

Construction of u Control Chart by Type-2 Fuzzy Sets

Ihsan KAYA

Abstract: Control charts are very useful tools to monitor process for evaluating variability. Although they are often used, traditional control charts have to design based on crisp values for the quality characteristics. It is also clear that the measurements of quality characteristics includes some uncertainty related with human judgments and evaluations in real case problems. When the data includes uncertainty or vagueness, the results about process variability obtained from traditional control charts are not correct. To overcome these problem, the fuzzy set theory (FST) can be successfully applied into control charts. One of fuzzy sets extensions named type 2 fuzzy sets that have fuzzy membership values can be used more effectively to design of control charts. The fuzzy control charts based on type-2 fuzzy sets are more usable in real case problems because of modelling uncertainties of human judgments and measurement systems. For this aim, one of attribute control charts named control chart for nonconformities per unit, or u control chart has been analyzed based on type-2 fuzzy sets. The design of u control chart based on type-2 fuzzy sets has been improved and it is applied on a real case application in automotive sector.

Keywords- Control charts, attributes, the fuzzy set theory, type-2 fuzzy sets, u control chart

I. Introduction

Control charts (CCs) are used to follow processes' variation and to perform different tests using a wide variety of information about causes of variability for related quality characteristics. It is known that Shewhart designed the first control charts for a unique quality characteristic. The strength of these charts is in their ability to detect abnormal situations and circumstances. CCs can be generally classified in two groups. If characteristics are measurable on numerical scales, then variable CCs (VCCs) are used. By the way, sometimes the quality characteristics can be defined as "conforming/nonconforming" or "defective/non defective". These types of quality characteristics are called attributes. In the traditional CCs, data are generally expressed of crisp values. But the measurement systems that include mainly operators, gage and environmental conditions can include "uncertainty" or "vagueness" on quality evaluations. These uncertainties are based on the process and measurement systems, which can lead to some difficulties in obtaining crisp values from the process.

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In these cases, the control charts based on the fuzzy set theory (FST) are more useful for evaluating the process and for constructing fuzzy control charts (FCCs). It is clear that FST can be successfully adopted for the CCs based on uncertainties. In this condition, FST supports the development of concepts and techniques for dealing with sources of uncertainty or imprecision. Major contribution of FST is its capability to represent and modeling linguistic and approximate data for quality control process. Although FST has great advantages to model uncertainty, sometimes type-1 fuzzy sets called classical fuzzy sets cannot successfully model the uncertainty as a result of crisp definitions for membership functions that consist of two dimensional. So, one of the fuzzy set extensions named type-2 fuzzy sets whose membership functions are three-dimensional and includes fuzzy membership functions proposed by Zadeh [1] to improve modeling quality of uncertainty. Thus type-2 fuzzy sets can successfully represent the uncertainty and can reduce its harmful effects [2-3]. For this aim, one of well-known attribute control charts named u control chart has been developed and re-formulated based on type-2 fuzzy sets in this paper. The main characteristic functions of u-control chart are re-designed by taking into account the human evaluations or uncertainties of measurement systems. Finally, there main parameters such as Upper Control Limit (UCL), Central Limit (CL) and Lower Control Limit (LCL) of the chart are re-formulated and out of control situations have been analyzed based on type-2 fuzzy control charts. The proposed u-control chart based on type-2 fuzzy (\tilde{u}) sets is also applied on a real case application from automotive sector. For this aim, manufacturing process of brake hose products is analyzed by using \tilde{u} control chart.

The rest of this paper has been organized as follows: Section 2 includes a summary related with type-2 fuzzy sets. The control chart for nonconformities per unit named u control chart based on type-2 fuzzy sets is detailed and the UCL, LCL and CL formulations have been derivate for type-2 fuzzy sets into Section 3. A real case application for the proposed methodology is shown into Section 4. Finally, the obtained results and future research suggestions are discussed into Section 5.

