

# Float Analysis in Line of Balance Scheduling

[Mohammad A. Ammar and Ayman Altuwaim]

**Abstract**—CPM has failed practically for scheduling repetitive projects and hence LSMs are used. LOB is the most commonly used LSM for scheduling typical repetitive projects. In CPM, non-critical activities have time float which is considered a critical asset or resource. In addition to time float, repetitive activities may have rate float. Extending traditional float and criticality analysis to repetitive activities is a remarkable demand in the literature. In this paper, various float types of typical repetitive activities are analyzed and calculated. Time float concept of non-repetitive activities is extended to repetitive ones. Rate float as an inherent attribute of repetitive activities is also defined and analyzed. Time and rate floats of typical repetitive activities are calculated in an easy analytical approach. An example repetitive project is used to demonstrate float calculations and obtained results are then analyzed and discussed. Introducing float concepts to LOB apparently extends its features over other LSMs. Float analysis in LOB scheduling opens the door to extend full range of CPM functionality to repetitive projects such as TCTO and resource allocation.

**Keywords**—Repetitive Projects, Scheduling, Typical Activities, Line of Balance, Time and Rate Floats

## I. Introduction

Construction projects that involve similar or identical units are usually referred to as repetitive projects [1]. Repetitive projects include repetition of a set of activities throughout the project such as multiple houses, whereas linear projects are repeated due to their geometrical layout (e.g. pipelines, highways, etc.) Repetitive and linear projects will be referred to, hereinafter, as repetitive projects. Repetitive projects contribute to the construction industry with a large portion and, therefore, the effective management of such projects is mandatory [2].

Repetitive activities can, generally, be classified as “typical” and “atypical”. For typical activities, it is assumed that all units have identical durations and, consequently, assume constant progress rates. On the other hand, units do not always have equal durations in atypical activities. Dealing with atypical repetitive activities is out of scope of the present study.

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Mohammad A. Ammar, Professor (On Leave)  
Faculty of Engineering, Tanta University  
Egypt,  
Ayman Altuwaim, Assistant Professor,  
Department of Civil Engineering, King Saud University,  
Riyadh, Saudi Arabia,

Since the late 1950s, networking techniques (e.g., CPM) have been widely used for scheduling construction projects. Information obtained from CPM (e.g., floats and criticality)

are extremely useful to project planners and managers. Despite its wide application in the construction industry, CPM has failed practically to schedule repetitive projects.

On the other hand, various Linear Scheduling Methods (LSMs) have been developed. Line of Balance (LOB) is commonly used to schedule typical repetitive activities [3]. Arditi et al. [4] concluded: “although variations of LOB have been proposed to suit different characteristics of repetitive projects, all are graphically similar”. LSMs lack the ability to specify floats/criticality similar to CPM [5]. El-Rayes [6] added that repetitive project scheduling can be significantly improved by integrating repetitive and non-repetitive scheduling techniques, e.g. LOB and CPM, respectively.

The concept that some activities are critical while others have float is not only beneficial as a management tool but is also useful in determining the effects of delays [7]. Float signifies the importance degree of an activity and, hence, has long been considered as an essential parameter [8]. Yang [9] concluded that “the critical path(s)/activities and floats are the key measures usually used to reflect management focus and flexibility of activities, especially in complex projects”.

Repetitive scheduling techniques should provide float/criticality information similar to that obtained from CPM. Harmelink and Rowings [10] advised that “this ability would provide an analytical or engineering foundation on which full range of functionality could be built”. A notable limitation of current repetitive scheduling methods is their inability to identify floats/criticality in a direct analytical approach similar to that of CPM.

Float determination makes most CPM functionality possible for repetitive projects (e.g., resource leveling, project crashing, delay analysis, etc.) Extending traditional float and criticality concepts to repetitive projects is a remarkable demand in the literature [1, 5, 8, 9, 11, 12]. In this paper, time float concept is extended to repetitive activities in a non-graphical analytical approach. Rate float of repetitive activities is also defined and calculated analytically. The LOB/CPM repetitive scheduling model developed by Ammar [13] is adopted as a scheduling platform.

