Influence of Train Speed on the Dynamic Behavior of the Bridge-Train system

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Abstract— The need to find a sustainable method easily understood and applied by a common construction engineer in less technically advanced countries compels us to start the work presented in this paper as a pushover method in bridge-vehicle interaction research. Handling limited scientific resources to solve complex structural engineering problems to predict and control the traffic induced vibration associated with high order differential equations is the main objective of this work. The proposed methodology models the bridge-vehicle system by considering the bridge sub model and the vehicle sub model in the system interaction model, each subsystem including interaction dynamic parametric elements associated with irregularities on the wheel-rail interface, and the resulted dynamic interaction equations are solved using higher-order and fixed-point methods. Graphical results obtained from the developed numerical approximation is easily applied and understood by local civil engineers with adequate computational wares, and can also be extended to the determination of the distribution of dynamic stresses in the bridge deck due to the bridge-train interaction effects.

Keywords— dynamic, bridge deck, vehicle, sub model, interface, wheel-rail, Interaction.

ı. Introduction

The basic purpose of modeling in bridge analysis is to provide the simplest mathematical formulation, easy applicable by structural engineers, of a bridge behavior that satisfies specified functional or design requirements for response evaluation [1].

Any sound approximation method in dynamical analysis starts with a simply supported case with different alteration parameters to assess their effect on the dynamic response. Influence of various parameters such as mass, stiffness, damping and span length are usually involved in dynamic analysis at various rates. A work produced by Hjelm and Karlsson [2] shows effects of mass, stiffness, damping, speed and span length on continuous and simply supported spans bridges ranging from the variability of deflections and accelerations to the spread of natural frequencies and of the dynamic performance level.

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The need to find a sustainable method easily understood and applied by a common construction engineer in less technically advanced countries compels us to start the work presented in this paper as a pushover method [3, 4] in bridgevehicle interaction research, handling limited resources and complex differential equations derived from the Galerkin and using the finite element method and the modal superposition method as a tool to predict and control the traffic induced vibration [5, 6]. The work outlines the proposed methodology in the bridge-vehicle system modelization by considering the bridge sub model and the vehicle sub model. With the theoretical formulation and the numerical approximation given at each stage of the system model formulation, the response motion is obtained from the obtained system dynamic equation resolution using the Wilson θ method and the Runge-Kutta fourth-order method.

The obtained numerical approximation is applied on the main span of a six-span continuous bridge build by the incremental launching method. The influence of the bridge parameters is computed using built-in contextualized Software such as MatLab [7] and CSI Bridge [8] with easier iterative and dynamic simulations adapted to each sub model. The train model includes at each dynamic node linear and angular degrees of freedom (DOF) with corresponding dynamic coefficients on the given variable [9]. The bridge structure is modeled with the Mindlin's plate theory considering irregularities on the wheel-rail interface, and the resulted equations are solved using the higher-order and fixed-point methods in order to clearly assess the vehicle-bridge interaction level.

Numerical analysis done on the proposed railway bridge on the Wouri river of Douala/Cameroon [10] for four reference speeds on the chosen spans shows graphical results of the displacement history as well as of the relative deflection during a standard train motion. From the results we can point out important facts that support the accuracy level of the proposed theoretical and approximation methodology, and that enables different actors of the construction and transport industry to implement it to easily. Even if the proposed methodology can be easily applied by local civil engineers with adequate computational wares, the work can be extended to the determination of the distribution of dynamic stresses in the bridge deck due to the bridge-train interaction in order to clearly define the variation of the critical coordinates of a system subjected dynamic loading.



п. Model Development

The bridge-train system model is obtained from individual sub models development first of the bridge structure, and there after of the vehicle model with its mechanical components, as well as its complex wheel-rail interface.

A. Bridge deck model

1) Design

The bridge is modeled using the Finite Element Method and has several different parts for which related elements are used. The bridge deck is modeled as a layered structure that includes rails, rail pads, ties, ballast and deck [11, 12].