II. The Fuzzy Sets and Type-2 fuzzy Sets

The fuzzy set theory (FST) has been successfully used to describe uncertainty and imprecision and it was introduced by Zadeh [1]. The quality characteristics that includes uncertainty can be successfully analyzed by chart control that designed by using FST. In this case, the fuzzy based control charts (FCCs) are more usable and more sensitive than traditional control charts, and the obtained results are more realistic. In recently, some of fuzzy set extensions have been proposed to improve capability of FST. One of them, type-2 fuzzy sets are very useful when a full membership function cannot be determined.

The membership function of them can be expressed by traditional fuzzy set named type-1 fuzzy sets. A type-2 fuzzy set

\tilde{A} in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}^x$, shown $\tilde{X} = \{(x, u), \mu(x, u) | \in A, \forall u \in J_x \subseteq [0, 1], 0 \leq \mu(x, u) \leq 1\}$ where J_x denotes an interval in $[0, 1]$. If $\mu_{\tilde{A}}^x(x, u)$ is equal to 1.00 for all values, \tilde{X} named as interval type-2 fuzzy set [1-10].

Type-1 membership functions are expressed on a two-dimensional plane, namely the set variable and membership degree. In the Type-2 membership functions, secondary membership functions that consist of an intermittent membership degree correspond to a set variable. Type-2 membership functions are expressed in three dimensions due to secondary membership function. Therefore, type-2 fuzzy sets are more effective in overcoming uncertainties than traditional FST. So it is very useful tool to design of control charts by managing uncertainties more effectively. For this aim, one of attribute control charts, u-control chart, has been analyzed based on type-2 fuzzy sets.

III. Control Chart for nonconformities per unit Based on Type-2 Fuzzy Sets

As we know, the control charts have been designed to monitor the process' variation. The charts have been successfully applied to many sectors such as manufacturing, engineering, health care, education, service industry for quality improvement. When the control charts have been designed, two types of quality characteristics such as variable and attribute charts are taken into account. If the characteristic is measureable such as thickness, length, conductivity, weight, etc., the variable control charts are used. If the related quality characteristic is non-measureable such as failure or success, good or defective, accept or reject, etc., the attribute control charts (ACCs) are used. One of ACCs based on the number of defects or nonconformities over time is called u charts or control chart for nonconformities per unit. The control chart for nonconformities per unit should be used with discrete or attribute defect data when the sample size are varies.

In this section, one of the ACCs for defects has been analyzed based on uncertainties. For this aim the control chart for nonconformities per unit, u control chart, has been designed based on type-2 fuzzy sets and the type-2 fuzzy u control charts has been arisen.

In the \tilde{u} control chart, the number of defects per unit (u) is the ratio of the number of defects in the sample. Additionally, the average number of defects per unit as type-2 fuzzy number

(\tilde{u}) should be obtained. Based on the arithmetic operations of type-2 fuzzy sets, the number of defects per unit (u) as type-2 fuzzy number, can be defined as follows:

$$\tilde{u}_1 = [(x_{11}^U, x_{12}^U, x_{13}^U, x_{14}^U, H_1(X_1^U), H_2(X_1^U)); (x_{11}^L, x_{12}^L, x_{13}^L, x_{14}^L, H_1(X_1^L), H_2(X_1^L))] \quad (1)$$

The mean value of defects per unit as defined by using type-2 fuzzy sets (\tilde{u}) can be obtained as follows:

$$\tilde{u}_1 = [(x_{11}^U, x_{12}^U, x_{13}^U, x_{14}^U, H_1(X_1^U), H_2(X_1^U)); (x_{11}^L, x_{12}^L, x_{13}^L, x_{14}^L, H_1(X_1^L), H_2(X_1^L))] \quad (2)$$