This paper, in essence, describes float calculation for repetitive activities under LOB scheduling thus makes LOB/CPM general scheduling model. LOB as a repetitive planning and scheduling technique is presented briefly. Time and rate float concepts are reviewed and various types of float in repetitive projects are defined. The adopted integrated CPM-LOB scheduling model is briefly reviewed along with an example project. The developed method for float analysis in LOB scheduling is then described in detail. Details of float calculations are then illustrated by an example project. The main features of the proposed float analysis method are listed, and conclusions drawn from the present development are finally outlined.

## II. Time and Rate Floats

Float analysis aims to determine the amount by which the duration of a non-critical activity can be prolonged, or the start time of the activity can be delayed before it becomes critical [12]. The presence of float allows for flexible scheduling of non-critical activities and, hence, deciding most efficient use of resources [14]. In CPM float computation, time and logical dependency constraints are only considered.

Said and Lucko [5] defined float as “the slack available for an activity to absorb delays in its scheduled dates or relax its scheduled production rate without delaying any of its successors”. Float, thus, expresses flexibility of executing an activity in accordance with its logical relationships. Yang [9] reviewed various types of float and concluded that “float is a resource that provides flexibility in the contractor’s operation”. Construction time-based claims have added yet another meaning to the popular expression “float is money” [15]. Float can, therefore, be considered a critical asset, which has increasingly concerned contractual parties [16].

In CPM scheduling, many types of time floats are applicable. According to Yang [9], time floats include six types namely: total, free, interfering, independent, relationship and as-built. In addition to time float, repetitive activities may have rate float [17]. In a CPM network, total float represents the flexibility one path has compared to other paths. Noncritical activities on a certain path (i.e., having total float) do not own time float but can share it. If an activity consumes some of its total float, this will reduce time float for the subsequent activities sharing with. As concluded by Lucko and Peña Orozco [18], “total float plays an important role for owners and contractors who wish to forecast if the project will finish on time or not”.

In contrast to total float, free float is not shareable between activities along the same path. Consuming some free float by an activity does not reduce available time float for other activities. Free float is an important parameter for subcontractors as their delays could cause a ripple effect on their further downstream [18]. Other types of time float are theoretically described in the literature and are of minor importance from the practical point of view. For more details on other types of time float, refer to [18].

For repetitive activities, the concept of float is somewhat different from that of traditional analysis [17]. In addition to activity duration and timings as being main parameters of time float, production rate is a fundamental attribute of repetitive activities. Harmelink [17] concluded that “for float to be meaningful, it should reflect repetitive activity’s major attribute (progress rate)”. In linear schedules, an activity can have one or all of three float types: shift float, rate float, and combined spatiotemporal float [5]. It differs from time float as it is defined in a three-dimensional environment.

Repetitive activities, therefore, may have rate float in addition to shift (i.e., time) float. Rate float results from variation of progress rates of consecutive repetitive activities and, hence, measures possible change in the progress rate of some repetitive activities. Harmelink [17] defined rate float as “the amount that the progress rate of a non-controlling linear

activity can be lowered before this activity becomes a controlling segment, and thus impact the project finish if delayed.” The lowest possible progress rate an activity can assume determines its rate float, which is controlled by progress rates of its immediate predecessors and successors.

Harmelink and Rowings [10] developed a method for determining controlling activity path for linear projects only; however, floats of non-controlling activities are not addressed. Harmelink [17] developed a graphical method for determining rate float of non-critical segments of geometrically serial activities of linear projects and did not address time floats in repetitive projects.

Arditi et al. [14] developed an LOB scheduling algorithm in which non-linear and discrete activities are incorporated to calculate time floats only for repetitive activities. Ammar [19] developed a graphical scheduling method to determine floats of non-serial repetitive activities. However, the method is applicable only for linear projects with activities utilizing only one crew. Ökmen [1] developed a procedure to determine logical and resource critical units, in which activities are represented by variable production rates and various types of logical relationships are allowed. Francis and Morin-Pepin [11] used the concept of margin calculation based on site occupation and chrono-graphical modelling to specify critical path. Zhang et al. [12] developed a float analysis method in RSM considering work interruption, by which total float, free float, and safety float are calculated.

Based on the previous literature review, float concepts and applications still have less explanatory power on LSMs than in CPM. There are no comparable applications for LSMs with that of CPM. LSMs has typically been regarded as a visual technique lacking CPM analytical qualities. In addition, LSMs lack an analytical capability to determine activities’ floats and hence critical path(s).

