

# Fluid flow and heat transfer characteristics of natural convection in square 3-D enclosure due to discrete sources

Mehdi Ahmadi

Department of mechanical engineering  
Islamic Azad University  
Shahrekord Branch, Ardal center- Iran

Behnaz Bagheri Dastgerdi

Department of mechanical engineering  
Islamic Azad University  
Shahrekord Branch- Iran

**Abstract**— A numerical analysis was carried out to study the laminar natural convection in a three-dimensional enclosure. Main efforts were focused on the Rayleigh number ( $Ra$ ), position of hot and cold sources on the flow structure and heat transfer characteristics are investigated in detail. Results reveal that by increasing in  $Ra$  from  $10^3$  to  $10^6$  the average Nusselt will be increase and the rate of heat transfer from hot and cold sources will be increase, also the maximum component of velocity will increase, The increase in the velocity component along the acceleration of gravity is higher than other component of velocity. According to the results by increasing the distance between heat sources rate of heat transfer between heat sources (average Nusselt) significantly decreased, also average temprature and maximum component of velocity in all direction will be increase.

**Keywords**— Natural convection, enclosure, heat transfer, velocity component, flow structure

## Introduction

The convection due to buoyancy forces i.e natural convection in a rectangular enclosures is a fundamental problem due to the wide range of applications in thermal insulation, heating and cooling of buildings, energy drying processes, cooling of electronics, lakes and geothermal reservoirs, solidification of casting, underground water flow, solar collector etc. [1]. Associated industrial applications include secondary and tertiary oil recovery, growth of crystals [2]. During these last three decades, numerous theoretical and experimental studies on natural convection in enclosures have been investigated. Understanding the dynamical evolution of heat and fluid flow due to natural convection, has received a considerable attention from many researchers. An extensive bibliography Natural convection in cavities up to 1988 may be found in the review article by Ostrach [3].

In the framework of the present paper, we refer to the literature pertinent to natural convection in an 3D enclosure whose all walls are adiabatic and sources have constant temperature. In the broad sense, the above class of problem can be assified into two major categories: (i) uniform heated enclosures, (ii) differentially heated enclosures. Above type of problems have been studied extensively by many researchers. A numerical investigation on the natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides has been performed by Ganzarolli and Milanez [4]. Ramos and Milanez [5] studied the natural convection in cavities heated from below by a thermal source which dissipated energy at a

constant rate. A study of three-dimensional natural convection from a discrete flush-mounted rectangular heat source on the bottom of a horizontal enclosure whose upper wall surface maintained at a cold temperature along with the two kinds of boundary conditions on the sidewalls has been presented by Sezai and Mohamad [6]. A steady laminar natural convection in 2D enclosures heated from below and cooled from above for a wide variety of thermal boundary conditions at the sidewalls has been carried numerically by Corcione [7]. An analysis on the combined effects of thermal and mass diffusion of viscous incompressible fluid through a finitely long vertical irregular wall and a smooth flat wall in the presence of internal heat source or sink is performed by Fasogbon, [8]. In his study walls are maintained at constant but different temperatures and species concentrations.

In the present paper a numerical analysis was carried out to study the laminar natural convection in a three-dimensional enclosure. Main efforts were focused on the Rayleigh number  $Ra$ , position of the hot and cold sources and size of them on the flow structure and heat transfer characteristics are presented. In this investigation The Special attention is given to understand the effect of aspect ratio and heat source intensity i.e Rayleigh number,  $Ra$ , on the fluid flow configuration as well as on the local and average heat transfer rates. The results are presented in terms of stream function  $\psi$ , temperature  $\theta$ , vertical velocity component and Nusselt numbers (local Nusselt number,  $Nu_L$ , and average Nusselt number,  $Nu_m$ ).

## I. Validation And Grid study

### A. Validation

For studying operation of computer code in 3-dimensional state, a comparison was performed with done work in references[9],[10],[11]. In these papers, a square – cubic enclosure with cold wall on the right and hot wall on the left with boundary conditions as fixed temperature has been considered, the other walls of enclosure are insulated Fig.1. The (Table.1) shows a sample of results of the others work and present study in which average Nusselt has been provided in lieu of different Rayleigh numbers. As you see, results of this study have very little difference with whatever has been provided from references. After assurance from program operation, first we describe a suitable grid with regard to boundary conditions change for computer calculations.