The rail is loaded by a number of forces P_{ji} at distances $x_{w,ji}$, where i=j=1,2 and each force corresponds to a set of wheels of the vehicle. The two rails are simplified into a two-dimensional Euler-Bernoulli beams in the x-z plane, discreetly supported by a number (N_{s1}) of spring-damper mechanisms at distances $x_{s1,k}$ which take into account the fact that the rail pads are connected to the sleepers [13] as shown in figure 1.

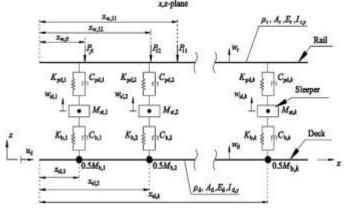


Figure 1: Computational bridge deck model

The rail has the following parameters: density ρ_r , cross sectional area A_r , Young modulus E_r , moment of inertia around the y axis $I_{r,y}$ and displacement in the z direction w_r . The spring-damper system used to model the rail pads is described by the stiffness $K_{pd,k}$ and the damping $C_{pd,k}$ coefficients, where i=j=1,2. The effects of concentrated forces and sleepers are generated using the Dirac delta $\delta(x)$ function, so the equation of motion for the rail is in the form:

$$\begin{split} &\rho_r A_r \frac{\partial^2 w_r(x,t)}{\partial t^2} + E_r I_{r,y} \frac{\partial^4 w_r(x,t)}{\partial x^4} + \sum_{k=1}^{N_{Sl}} \delta \Big(x - x_{Sl,k} \Big) K_{pd,k} \Big(w_r(x,t) - w_{Sl,k}(t) \Big) + \sum_{k=1}^{N_{Sl}} \delta \Big(x - x_{Sl,k} \Big) C_{pd,k} \Big(\frac{\partial w_r(x,t)}{\partial t} - \frac{\partial w_{Sl,k}(t)}{\partial t} \Big) = \sum_{j=1}^2 \sum_{i=1}^2 \delta \Big(x - x_{w,ji}(t) \Big) P_{ji}(t) \end{split}$$

In addition, each rail pad is connected to a cross member which has a degree of freedom $w_{sl,k}$ in the z-direction, and there are N_{sl} sleepers in the system. Each cross member has a

mass $M_{sl,k}$ and is connected to the platform by another spring damping system which takes into account the stiffness of the ballast $K_{b,k}$ and the damping $C_{b,k}$. It assumes that half of the mass of ballast $M_{b,k}$ moves with the crossbar, and the other with the bridge deck. Therefore, the total mass moving with the crossbar becomes $M_{sl,k} = M_{sl,k} + 0.5M_{b,k}$. The equation of motion for a traverse becomes:

$$\begin{split} & (K_{pd,k} + K_{b,k}) w_{Sl,k}(t) - K_{pd,k} w_r (x_{Sl,k}, t) - \\ & K_{b,k} w_d (x_{Sl,k}, t) + (C_{pd,k} + C_{b,k}) \frac{dw_{Sl,k}(t)}{dt} - \\ & C_{pd,k} \frac{\partial w_r (x_{Sl,k}, t)}{\partial t} - C_{b,k} \frac{\partial w_d (x_{Sl,k}, t)}{\partial t} + M_{sl,k} \frac{d^2 w_{Sl,k}(t)}{dt^2} = 0 \end{split}$$