In conclusion, floats in repetitive projects can be classified into two basic types: time and rate floats. Total and free floats are familiar for CPM users while other types are of minor importance. Accordingly, only total and free time floats of repetitive activities will be addressed in this paper, in addition to rate float. Calculations of different float types in repetitive projects, in an easy analytical approach, are discussed subsequently in details.

### III. Line of Balance Technique

LOB technique was essentially developed for the management of manufacturing processes in the early 1940s; however, it properly used to schedule repetitive projects [20]. Dolabi et al. [21] enumerated several advantages of LOB over other LSMs; the notable one is allowing for multiple crews’ usage per activity which allows a project to be rescheduled efficiently and, hence, enhances the practicability of obtained schedules [22].

LOB is used, basically, to schedule typical repetitive activities. In LOB, it is assumed that only one crew works in a single unit spending ( $d$ ) time on the unit before moving to the next corresponding unit [23]. It is also assumed that quantity of work of each activity is identical in all repetitive units (i.e., constant duration). The objective of LOB is to determine the number of crews to be employed for each typical activity. such that (i) units are delivered with a progress rate meeting a prespecified deadline, (ii) logical dependency within network of each unit is respected, and (iii) work continuity of crews is maintained [3].

A typical LOB of a repetitive activity is shown in Fig.1. Activities in LOB schedule are represented by a sloping bar, the width of which is activity unit duration. The activity progress or production rate is represented by the slope of LOB bar ( $R$ ). To represent crew movement among repetitive units, different crew configurations can be assumed. The crew configuration shown in Fig.1 ensures synchronization and work continuity [3].

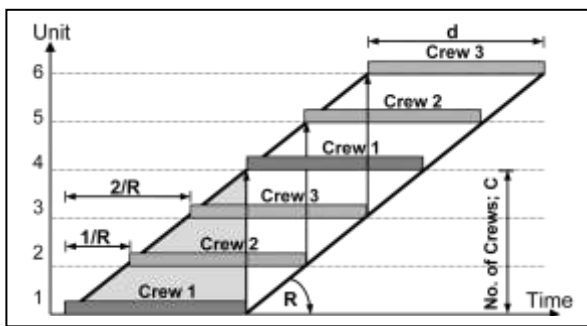


Figure 1. Synchronization and Work Continuity of Crews in LOB [3]

Using LOB (with reference to Fig.2), it is possible to formulate a strategy to meet a pre-specified project duration. Assuming ideal case of parallel LOB of activities, the desired project rate of delivery ( $R_T$ ) can be by calculated as  $(N-1)/(T_P - T_1)$ , where  $T_1$  is the CPM duration of the first unit,  $T_P$  is the desired project deadline, and  $N$  is the number of typical units. The use of total float of non-critical activities enables more reasonable resource assignment. The target progress rate of an activity  $i$  ( $R_{Ti}$ ) can then be calculated using Eq.1, where  $TF_i$  is total float of activity  $i$  (at first unit only).

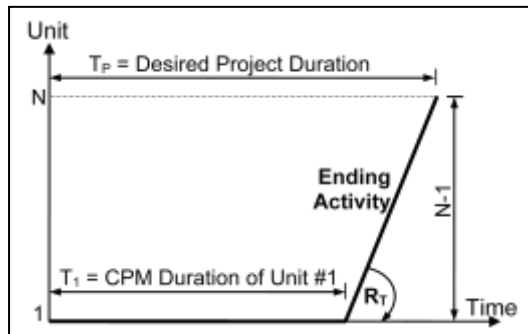


Figure 2. Desired Project Rate of Delivery [3]

To maintain the target activity progress rate, the required number of crews can be calculated using Eq.2 (with reference to Fig.1), where  $d_i$  is the activity unit duration and  $C_{ti}$  is the theoretical number

of crews deployed by the activity. From practical point of view, fractions of crews are not allowed. Eq.3 ensures practicality and availability of crews, in which  $C_{ai}$  is the actual number of crews assigned for activity  $i$  and  $C_{iav}$  is the number of crews available for that activity. Having actual number of crews per activity calculated or decided, the actual progress rate ( $R_{ai}$ ) of an activity  $i$  can be recalculated using Eq.4. For detailed information on LOB characteristics and calculations, refer to [23].