$$\tilde{u}_2 = [(x_{21}^U, x_{22}^U, x_{23}^U, x_{24}^U, H_1(X_1^U), H_2(X_1^U)); (x_{21}^L, x_{22}^L, x_{23}^L, x_{24}^L, H_1(X_1^L), H_2(X_1^L))] \quad (3)$$

$$\tilde{u}_n = [(x_{n1}^U, x_{n2}^U, x_{n3}^U, x_{n4}^U, H_1(X_1^U), H_2(X_1^U)); (x_{n1}^L, x_{n2}^L, x_{n3}^L, x_{n4}^L, H_1(X_1^L), H_2(X_1^L))] \quad (4)$$

$$E[\tilde{u}] = \tilde{u} \quad (5)$$

$$\tilde{u} = \frac{\sum_{i=1}^m \tilde{u}_i}{m} \quad (6)$$

$$\tilde{u} = (\bar{u}_{x_{1,1}}^u, \bar{u}_{x_{2,2}}^u, \bar{u}_{x_{3,3}}^u, \bar{u}_{x_{4,4}}^u; \bar{u}_{x_{1,1}}^l, \bar{u}_{x_{2,2}}^l, \bar{u}_{x_{3,3}}^l, \bar{u}_{x_{4,4}}^l) \quad (7)$$

$$\bar{u}_{x_{1,1}}^u = \frac{\sum_{i=1}^n x_{i1}^u}{n}, \bar{u}_{x_{2,2}}^u = \frac{\sum_{i=1}^n x_{i2}^u}{n}, \bar{u}_{x_{3,3}}^u = \frac{\sum_{i=1}^n x_{i3}^u}{n}, \bar{u}_{x_{4,4}}^u = \frac{\sum_{i=1}^n x_{i4}^u}{n} \quad (8)$$

$$\bar{u}_{x_{1,1}}^l = \frac{\sum_{i=1}^n x_{i1}^l}{n}, \bar{u}_{x_{2,2}}^l = \frac{\sum_{i=1}^n x_{i2}^l}{n},$$

$$\bar{u}_{x_{3,3}}^l = \frac{\sum_{i=1}^n x_{i3}^l}{n}, \bar{u}_{x_{4,4}}^l = \frac{\sum_{i=1}^n x_{i4}^l}{n} \quad (9)$$

Based on these equations, the limit values for the \tilde{u} control charts can be calculated as follows:

$$UCL_{\tilde{u}} = \tilde{u} \oplus 3\sqrt{\frac{\tilde{u}}{n}} \quad (10)$$

$$\begin{aligned} \overline{UCL}_{\tilde{u}} = & \left[\bar{u}_{x_{1,1}}^u + 3\sqrt{\frac{\bar{u}_{x_{1,1}}^u}{n}}, \bar{u}_{x_{2,2}}^u + 3\sqrt{\frac{\bar{u}_{x_{2,2}}^u}{n}}, \bar{u}_{x_{3,3}}^u + 3\sqrt{\frac{\bar{u}_{x_{3,3}}^u}{n}}, \bar{u}_{x_{4,4}}^u + 3\sqrt{\frac{\bar{u}_{x_{4,4}}^u}{n}}; \right. \\ & \left. \min(H_1(X_1^U), H_2(X_1^U)); \bar{u}_{x_{1,1}}^l + 3\sqrt{\frac{\bar{u}_{x_{1,1}}^l}{n}}, \bar{u}_{x_{2,2}}^l + 3\sqrt{\frac{\bar{u}_{x_{2,2}}^l}{n}}, \bar{u}_{x_{3,3}}^l + 3\sqrt{\frac{\bar{u}_{x_{3,3}}^l}{n}}, \bar{u}_{x_{4,4}}^l + 3\sqrt{\frac{\bar{u}_{x_{4,4}}^l}{n}}; \right. \\ & \left. \min(H_1(X_1^L), H_2(X_1^L)) \right] \quad (11) \end{aligned}$$