The bridge structure is modeled in the x, z plane using the Euler-Bernoulli flexural beam theory. In this work we focus on plate type models. To account for the displacement of the bridge in the z-direction w_r , the following parameters are used: density ρ_d , section area A_d , Young modulus E_d , moment of inertia around the y axis $I_{d,y}$, Half of the ballast mass $M_{b,k}$ and the effects of the sleepers are taken into account by using the Dirac delta function $\delta(x)$ so as to obtain the following differential dynamic motion equation:

$$\begin{split} \rho_d A_d \frac{\partial^2 w_d(x,t)}{\partial t^2} + E_d I_{d,y} \frac{\partial^4 w_d(x,t)}{\partial x^4} + \sum_{k=1}^{N_{Sl}} \delta \left(x - x_{Sl,k} \right) K_{pd,k} \left(w_d(x,t) - w_{Sl,k}(t) \right) + \sum_{k=1}^{N_{Sl}} \delta \left(x - x_{Sl,k} \right) C_{b,k} \left(\frac{\partial w_d(x,t)}{\partial t} - \frac{\partial w_{Sl,k}(t)}{\partial t} \right) + \sum_{k=1}^{N_{Sl}} \delta \left(x - x_{Sl,k} \right) \frac{M_{b,k}}{2} \frac{\partial^2 w_d(x,t)}{\partial t^2} = 0 \end{split}$$

$$(3)$$

Moreover when considering the axial deformation of the bridge u_d , the equation of the given motion becomes:

$$E_d A_d \frac{\partial^2 u_d(x,t)}{\partial x^2} + \rho_d A_d \frac{\partial^2 u_d(x,t)}{\partial t^2} + \sum_{k=1}^{NSl} \delta(x - x_{Sl,k}) \frac{M_{b,k}}{2} \frac{\partial^2 u_d(x,t)}{\partial t^2} = 0$$

$$(4)$$

2) Formulation

We combine the three equations of rails, sleepers and ballast with the deck, taking into account the differentiation of related variables and the action of the applied moving point load $P_{i,j}$, to obtain the numerical sub model of the bridge as the following expression:

$$\begin{split} &\rho_{r}A_{r}\frac{\partial^{2}\omega_{r}(x,t)}{\partial t^{2}}+E_{r}I_{r,y}\frac{\partial^{4}\omega_{r}(x,t)}{\partial x^{4}}+\rho_{d}A_{d}\frac{\partial^{2}\omega_{d}(x,t)}{\partial t^{2}}+\\ &E_{d}I_{d,y}\frac{\partial^{4}\omega_{d}(x,t)}{\partial x^{4}}+\sum_{k=1}^{N_{S}l}\Bigl(\delta\bigl(x-x_{Sl,k}\bigr)\frac{1}{2}M_{b,k}\frac{\partial^{2}\omega_{d}(x,t)}{\partial t^{2}}+\\ &M_{Sl,k}\frac{d^{2}\omega_{Sl,k}(t)}{dt^{2}}\Bigr)=\sum_{j,i=1}^{2}\Bigl(\delta\Bigl(x-x_{\omega_{ji}}(t)\Bigr)P_{ji}\Bigr) \end{split} \tag{5}$$



B. Train model

1) Design

The train is modeled by a locomotive pulling identical wagons. The locomotive (or wagon) is simplified to a two-dimensional system consisting of two layers of shock-spring suspension systems in which the vertical and pitch rotation of the bogies and the vehicle body are included [9, 12, 14].

The body of the vehicle has a mass M_c , and a moment of inertia I_c , and two degrees of freedom: vertical displacement w_c and pitch angle φ_c . It is connected to the bogies by a secondary suspension with rigidity $K_{su,2}$ and damping $C_{su,2}$, at distances L_2 from the axis of the vehicle.