### B. Grid study

For choosing a solvable network, first according to enclosure dimensions, effect of changing flow and temperature parameters and the number of grids were studied.

We consider a state of enclosure based on Fig.2 with geometrical dimensions;  $L=1, H=1, W=0.875, H_1=0.125, H_2=0.20, H_3=0.25, H_4=0.30, L_1=L_2=0.30, L_3=L_4=0.2$ . Rayleigh number is considered  $10^5$ .

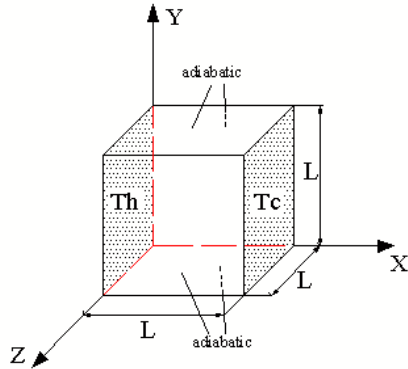


Fig1. Problem geometry for evaluating validity of computer code

Table1. Comparing average number resulting from computer program and the others works in 3-dimensional enclosure.

Ra	$Nu_m$			
	Fusegi et al.[12]	Leutree and Lauriat[13]	Ho and Tu [27]	Present research
$10^3$	085.1	-	057.1	066.1
$10^4$	001.2	-	074.2	039.2
$10^5$	361.4	348.4	367.4	300.4
$10^6$	770.8	651.8	755.8	540.8

After finding out optimal parameters, average Nusselt of hot source and maximum vertical component of velocity were calculated in lieu of different grid points.

Results of this study have been shown in the Fig.3 and Fig.4. Although, there is a trivial difference (Result of grid study are contain; average Nusselt numbers and maximum vertical component of velocity) between  $50*50*50$  grid with  $60*60*60$  grid points therfor Non-uniform grid  $60*60*60$  had some acceptable errors for performing program.

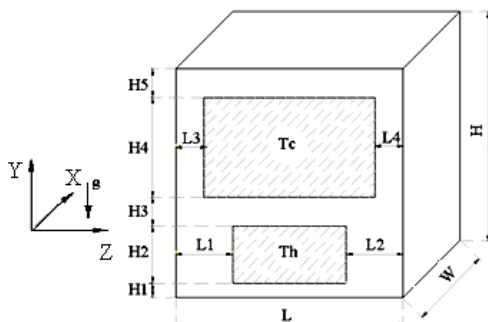


Fig 2.scheme of 3D enclosure

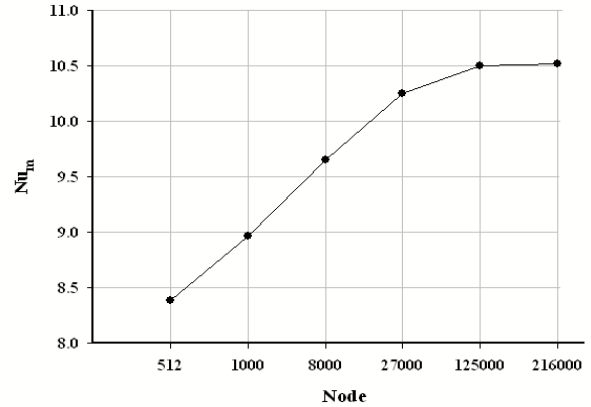


Fig 3. average Nusselt of hot source based on number of grid points.

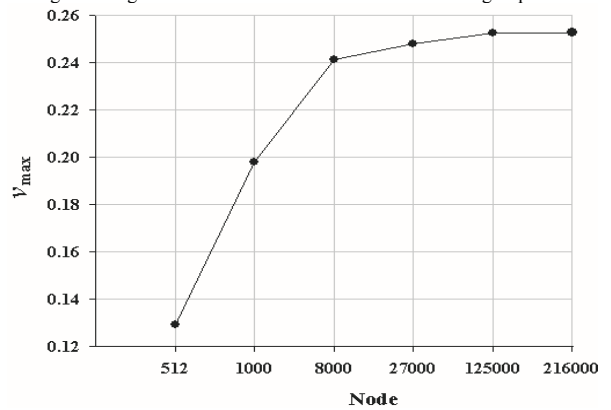


Fig 4. maximum vertical component of velocity based on number of grid points

After assurance from operation of computer code and grid selection, different performances were done in different states.