$$CL_{\tilde{u}} = \tilde{u} \quad (12)$$

$$\overline{CL}_{\tilde{u}} = [\bar{u}_{x_{1,1}}^u + \bar{u}_{x_{2,2}}^u + \bar{u}_{x_{3,3}}^u + \bar{u}_{x_{4,4}}^u; \min(H_1(X_1^U), H_2(X_1^U)), \bar{u}_{x_{1,1}}^l + \bar{u}_{x_{2,2}}^l + \bar{u}_{x_{3,3}}^l + \bar{u}_{x_{4,4}}^l; \min(H_1(X_1^L), H_2(X_1^L))] \quad (13)$$

$$LCL_{\tilde{u}} = \tilde{u} \ominus 3\sqrt{\frac{\tilde{u}}{n}} \quad (14)$$

$$LCL_{\tilde{u}} = \left[\begin{array}{l} \tilde{u}_{x_1^U} - 3\sqrt{\frac{\tilde{u}_{x_1^U}}{n}}, \tilde{u}_{x_2^U} - 3\sqrt{\frac{\tilde{u}_{x_2^U}}{n}}, \tilde{u}_{x_3^U} - 3\sqrt{\frac{\tilde{u}_{x_3^U}}{n}}, \tilde{u}_{x_4^U} - \\ 3\sqrt{\frac{\tilde{u}_{x_1^U}}{n}}; \min(H_1(X_1^U), H_2(X_1^U)); \tilde{u}_{x_1^L} - 3\sqrt{\frac{\tilde{u}_{x_1^L}}{n}}, \tilde{u}_{x_2^L} - 3\sqrt{\frac{\tilde{u}_{x_2^L}}{n}}, \tilde{u}_{x_3^L} - \\ 3\sqrt{\frac{\tilde{u}_{x_1^L}}{n}}, \tilde{u}_{x_4^L} - 3\sqrt{\frac{\tilde{u}_{x_1^L}}{n}}; \min(H_1(X_1^L), H_2(X_1^L)) \end{array} \right] \quad (14)$$

Finally, \tilde{u} control chart whose limit values consist of type-2 fuzzy number can be obtained as shown in Fig. 1.

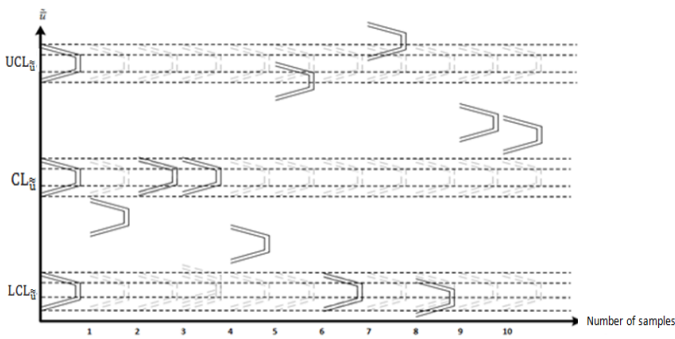


Fig. 1. A Representation of \tilde{u} control chart

iv. A Real Case Application for Control chart for nonconformities per unit Based on Type-2 Fuzzy Sets from Automotive Industry

In this paper, one of the fuzzy extensions named type-2 fuzzy sets that have fuzzy membership degrees has been integrated with control chart for nonconformities per unit. For this aim, the \tilde{u} control chart has been obtained. The obtained type-2 fuzzy based control chart has been applied on a real case application from automotive industry. The firm produces rubber hoses; metal, water injection and thermoforming tubes; suction, hot gas, and liquid lines; high pressure brake hose; brake booster line; battery, transmission and fuel cell cooling lines; engine coolant / heater lines; dirty and clean air ducts and other lines for automobiles. An application for brake hose that shown in Figure 2 has been managed based on quality control data.



Figure 2 An Example for brake hose

According to quality control results, 16 different failure modes are defined for brake hoses and these failures are classified according to severity of the failures that are defined in the company quality procedure. For all classes, the weights of defects are defined based on pairwise comparisons as shown in Table 1.