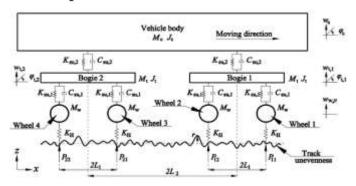


Figure 2: The multi-degree of freedom vehicle model

The equation of motion for the vertical displacement of the vehicle body is as follows:

$$\begin{split} 2K_{su,2}w_c(t) - K_{su,1}w_{t,1}(t) - K_{su,2}w_{t,2}(t) + \\ 2C_{su,2}\frac{dw_c(t)}{dt} - C_{su,2}\frac{dw_{t,1}(t)}{dt} - C_{su,2}\frac{dw_{t,2}(t)}{dt} - \\ M_c\frac{d^2w_c(t)}{dt^2} = 0 \end{split}$$

In addition, the first and second bogies are composed with the primary and secondary suspensions, with primary suspension having the stiffness $K_{\mathfrak{su},1}$ and damping $C_{\mathfrak{su},1}$. Each bogie has a mass $M_{\mathfrak{t}}$, a moment of inertia $J_{\mathfrak{t}}$, a vertical displacement $w_{\mathfrak{t},j}$ and a pitch angle $\varphi_{\mathfrak{t},j}$. The vehicle running of the vehicle on the bridge is governed by:

$$\begin{split} & \left(2K_{su,1}+K_{su,2}\right) w_{t,j}(t) - K_{su,1} w_{w,j1}(t) - \\ & K_{su,1} w_{w,j2}(t) + \left(2C_{su,1}+C_{su,2}\right) \frac{dw_{t,j}(t)}{dt} - \\ & C_{su,1} \frac{dw_{w,j1}(t)}{dt} - C_{su,1} \frac{dw_{w,j2}(t)}{dt} - M_t \frac{d^2 w_c(t)}{dt^2} = 0 \quad j = 1,2 \end{split}$$

Each wheel is attached to the primary suspension and has a vertical degree of freedom $w_{w,ji}$ and mass M_w . Each wheel is also affected by the wheel-rail interaction force P_{ji} and a quarter of the force $F_{total} = M_{total}g$, where M_{total} is the total mass of the vehicle and g is the gravitational constant [13]. Thus, the equations of motion of the wheels become:

$$\begin{split} K_{su,1}w_{w,ji}(t) - K_{su,1}w_{t,j}(t) + (-1)^{i}L_{1}K_{su,1}\varphi_{t,j}(t) + \\ C_{su,1}\frac{dw_{w,ji}}{dt} - C_{su,1}\frac{dw_{t,j}(t)}{dt} + (-1)^{i}L_{1}C_{su,1}\frac{d\varphi_{t,j}(t)}{dt} - \\ M_{w,ji}\frac{d^{2}w_{w,ji}(t)}{dt^{2}} = P_{ji}(t) + \frac{1}{4}F_{total}, \ \ j,i = 1,2 \end{split}$$
(8)

2) Formulation

As before, we combine the equations of vehicle body parts connected to suspensions and to the wheels to order obtain the following numerical sub model equations of the vehicle motion:

$$2K_{su,2}\omega_c(t) + 2C_{su,2}\frac{d}{dt}\omega_c(t) - (M_c + 2M_t)\frac{d^2}{dt^2}\omega_c(t) - M_{\omega_{ji}}\frac{d^2}{dt^2}\omega_{\omega_{ji}}(t) = P_{ji} + F_{total}$$
(9)

c. The Bridge-Train System Model

The system modeling is performed using the Finite Element Method (FEM) [15]. The displacement and velocity are expressed as a linear function of the acceleration according to the Newmark method (with $\delta = 1/2$ and $\theta = 1/6$). Thus we are interested in the overall system of the bridge-train interaction as:

$$\begin{cases} \dot{Z}_{i+1} = \dot{Z}_i + \frac{\Delta t}{2} \left(\ddot{Z}_{i+1} + \ddot{Z}_i \right) \\ Z_{i+1} = Z_i + \Delta t \dot{Z}_i + \frac{\Delta t^2}{6} \left(\ddot{Z}_{i+1} + 2 \ddot{Z}_i \right) \end{cases}$$
(10)

where all variables are known at the instant i, therefore these equations can be rewritten in the discrete form as:

$$F_{v}(\ddot{Z}_{v,i+1}) = A_{v}\ddot{Z}_{v,i+1} + f_{v,i+1}(\ddot{Z}_{v,i+1}, \ddot{Z}_{str,i+1}) - b_{v,i} = 0$$

$$F_{str}(\ddot{Z}_{str,i+1}) = A_{str}\ddot{Z}_{str,i+1} + f_{str,i+1}(\ddot{Z}_{v,i+1}, \ddot{Z}_{str,i+1}) - b_{str,i} = 0$$

$$A_{\alpha} = \left(M_{\alpha} + \frac{\Delta t}{2}C_{\alpha} + \frac{\Delta t^{2}}{6}K_{\alpha}\right)$$

$$b_{\alpha,i} = \left[q_{\alpha} - C_{\alpha}\left(\dot{Z}_{\alpha,i} + \frac{\Delta t}{2}(\ddot{Z}_{\alpha,i+1} + \ddot{Z}_{\alpha,i})\right) - K_{\alpha}\left(Z_{\alpha,i} + \Delta t\dot{Z}_{\alpha,i} + \frac{\Delta t^{2}}{6}(\ddot{Z}_{\alpha,i+1} + 2\ddot{Z}_{\alpha,i})\right)\right]$$
(11)

For $\alpha \equiv v \text{ or str}$, the two previous systems of equations are coupled by the contact vector. Indeed, the contact forces P_{ji} are inside the two vectors $f_{v,i+1}$ and $f_{str,i+1}$ with opposite signs.

The general forms of the differential equations selected from the models are written for the bridge and vehicle/train structures respectively as:

$$M_{str}\ddot{Z}_{str} + C_{str}\dot{Z}_{str} + K_{str}Z_{str} - f_{\frac{v}{str}}(Z_v, Z_{str}) = q_{str}$$
 ad

$$M_v \ddot{Z}_v + C_v \dot{Z}_v + K_v Z_v - f_{\frac{str}{v}}(Z_v, Z_{str}) = q_v$$

$$\tag{11}$$



And the relative displacement taken into account the interaction is given by the following expression:

$$w_{rel,ji}(t) = w_{w,ji}(t) - w_r(x_{w,ji}(t), t) - r(x_{w,ji}(t))$$
(12)

where:

 $\mathbf{w}_{rel,fi}(\mathbf{t})$ Relative movement of the train with respect to the bridge;

 $x_{\omega_{ji}}(t)$ Position/abscissa of wheel *i* attached to suspension *j*;

 $\omega_r(x,t)$ Vertical movement of the rail pad at the x position and the date t;

 $\omega_{\omega_{ji}}(t)$ Vertical displacement of the wheel *i* attached to the suspension *j*;

 $\mathbf{r}(\mathbf{x})$ Rail surface imperfection at the x-axis of the bridge.

This formula highlights the movement of the train, the displacement of the deck and irregularities of the rails.

The positive sign is obtained for: pulling suspensions of the train; the excessive bending of the bridge deck relative to the position of the train and the presence of hollows at the interface contact of the wheels with the rails. These situations must be combined to obtain the overall case of the positive sign, which expresses a zero interaction energy between the vehicle and the structure because the two are not in contact at this moment.

The negative sign is obtained for: the compression of the suspensions of the train; the lower bending of the bridge deck relative to the position of the train and the presence of bumps at the contact of the wheels with the rails. These situations must be combined in order to obtain the global case of the sign, which expresses a positive interaction energy between the vehicle and the structure because the two are in contact at that moment. This contact is stabilizing when it tends towards the steady state

The null value is obtained by combining previous cases so that the wheels brush the rails. More generally, we say that there are compensation effects. This state is comparable to that of an unstable equilibrium of the heavy pendulum.