In all calculations, fluid properties and enclosure dimensions ( $L=1, H=1, W=0.875, L_1=L_2=0.30, L_3=L_4=0.2$ .) were considered fixed. First with changing Rayleigh number from  $10^3$  to  $10^6$  effect of isothermal lines are studied.

### c. Equations and Solution procedure

We consider a three dimensional fluid flow in a rectangular enclosure of length L and height H and W as shown schematically in Fig. 2. The flow is induced by the fix temprature of sources and other walls are adiabatic. The Boussinesq approximation is involved for the fluid properties to relate density changes to temperature changes, and so to couple in this way the temperature field to the flow field. Our main focus is on influence of aspect ratio and heat source intensity on the fluid flow configuration as well as on the heat transfer rates. The governing equations for the steady-state flow as well as heat transfer in the Cartesian coordinate system are given by using non-dimensional equations:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{2-1}$$

$$\tag{2-2}$$

$$\frac{\partial}{\partial x_j}(u_j \theta) = \frac{1}{Pr Gr_h^{1/2}} \nabla^2 \theta \quad (2-3)$$

With the boundary conditions, on the walls  $u=v=w=0$ , on the surface of hot source  $\theta=1$  and cold source  $\theta=0$  and for adiabatic walls

$$\frac{\partial \theta}{\partial x} \Big|_{x=0} = 0 \quad \frac{\partial \theta}{\partial y} \Big|_{y=0} = 0 \quad \frac{\partial \theta}{\partial z} \Big|_{z=0} = 0$$

Where  $x, y$  and  $z$  are dimensionless coordinates along horizontal and vertical directions, respectively;  $u, v, w$  are, dimensionless velocity components in the  $x-y$  and  $Z$  directions, respectively;  $\theta$  is the dimensionless temperature;  $P$  is the dimensionless Pressure;  $Ra$  and  $Pr$  are Rayleigh and Prandtl numbers, respectively.

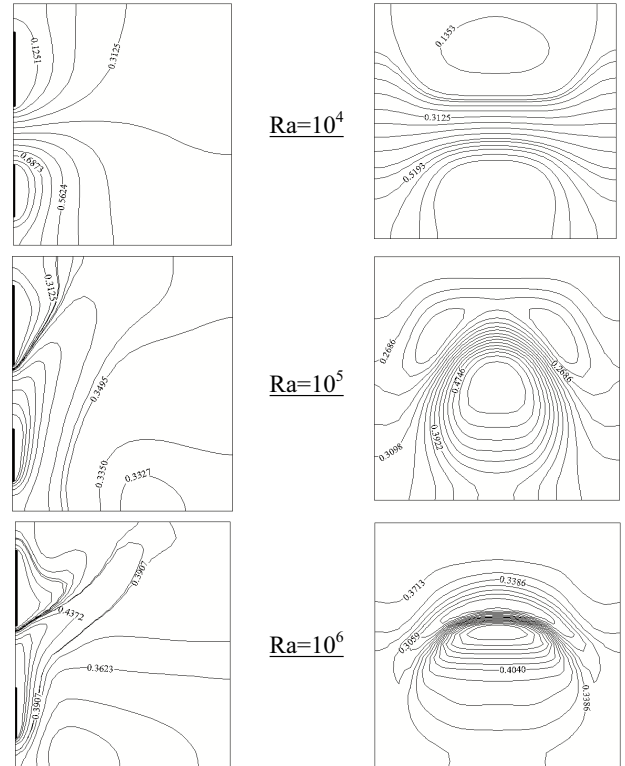
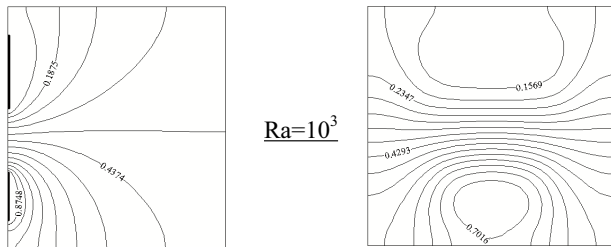
The governing equations, Eq.; (2-1)– (2-3), are discretized by the finite volume method (FVM) on non-uniform grid system. The third-order QUICK scheme and the second-order central difference scheme are, respectively, implemented for the convection and diffusion terms. The set of discretized equations for each variable are solved by a line-by-line procedure, combining the tri-diagonal matrix algorithm (TDMA) with the successive over-relaxation iteration (SOR) method. The coupling between velocity and pressure is solved by SIMPLE algorithm. The solution is terminated until the convergence criterion is reached, i.e., the maximal residual of all the governing equations is less than  $10^{-8}$ .