$$R_{Ti} = (N-1)/(T_P - T_1 + TF_i) \quad (1)$$

$$C_{ti} = d_i \times R_{Ti} \quad (2)$$

$$C_{ai} = \text{Round-Up}(C_{ti}), C_{ai} \leq C_{iav} \quad (3)$$

$$R_{ai} = C_{ai}/d_i \quad (4)$$

#### IV. CPM-LOB Scheduling Model

Integrating CPM and LOB merges their merits and reduces their individual flaws [24]. The CPM-LOB repetitive scheduling method developed by Ammar [13] will be used as scheduling platform in this analysis. Having desired project duration specified, actual number of crews ( $C_a$ ) and actual progress rate ( $R_a$ ) are calculated using Equations (1-4). The duration of an activity  $i$  along its all-repetitive units ( $D_i$ ) is calculated using Eq.5, in which  $ST_{iN}$  is the start time of activity  $i$  at the last unit ( $N$ ) and  $ST_{i1}$  is its start time at the first unit.

$$D_i = d_i + ST_{iN} - ST_{i1} = d_i + (N-1)/R_{ai} \quad (5)$$

To specify logical relationship between activities as well as associated lag values, actual progress rate of each activity is compared with that of its succeeding activities. Having activities' durations calculated and logical relationship between consecutive activities specified, time analysis is easily performed using overlapping activities very similar to that of CPM.

The example project introduced by Ammar [13] will be used to illustrate the developed float analysis approach. The project consists of 10 typical units with a desired deadline of 70 working days. The planning data for a typical unit are given in Table 1, including work breakdown (activities), logical relationships and estimated unit duration. A minimum buffer time of one day is assumed.

Considering unit duration of activities and a minimum buffer time of one day between consecutive activities, the time analysis for the first unit is done (if it is handled separately). The resulting duration of the first unit only ( $T_1$ ) is 34 days. Total float values of non-critical activities (for first unit only) are given in Table 1. Theoretical and actual number of crews and, hence, actual progress rates are calculated for projects activities. Activities' duration along all units are also calculated as given in Table 1.

TABLE I. LOB CALCULATIONS FOR THE EXAMPLE PROJECT [13]

Act. (i)	Pred.s	$d_i$	$TF_i$	$R_{Ti}$	$C_{ti}$ ( $d_i \times R_{Ti}$ )	$C_{ai}$	$R_{ai}$	$\frac{ST_{i10} - ST_{i1}}{R_{ai}}$	$D_i$
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Start	---	0	---	---	---	---	0.50 <sup>a</sup>	18	18
A	Start	4	9	0.20	0.80	1	0.25	36	40
B	Start	6	0	0.25	1.50	2	0.33	27	33
C	Start	2	14	0.18	0.36	1	0.50	18	20
D	A	8	9	0.20	1.60	2	0.25	36	44
E	B	10	1	0.24	2.43	3	0.30	30	40
F	B	16	0	0.25	4.00	4	0.25	36	52
G	C	6	14	0.18	1.08	2	0.33	27	33
H	D	4	9	0.20	0.80	1	0.25	36	40
J	E	8	1	0.24	1.94	2	0.25	36	44
K	F, G	10	0	0.25	2.50	3	0.30	30	40
L	H, J	6	1	0.24	1.46	2	0.33	27	33
End	K, L	0	---	---	---	---	0.33 <sup>b</sup>	27	27

a: Largest progress rate of succeeding activities,  
 b: Largest progress rate of preceding activities.

Relationship type and associated lag value are shown in Fig.3, along links connecting activities. CPM time analysis are given in Fig.3, in which forward calculations are shown in the upper cells of activities' boxes while backward pass calculations are shown in the bottom cells. The resulting project duration is 70 days.