TABLE 1 FAILURE MODES AND WEIGHTS FOR BRAKE HOSE

No	Failure Class	Failure Mode	Failures weights
1	1	Marking failure	0.1
2	1	Packaging	0.1
3	2	Grommet deformation	0.2
4	2	Hose surface problem	0.2
5	2	Fitting length problem	0.2
6	2	Missing cap	0.2
7	2	Grommet length	0.2
8	3	Reverse direction of fitting	0.5
9	3	Fitting thread faulty	0.5
10	3	Wrong fitting used	0.5
11	3	Missing component	0.5
12	3	Fitting deformation	0.5
13	3	Hose length failure	0.5
14	4	Torn/Split on the hose	0.9
15	4	Leakage	0.9
16	4	Knitting failure	0.9

According to total defect weights, the parts are defined as “completely defect, high defect, medium defect, no high defect and no completely defect” as shown in Table 2.

TABLE 2 DEFECT STATUS FOR QUALITY CONTROL FOR BRAKE HOSE

Defect Status	Detail
Completely Defect (CD)	$0.9 \leq \text{Defect weight} \leq 1.00$
High Defect (HD)	$0.7 \leq \text{Defect weight} < 0.9$
Medium Defect (MD)	$0.3 \leq \text{Defect weight} < 0.7$
No High Defect (NHD)	$0 < \text{Defect weight} < 0.3$
No Completely Defect (NCD)	Defect weight = 0

Based on failure weights, trapezoidal type-2 fuzzy numbers are determined for each classification as shown in Table 3 and all of the samples have been evaluated based on defect status as shown in Table 4.

TABLE 3 TYPE-2 FUZZY VALUES FOR DEFECT STATUS OF BRAKE HOSES

Defect Status	a_{11}^u	a_{12}^u	a_{13}^u	a_{14}^u	$H_1(A^U)$	$H_2(A^U)$	a_{11}^l	a_{12}^l	a_{13}^l	a_{14}^l	$H_1(A^L)$	$H_2(A^L)$
Completely Defect	0.82	0.87	0.93	1.00	1	1	0.84	0.88	0.92	0.96	0.85	0.85
High Defect	0.62	0.67	0.73	0.80	1	1	0.64	0.68	0.72	0.76	0.85	0.85
Medium Defect	0.42	0.47	0.53	0.60	1	1	0.44	0.48	0.52	0.56	0.85	0.85
No High Defect	0.22	0.27	0.33	0.40	1	1	0.24	0.28	0.32	0.36	0.85	0.85
No Completely Defect	0.02	0.07	0.13	0.20	1	1	0.04	0.08	0.12	0.16	0.85	0.85