## II. Results and Discussions

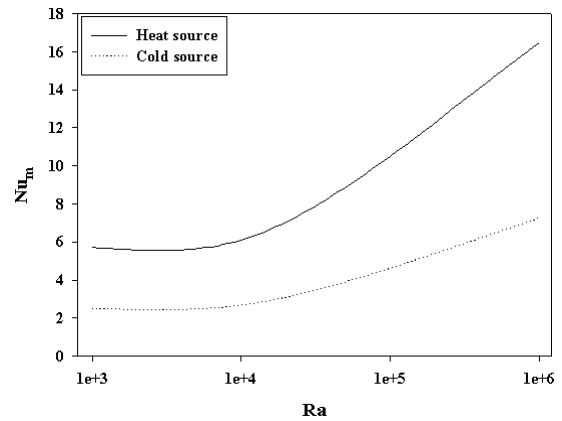
### A. Studying effect of Rayleigh number

In this part, we just change Rayleigh number. Geometrical parameters  $H_1=0.125, H_2=0.20, H_3=0.25, H_4=0.30$  are considered fixed. Isothermal lines have been presented in this state in lieu of different Rayleigh numbers in the middle planes  $x=0.4375, z=0.50$ , see Fig.5. As you can see, flows resulting from buoyancy force are very weak for small Rayleigh numbers, Isothermal lines are more regular and it shows that heat transfer mostly happens with conduction. But gradually with increasing Rayleigh number due to increasing velocity of fluid flow near thermal surface which causes to increase heat transfer by convection, curvature of isothermal lines becomes more. In addition, as we can see from figures, there is a cold area near floor of enclosure between Rayleigh numbers  $10^5-10^6$ . Existence of this area is because of velocity components in third dimension.

Then, we study effect of changing  $Ra$  on rate of heat transfer and flow parameters. In the Fig.6 and Fig.7, average changes of thermal sources and maximum velocity components have been provided based on charges of  $Ra$ . The figures show that increasing  $Ra$  causes to increase  $Nu_m$  that shows increasing rate of heat conduction on cold and hot sources.

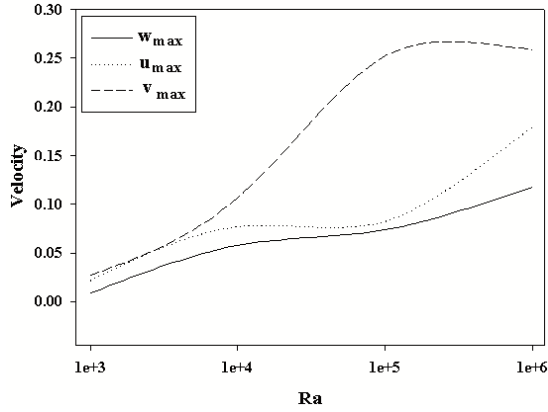


In small  $Ra$  which are under control of heat conduction mechanism, increasing  $Ra$  does not influence on rate of heat transfer, but from  $Ra=10^4$  to higher that natural convection is important, increasing  $Ra$  influences on  $Nu_m$  significantly. Also, we see that maximum velocity components increase with increasing  $Ra$ . This increasing in velocity component along gravity acceleration  $v$  is more significant.



6. Average Nusselt changes of hot and cold sources based on changes of Rayleigh number

Therefore, it is concluded that increasing maximum velocity components in the enclosure of a critical parameter is due to increasing heat transfer from a hot source to a cold source, which its consequence can be observed in increasing  $Nu_m$  in the enclosure.



7. Maximum changes of velocity components based on changes of Rayleigh number.

For more accurate comparison of above results Table.2 has been provided. In this table we can see not only average values of hot and cold sources and maximum velocity components but also average temperature of fluid in the enclosure is seen, too.

