number with different Reynolds number for different nanoparticles volume concentration and nanofluids under transition flow regime

the correlation of Pak and Cho [16] and a strong agreement is found with the correlation of Maiga *et al.* [14]. Also for  $\chi = 4\%$  and  $6\%$  and  $10 \times 10^3 \leq Re \leq 40 \times 10^3$ , it is also observed that the results of average Nusselt number are in good agreement with Pak and Cho [16], while for  $Re > 40 \times 10^3$ , present results deviates from the results of Pak and Cho [16] and moves closer to the results of Maiga *et al.* [14]. But for  $\chi = 6\%$  and  $Re > 80 \times 10^3$ , a significant deviation is observed with the correlations proposed by Pak and Cho [16] and Maiga *et al.* [14]. Moreover, a similar trend is observed for  $\text{TiO}_2$  nanoparticles using Eulerian-Lagrangian discrete phase model. The reason behind such behaviour of both  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$  nanoparticles is due to augmentation of velocity of the different phases for the increase of Reynolds numbers and the strong interaction between the fluid particles and nanoparticles. This is also due to the strong coupling between the fluid and nanoparticles phases as well.

### C. Conclusion

In this present work, Eulerian-Lagrangian discrete phase model has been introduced to investigate the thermal performance of  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$  nanoparticles in pipe with temperature dependent properties under transition to turbulent

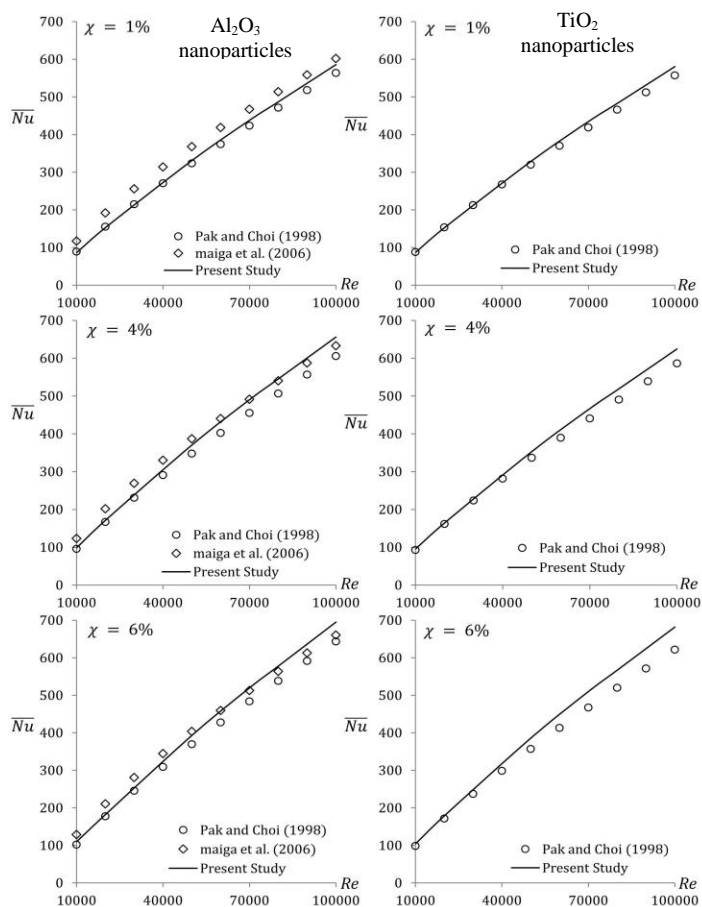


Figure 6: Variation of average Nusselt number with different Reynolds number for different nanoparticles volume concentration and nanofluids under turbulent flow regime

flow regimes. According to our findings, the following conclusion are made and summarised as follows:

(1) It was seen that for  $\chi = 4\%$  and  $6\%$  and  $10 \times 10^3 \leq Re \leq 40 \times 10^3$ , the results of heat transfer rate are very close to the experimental results of Pak and Cho [16], however for  $Re > 40 \times 10^3$ , results deviates from Pak and Cho [16] correlation and moves close to the numerical correlation of Maiga *et al.* [14].

(2) It was also seen that average shear stress coefficient ratio becomes inferior for high nanoparticles volume concentration compared with the results of single and multi-phase mixture models.

(3) Higher heat transfer enhancement was observed for  $Al_2O_3$  nanoparticles than  $TiO_2$  nanoparticles for all  $Re$  and  $\chi$ .

Finally, it is seen that performance of DPM is excellent without using information about the behaviour of nanofluid and its thermophysical properties. Since this model only requires physical properties of base fluid (water) and nanoparticles, this approach has opened a new platform to study the behaviour of new mixture which is used in this model. At the end, more experimental research is necessary to understand the performance of new mixture.

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