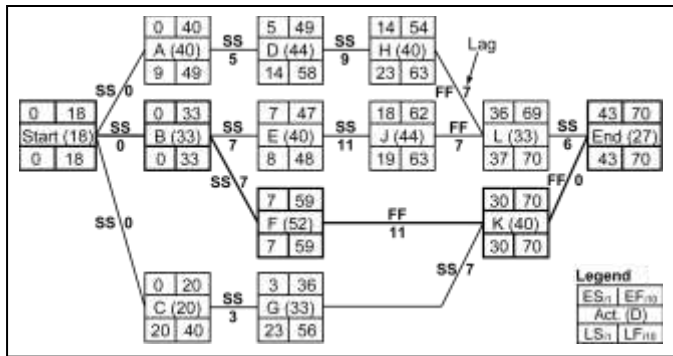


Figure 3. Time Schedule of the Example Project [13]

It should be noted that early and late start times of activities belong to the first unit only, while early and late finish times of activities belong to the last unit only. Having these values determined, early and late start and finish times for all other units of each activity (2~N-1) can be calculated. For detailed information of the CPM-LOB scheduling model, refer to [13].

## v. Float Analysis Model

In the proposed method for calculating floats in LOB scheduling, activities are classified as critical or non-critical. Critical activities do not have float, while non-critical activities must have. Non-critical repetitive activities have time float and may have rate float, while critical ones have neither time nor rate floats. The developed concepts for float analysis in LOB scheduling will be presented with reference to the example project on hand.

### A. Total Float

Total float of a repetitive activity is defined as “the amount of time by which an activity can be delayed without delaying project completion time but may delay

succeeding activities.” Total float is traditionally calculated using Eq.6, in which  $TF_i$  is the total float of activity  $i$ ,  $LS_i$  its late start time,  $ES_i$  its early start time,  $LF_i$  its late finish and  $EF_i$  its early finish.

$$TF_i = LS_i - ES_i = LF_i - EF_i \quad (6)$$

Equation 6 has been driven, basically, for traditional (non-repetitive) activities. Nevertheless, it can be extended for non-critical repetitive activities by Eq.7, in which  $TF_i$ ,  $LS_{i1}$ ,  $ES_{i1}$ ,  $LF_{iN}$ , and  $EF_{iN}$  are Total Float, Late Start at the first unit; (1), Early Start at the first unit (1), Late Finish at the last unit (N), and Early Finish at the last unit (N) of activity  $i$ , respectively.

$$TF_i = LS_{i1} - ES_{i1} = LF_{iN} - EF_{iN} \quad (7)$$

With reference to Fig.3, total float of activity G (for instance) is calculated as  $TF_G = LS_{G1} - ES_{G1} = 23 - 3 = 20$  or  $LF_{G10} - EF_{G10} = 56 - 36 = 20$ . Detailed total float calculation for project activities are given in Table 2.

TABLE II. DETAILED CALCULATION OF TIME FLOAT FOR THE EXAMPLE PROJECT

Act.	TF (days)	Succeeding Activities	Free Float (days)	
			SS Relation	FF Relation
A	9-0 = 9, or 49-40 = 9	D	5-0-5 = 0	---
B*	0-0 = 0, or 33-33 = 0	E	---	---
		F	---	---
C	20-0 = 20, or 40-20 = 20	G	3-0-3 = 0	---
D	14-5 = 9, or 58-49 = 9	H	14-5-9 = 0	---
E	8-7 = 1, or 48-47 = 1	J	18-7-11 = 0	---
F*	7-7 = 0, or 59-59 = 0	K	---	---
G	23-3 = 20, or 56-36 = 20	K	30-3-7 = 20	---
H	23-14 = 9, or 63-54 = 9	L	---	69-54-7 = 8
J	19-18 = 1, or 63-62 = 1	L	---	69-62-7 = 0
K*	30-30 = 0, or 70-70 = 0	End	---	---
L	37-36 = 1, or 70-69 = 1	End	43-36-6 = 1	---

\* Critical Activities

Having total float calculated, criticality can now be extended to repetitive activities. For the example project on hand, activities B, F and K are critical, while activities A, C, D, E, G, H, J and L are non-critical. By definition, the critical activities (B, F, and K) neither have time nor rate floats.

### B. Free Float

Free float of a repetitive activity is defined as “the amount of time by which an activity can be delayed without delaying succeeding activities.” Non-critical activities (having total float) may have free float (i.e., free float ≤ total float). Free float is traditionally calculated using Eq.8, in which  $FF_i$  is free float of activity  $i$ ,  $ES_j$  early start of its successor (s) and  $NS_i$  number of succeeding activities to that activity.

$$FF_i = \text{Min.} (ES_s - EF_i), \quad s = 1, 2, \dots, NS_i \quad (8)$$

As SS or FF logical relationship (or both) may exist between consecutive repetitive activities, Eq.8 cannot be applied directly to repetitive activities. Free float of noncritical

repetitive activities is calculated based on the logical relationship with succeeding activities as follows.