Table 2. Compare thermal and flow parameter in lieu of different Ra in 3-D enclosure

Ra	Nu <sub>m,h</sub>	Nu <sub>m,c</sub>	$\theta_{mean}$	W <sub>max</sub>	u <sub>max</sub>	v <sub>max</sub>
10 <sup>3</sup>	5.721	-2.517	0.3692	0.0085	0.0217	0.0269
10 <sup>4</sup>	6.085	-2.678	0.3659	0.0577	0.0768	0.1067
10 <sup>5</sup>	10.499	-4.620	0.3327	0.0735	0.0821	0.2526
10 <sup>6</sup>	16.493	-7.257	0.3099	0.1177	0.1793	0.2587

Increasing Rayleigh number causes to decrease mean temperature, it is because of temperature distribution in enclosure. As we know, in low Ra, role of heat transfer by conduction more important than high Ra, If effect of heat transfer by conduction is more, then mean temperature will increase in enclosure (because of linear distribution of temperature).

**B. Studying situation of hot and cold sources**

We consider enclosure of the Fig.2 again. In this part, in addition to changing distance between hot and cold sources H<sub>3</sub>, we put sources on opposite and adjacent walls in 2 states. Geometrical parameters are; H<sub>1</sub>=H<sub>5</sub>, H<sub>2</sub>=0.20, H<sub>4</sub>=0.30 and computations are done in lieu of Ra=10<sup>5</sup>.

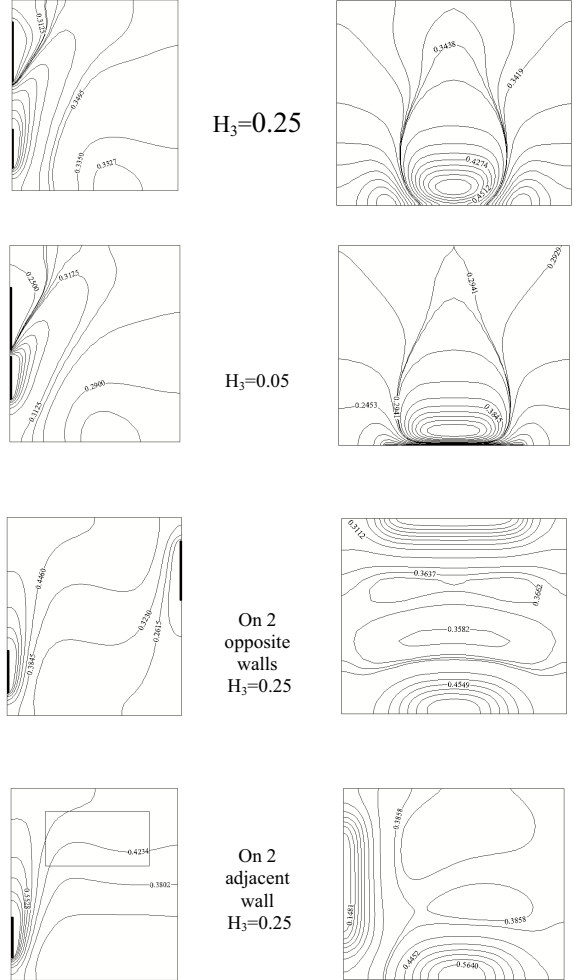
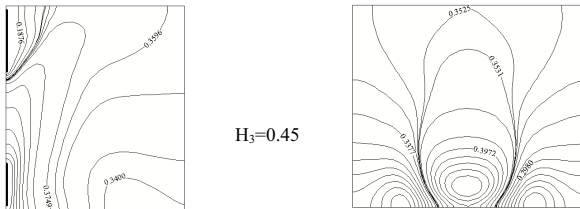


Fig 8. Isothermal lines of the sources at y=0.5(right) and isothermal lines of the sources at z=0.5(left) in different situations of sources at Ra=10<sup>5</sup>.

In this state, isothermal lines have been shown in lieu of situation change of cold and hot sources in the middle planes z=0.50, y=0.50 in the Fig.8. As we can see from the figures, when thermal sources are on a wall, with decreasing distance of two sources and inclining two sources towards center of plane, isothermal lines will be more condensed around thermal sources. In this state, isothermal lines have been shown in lieu of situation change of cold and hot sources in the middle planes z=0.50, y=0.50 in the Fig.8. As we can see from the figures, when thermal sources are on a wall, with decreasing distance of two sources and inclining two sources towards center of plane, isothermal lines will be more condensed around thermal sources.

As you can see from the figures, increasing distance between thermal sources causes to significantly decrease heat transfer in the form of average Nusselt. In addition, average temperature and maximum components of fluid velocity in y and z directions increase with increasing distance between thermal sources.