TABLE 5 TYPE-2 FUZZY VALUES FOR RESULT OF QUALITY PROCESS OF BRAKE HOSES

i	\bar{a}_{11}^u	\bar{a}_{12}^u	\bar{a}_{13}^u	\bar{a}_{14}^u	$H_1(A^U)$	$H_2(A^U)$	\bar{a}_{11}^l	\bar{a}_{12}^l	\bar{a}_{13}^l	\bar{a}_{14}^l	$H_1(A^L)$	$H_2(A^L)$
1	0.18	0.23	0.29	0.36	1.00	1.00	0.14	0.18	0.22	0.26	0.85	0.85
2	0.16	0.21	0.27	0.34	1.00	1.00	0.18	0.22	0.26	0.34	0.85	0.85
3	0.08	0.13	0.19	0.26	1.00	1.00	0.10	0.14	0.18	0.22	0.85	0.85
4	0.18	0.23	0.29	0.36	1.00	1.00	0.20	0.24	0.28	0.32	0.85	0.85
5	0.02	0.07	0.13	0.20	1.00	1.00	0.04	0.08	0.12	0.16	0.85	0.85
6	0.10	0.15	0.21	0.28	1.00	1.00	0.12	0.16	0.20	0.24	0.85	0.85
7	0.20	0.25	0.31	0.38	1.00	1.00	0.22	0.26	0.38	0.34	0.85	0.85
8	0.02	0.07	0.13	0.20	1.00	1.00	0.04	0.08	0.12	0.16	0.85	0.85
9	0.14	0.19	0.25	0.32	1.00	1.00	0.16	0.02	0.24	0.28	0.85	0.85
10	0.10	0.15	0.21	0.28	1.00	1.00	0.12	0.16	0.20	0.24	0.85	0.85
11	0.12	0.17	0.23	0.30	1.00	1.00	0.14	0.18	0.20	0.26	0.85	0.85
12	0.10	0.15	0.21	0.28	1.00	1.00	0.12	0.16	0.20	0.24	0.85	0.85
13	0.02	0.07	0.13	0.20	1.00	1.00	0.04	0.08	0.12	0.16	0.85	0.85
14	0.08	0.13	0.19	0.26	1.00	1.00	0.10	0.14	0.18	0.22	0.85	0.85
15	0.18	0.23	0.29	0.36	1.00	1.00	0.20	0.24	0.28	0.32	0.85	0.85
16	0.10	0.15	0.21	0.28	1.00	1.00	0.12	0.16	0.20	0.24	0.85	0.85
17	0.02	0.07	0.13	0.20	1.00	1.00	0.04	0.08	0.12	0.16	0.85	0.85
18	0.02	0.07	0.13	0.20	1.00	1.00	0.04	0.08	0.12	0.16	0.85	0.85
19	0.06	0.11	0.17	0.24	1.00	1.00	0.08	0.12	0.16	0.20	0.85	0.85
20	0.04	0.09	0.15	0.22	1.00	1.00	0.06	0.10	0.14	0.18	0.85	0.85
Ave.	0.10	0.15	0.21	0.28	1.00	1.00	0.11	0.14	0.20	0.24	0.85	0.85

TABLE 4 QUALITY CONTROL RESULTS OF BRAKE HOSES FOR TWENTY SAMPLES

Sample	Total	Defect Types	Defect Status
1	0.00		NCD
2	0.00		NCD
3	0.90	Knitting failure	CD
4	0.00		NCD
5	0.00		NCD
6	0.00		NCD
7	0.20	Grommet deformation	NHD
8	0.00		NCD
9	0.00		NCD
10	0.00		NCD
11	0.00		NCD
12	0.70	Fitting deformation,	HD
13	0.00		MD
14	0.00		NCD
15	0.00		NCD
16	0.00		NCD
17	0.00		NCD

Sample	Total	Defect Types	Defect Status
18	0.00		NCD
19	0.50	Hose length failure	MD
20	0.00		NCD

The average type-2 fuzzy values have been obtained as shown in Table 5. Then the limit values of the \tilde{u} control chart for brake hoses have been obtained as shown in Table 6.

TABLE 6 THE CONTROL LIMIT VALUES OF \tilde{u} CONTROL CHART FOR BRAKE HOSE

	a_{11}^u	a_{12}^u	a_{13}^u	a_{14}^u	$H_1(A^U)$	$H_2(A^U)$
\bar{UCL}	0.304	0.402	0.510	0.628	1.000	1.000
\bar{CL}	0.096	0.146	0.206	0.276	1.000	1.000
\bar{LCL}	0.000	0.000	0.000	0.000	1.000	1.000
	a_{11}^l	a_{12}^l	a_{13}^l	a_{14}^l	$H_1(A^L)$	$H_2(A^L)$

	α_{11}^u	α_{12}^u	α_{13}^u	α_{14}^u	$H_1(A^U)$	$H_2(A^U)$
\overline{UCL}	0.338	0.399	0.493	0.560	0.850	0.850
\overline{CL}	0.113	0.144	0.196	0.235	0.850	0.850
\overline{LCL}	0.000	0.000	0.000	0.000	0.850	0.850

Then the type-2 fuzzy values are defuzzified by using modifying the Best Nonfuzzy Performance (BNP) by Kahraman et al. [11] as shown in Table 7.