### SS Relationship

In this case, free float of a non-critical repetitive activity is calculated (with reference to Fig.4.a) using Eq.9, in which  $ES_{s1}$  is Early Start of succeeding activity (s) at the first unit and  $Lag_{SS}$  is time lag associated with SS relationship.

$$FF_i = \text{Min.} (ES_{s1} - ES_{i1} - Lag_{SS}), \quad s = 1, 2, \dots, NS_i \quad (9)$$

For instance, the free float of activity G (SS with activity K and 7 days lag) is calculated as  $FF_G = ES_{K1} - ES_{G1} - Lag_{SS} = 30-3-7 = 20$  days.

### FF Relationship

In this case, free float of non-critical repetitive activities is calculated (with reference to Fig.4.b) using Eq.10, in which  $EF_{sN}$  is Early Finish of succeeding activity (s) at the last unit and  $Lag_{FF}$  is time lag associated with FF relationship.

$$FF_i = \text{Min.} (EF_{sN} - EF_{iN} - Lag_{FF}), \quad s = 1, 2, \dots, NS_i \quad (10)$$

If both SS and FF relationships exist (where progress rates of activities are equal), the free float of a non-critical activity can be calculated using either Eq.9 or Eq.10.

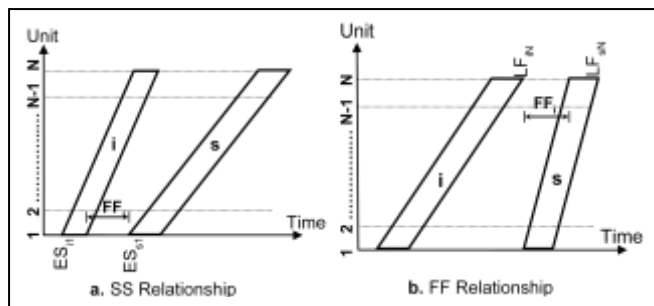


Figure 4. Free Float Calculation for Repetitive Activities

For instance, free float of activity H (FF relationship with activity L and 7 days lag) is calculated as  $FF_H = EF_{L10} - EF_{H10} - Lag_{FF} = 69-54-7 = 8$  days. Detailed free float calculation for non-critical activities are also given in Table 2.

### C. Rate Float

Rate float of a repetitive activity is defined as “the amount by which the progress rate of a non-critical repetitive activity can be lowered before that activity becomes critical.” As discussed previously, rate float results from variation in progress rates between consecutive activities. Rate float value is constrained by the lowest progress rate an activity can assume, which is controlled by the progress rates of its immediately succeeding activities. Accordingly, rate float of a repetitive activity can be calculated as the difference between the progress rate of the activity and the minimum progress rate of its succeeding activities.

To calculate rate float of a repetitive activity, its progress rate is compared with that of its immediate successors. If there

is more than one succeeding activity, the one with largest progress rate is dominating. Different possible cases that can exist are given in Table 3, by which rate float of repetitive activities can be calculated.

TABLE III. RULES FOR CALCULATING RATE FLOAT OF TYPICAL REPETITIVE ACTIVITIES

Rule #	Case	Rate Float (RF)	Notes
1	$R_{ai} \leq R_{as}$	0	
2	$R_{ai} > R_{as}$	$R_{ai} - R_{as}$	
3	$R_{END} > R_{ap}$	$R_{END} - R_{ap}$	Ending Activities Only

$R_{ap}$ : Actual progress rate of preceding activity p to ending activity(s)

Applying the rules given in Table 3 to the example project on hand, detailed calculations of rate float for non-critical activities are given in Table 4. For instance, the rate float of activity E (Case #2) is calculated as  $R_{aE} - R_{aJ} = 0.30 - 0.25 = 0.05$ .