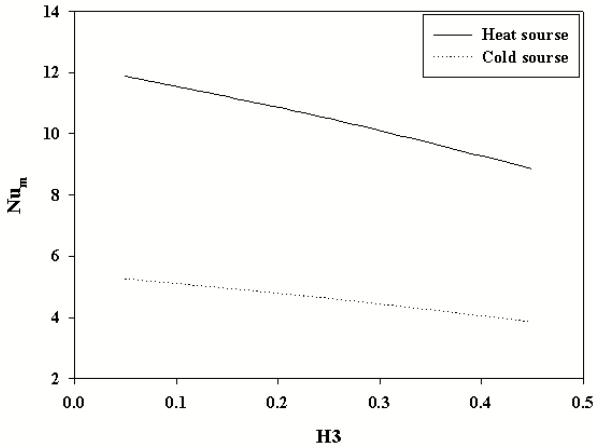


Fig 9. Num changes of cold and hot sources based on distance changes between thermal sources.

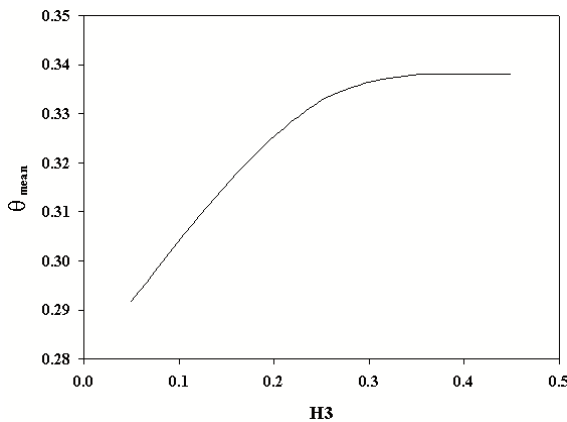


Fig 10. Average temperature changes in enclosure based on distance changes between thermal sources.

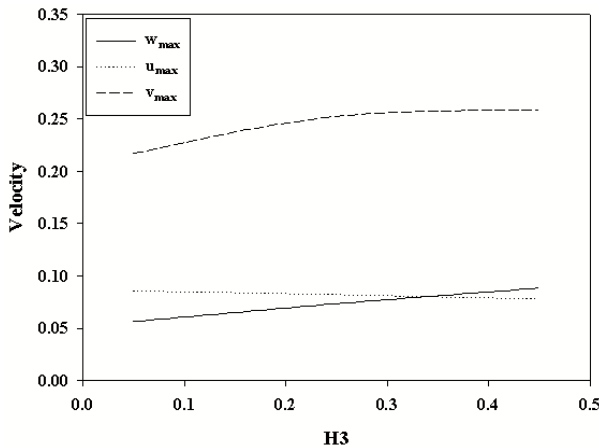


Fig 11. Maximum changes of velocity components in enclosure based on distance changes between thermal sources

walls. In this table, we can see increasing  $Nu_m$  in two opposite walls than their place on the two adjacent walls.

Table 3. Comparing heat conduction in different situations of thermal sources in three dimensional enclosure

situations of thermal sources	$Nu_{m,h}$	$Nu_{m,c}$	$\theta_{mean}$	$W_{max}$	$u_{max}$	$v_{max}$
One wall 45.0 H <sub>3</sub> =	861.8	861.-3	3382.0	0883.0	0779.0	2587.0
One wall 25.0 H <sub>3</sub> =	499.10	620.-4	3327.0	0735.0	0821.0	2526.0
One wall 05.0 H <sub>3</sub> =	876.11	261.-5	2918.0	0564.0	0856.0	2170.0
two walls opposite each other 25.0 H <sub>3</sub> =	252.11	951.-4	4516.0	0417.0	1357.0	2465.0
adjacent walls 25.0 H <sub>3</sub> =	621.9	622.-4	3792.0	1205.0	1245.0	2511.0

It is because of regular vortex movement of fluid in which fluid adjacent hot sources goes up and the fluid adjacent cold sources goes down in enclosure. being thermal sources on two opposite walls these two flow of fluid reinforce each other, Therefore, average velocity on thermal sources be increase, increasing velocity parameters on thermal sources caused to increase heat transfer due to fluid convection too.

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Table.3 has been provided for more comparison between above results. In addition, a comparison has been done between heat transfer in cold and hot thermal sources in two opposite and adjacent