TABLE 7 THE DEFUZZIFIED MEASUREMENTS FOR BRAKE HOSE

Sample	DIT2U	DIT2L	DIT2 TRAP
1	0.27	0.19	0.23
2	0.25	0.23	0.24
3	0.17	0.15	0.16
4	0.27	0.24	0.25
5	0.11	0.09	0.10
6	0.19	0.17	0.18
7	0.29	0.28	0.28
8	0.11	0.09	0.10
9	0.23	0.17	0.20
10	0.19	0.17	0.18
11	0.21	0.18	0.19
12	0.19	0.17	0.18
13	0.11	0.09	0.10
14	0.17	0.15	0.16
15	0.27	0.24	0.25
16	0.19	0.17	0.18
17	0.11	0.09	0.10
18	0.11	0.09	0.10
19	0.15	0.13	0.14
20	0.13	0.11	0.12
UCL	0.461	0.414	0.438
CL	0.231	0.207	0.219
LCL	0.000	0.000	0.000

The \tilde{u} control chart is obtained as shown in Figure 3.

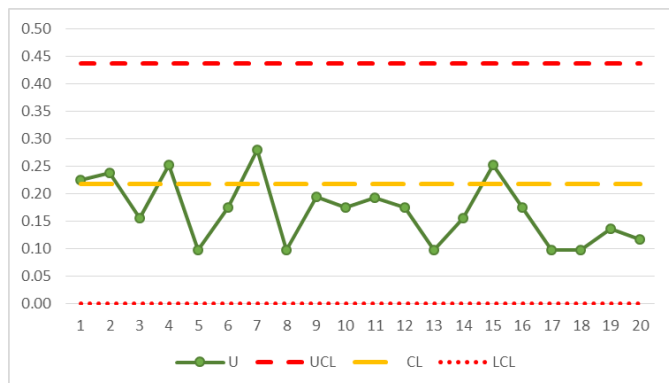


Figure 3 The \tilde{u} control chart for Brake Hoses process

As shown in Figure 3, the obtained results indicate the variability of the process is located between control limits and

the Brake Hoses process is in statistically control based on defect status.

v. Conclusions

The control charts are completely useful techniques to monitor variability of the process. It is an important issue to confirm customer expectation by decreasing process variability. Despite this effective use, knowing all the quality characteristics that are considered to be taken into account for variability is one of the most important disadvantages of control diagrams. It is clear that this cannot be possible in real case problems. It is likely to be faced with many uncertainties caused by the operator, measuring instrument, measuring system, machine or human. And in these cases, using traditional control chart will not yield accurate results. So, the fuzzy set theory can be successfully used to overcome this problem and to integrate the uncertainties into control charts. Recently, some fuzzy extensions have been proposed in order to take more advantage of the capabilities of fuzzy clusters. One of them named type-2 fuzzy sets and includes a fuzzy set for membership function has been used to improve control charts. For this aim, the u chart or control chart for nonconformities per unit has been re-designed based on type-2 fuzzy sets in this paper. The main parameters of this chart has been derivate based on type-2 fuzzy sets. The obtained fuzzy based control chart is applied on a real case problem. The obtained results show that the fuzzy based control charts provide more sensitive and flexible evaluations to follow the variability.

As a future research suggestion, the proposed methodology can be adapted other type of attribute control charts. Additionally, the other fuzzy extensions such as hesitant, intuitionistic, Pythagorean etc. can be analyzed to design of control charts.

Acknowledgement

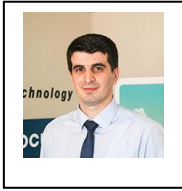
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