TABLE IV. RATE FLOAT CALCULATION FOR THE EXAMPLE PROJECT

Act.	$R_{ai}$	Successor	$R_{as}$	Rule #	Rate Float Value
A	0.25	D	0.25	1	0
C	0.50	G	0.33	2	$0.50 - 0.33 = 0.17$
D	0.25	H	0.25	1	0
E	0.30	J	0.25	2	$0.30 - 0.25 = 0.05$
G	0.33	K	0.30	2	$0.33 - 0.30 = 0.03$
H	0.25	L	0.33	1	0
J	0.25	L	0.33	1	0
L	0.33	End	0.33	1	0

### D. Discussion of Results

Figure 5 shows the LOB of the non-critical path C-G-K, in which continuous LOBs represent early schedule of activities while dashed LOBs represent late schedule. Time float of activity G and rate float of activity C (for instance) are shown graphically in Fig.5. Figure 6 shows, however, the LOB of the critical path B-F-K.

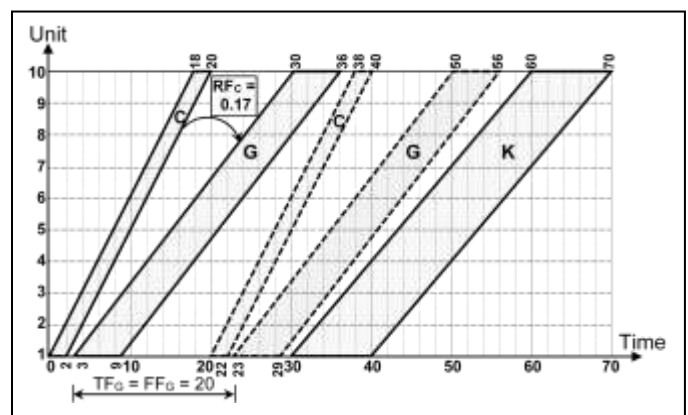


Figure 5. LOB of the Non-Critical Path C-G-K

With reference to Fig.6, two important issues have to be declared. From first impression, units #2 to #10 of activity B (Fig.6) apparently have time float. For instance, unit #10 can be theoretically delayed by 9 days (considering a buffer time of one day) without delaying the corresponding succeeding

unit (i.e. unit #10 of activity F). This is, however, infeasible as crew work continuity should not be violated as resource continuity is a primarily advantage of LOB. This applies also for units #1 to #9 in activity F.

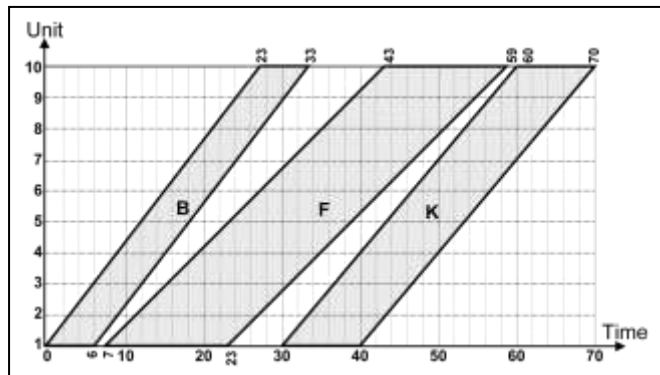


Figure 6. LOB of the Critical Path B-F-K

Secondly, activities B and K have apparently rate float. Theoretically, progress rate of activities B and K may be lowered to cope with that of activity F. This is, however, infeasible from the practical point of view. The difference in progress rate between activities B and F is 0.08 (0.33-0.25). The progress rate of activity B can be lowered only at discrete intervals of 0.167; 1/unit duration (6). Either two crews ( $R_B = 2/6 = 0.33$ ) or a single crew ( $R_B = 1/6 = 0.167$ ) can be used by activity B. Any other value for the progress rate of activity B between these two values is infeasible. In addition, the selected number of crews per activities satisfies the desired project duration (70 days). Changing the number of crews assigned for project activities (if possible) may violate the pre-specified project duration.

### E. Computational Experience

To simplify the calculations of the proposed method, a spreadsheet tool (Excel 2016) is used. Spreadsheets have many powerful features such as simple interface and ease of use even for first time users. In addition, data entry, manipulation, large number of built-in functions, and customized charts are popular features of spreadsheets. The designed Excel sheets are used to perform all LOB calculations, CPM-LOB integrated model analysis, and float calculations.

In addition to the example project discussed earlier, other case studies with varying project parameters are analyzed using the designed Excel sheets. The planning data of the example project for a repetitive unit (Table 1) are used with various number of repetitive units and different project durations. The case studies include: 20 Units-100 days, 30 Units-120 days, 40 Units-150 days, in addition to the basic case of 10 Units-70 days. Summary of the obtained float analysis results for these case studies are given in Table 5.

TABLE V. FLOAT SUMMARY REPORT OF THE EXAMPLE PROJECT: CASE STUDIES

Case	(2) 20 Units,	(3) 30 Units,	(4) 40 Units,
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Act.	100 days			120 days			150 days		
	TF	FF	RF	TF	FF	RF	TF	FF	RF
A	7.9	0	0	Critical			Critical		
B	Critical			11.3	0	0.13	18.0	0	0.13
C	19.3	0	0.17	24.3	0	0.17	30.0	0	0.17
D	7.9	0	0	Critical			Critical		
E	Critical			11.3	0	0	18.0	0	0
F	5.3	0	0	20.0	0	0	35.5	5.5	0
G	19.3	14	0.08	24.3	4.3	0	30.0	0	0
H	7.9	0	0	Critical			Critical		
J	Critical			11.3	11.3	0	18.0	18.0	0.05
K	5.3	5.3	0	20.0	20.0	0.05	30.0	30.0	0
L	0	0	0	Critical			Critical		
Dur.	102.6 days			131.3 days			168 days		

It should be noted that resulting project duration may exceed stipulated one. This is not a surprise as traditional LOB calculations do not ensure stipulated project duration [23]. For instance, the resulting project duration in case #4 is 168 days while the stipulated one is 150 days. To retain stipulated project duration, one has to increase number of crews on some selected activities (not necessarily the critical ones). For more details on resource optimization in LOB scheduling, refer to [25].

It is apparent, from the results given in Table 5, that critical activities, and hence critical path(s), change from case to another. While the critical activities in Case #1 are B, F and K, the critical ones in Case #3 are A, D, H and L. It can also be noted that the values of time float increase as the project size increase (as in cases #3 and #4). Time floats enhance the project performance which gives planner and project manager flexibility to manage his/her project effectively. This implies the significance of calculating floats of repetitive activities using the proposed method.

In conclusion, the developed model is more general and applicable for both linear and repetitive project in which multiple crews per activity can be employed. In addition, various types of floats are determined in an easy analytical manner.

## VI. Summary and Conclusions

LSMs have not been widely applied to construction industry as they are unable to specify floats and criticality in repetitive projects. Activities' floats represent a challenging scheduling issue in the construction industry. Specifying critical activities and float of non-critical ones is crucial for project managers. In addition to time float, non-critical repetitive activities may have rate float. In this paper, the time float concept is extended to both linear and repetitive activities and rate float of which is also addressed. A structured analytical approach is developed to identify floats of repetitive activities similar to that of CPM. The main features of the proposed float analysis in LOB scheduling include: (1) Only planning data for a typical unit are needed, (2) An efficient integrated LOB-CPM repetitive scheduling model has been used considering both precedence and resource continuity constraints, (3) Time and rate floats of repetitive activities are calculated in an easy analytical approach, and (4) Non-serial activities in both linear and repetitive projects are considered.

Having various types of floats of repetitive activities calculated, management tools can now be applied to repetitive projects such as Resource Allocation and Time-Cost Trade-Off. However, float calculations developed here are applicable only for typical repetitive activities with Finish-to-Start relationship.

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### About Author (s):



Dr. Ammar is Professor of Construction Engineering and Management, Faculty of Engineering, Tanta University, Egypt (On Leave). Dr. Ammar joined the Building, Civil, and Environmental Engineering Department, Faculty of Engineering and Computer Science (ENCS), Concordia University, Canada, (2008) as a Visiting Scholar (Post Doctor Fellow). Dr. Ammar supervised and examined more than 25 M.Sc. and Ph.D. theses. He is a reviewer for more than 17 reputable international journals. During his graduate study, a wide range of research topics are of interest including, but not limited to, Fuzzy Scheduling of Construction Projects, Planning and Scheduling of Linear/Repetitive Projects, Simulation and Modeling of Construction Projects, Uncertainty and Risk Management, Analysis of Earthmoving Systems, etc.