We thus propose a more general expression of the system model of the bridge-train interaction as:

$$\begin{split} &\rho_{r}A_{r}\frac{\partial^{2}\omega_{r}(x,t)}{\partial t^{2}}+E_{r}I_{r,\mathcal{V}}\frac{\partial^{4}\omega_{r}(x,t)}{\partial x^{4}}+\rho_{d}A_{d}\frac{\partial^{2}\omega_{d}(x,t)}{\partial t^{2}}+\\ &E_{d}I_{d,\mathcal{V}}\frac{\partial^{4}\omega_{d}(x,t)}{\partial x^{4}}+\sum_{k=1}^{NSl}\left(\delta\left(x-x_{Sl,k}\right)\frac{1}{2}M_{b,k}\frac{\partial^{2}\omega_{d}(x,t)}{\partial t^{2}}+\\ &M_{Sl,k}\frac{d^{2}\omega_{Sl,k}(t)}{dt^{2}}\right)=\sum_{j,i=1}^{2}\left\{\delta\left(x-x_{Sl,k}\right)\left[2K_{Su,2}\omega_{c}(t)+2C_{Su,2}\frac{d}{dt}\omega_{c}(t)-(M_{c}+2M_{t})\frac{d^{2}}{dt^{2}}\omega_{c}(t)-M_{\omega_{ji}}\frac{d^{2}}{dt^{2}}\omega_{\omega_{ji}}(t)-F_{Total}\right]\right\} \end{split}$$

ш. The Resolution of the Model

There are several numerical soft wares to approximate the Vehicle-Structure interaction at different accuracy level

depending on the static/dynamic parameter under consideration. Approximation methods of dynamic differential equations of such interaction can be divided into two groups: the superposition method and the direct integration methods [16]. This work, based on the second group, have been developed and applied on the resolution of the dynamic equation considering the non-linearity of the vehicle input force. The Newton-Raphson method as a relevant iterative model is used to solve the system equation and to construct a singular equation that includes the variables of both vehicle and bridge:

$$F(\ddot{Z}_{v,i+1}) = A\ddot{Z}_{i+1} + f_{i+1}(\ddot{Z}_{i+1}, \ddot{Z}_{i+1}) - b_i = 0$$
where:
$$f_{i+1} = \begin{bmatrix} -f_{v,i+1}(\ddot{Z}_{v,i+1}, \ddot{Z}_{str,i+1}) \\ -f_{v,i+1}(\ddot{Z}_{v,i+1}, \ddot{Z}_{str,i+1}) \end{bmatrix}$$
(15)

This system simplifies the data of a system to $N_{eq} = 10 + 2n + 2m$ equations, 10 being the number of degrees of freedom of the vehicle, n the number of nodes considered on the rail and m the number of sleepers taken into account.

For the rest of the procedure we must first determine the Jacobian of the system variables in order to apply further transformations:

transformations:
$$J(\ddot{Z}_{i+1}^k) = A + \begin{bmatrix} 0_{6\times 6} & 0_{6\times (4+2n)} & 0_{6\times 2m} \\ 0_{(4+2n)\times 6} & R(\ddot{Z}_{i+1}^k)_{(4+2n)\times (4+2n)} & 0_{(4+2n)\times 2m} \\ 0_{2m\times 6} & 0_{2m\times (4+2n)} & 0_{2m\times 2m} \end{bmatrix}$$

$$(16)$$

with
$$R_{l,c} = \frac{\partial f_{l+6}}{\partial \ddot{z}_{l+1,c+6}}, \quad 1 \le l \le (4+2n), \quad 1 \le c \le (4+2n).$$

Once the Jacobian is determined the value of the acceleration can be deduced as the ultimate value of the following iterative sequence:

$$J(\ddot{Z}_{i+1}^{k})(\ddot{Z}_{i+1}^{k+1} - \ddot{Z}_{i+1}^{k}) = -F(\ddot{Z}_{i+1}^{k})$$
 (17)

where k represents the Raphson iteration order and the corresponding instant.

The calculation time of this method is considerable, especially if the dimensions of the elements are high. Thus, to reduce the machine time required by the Jacobian factorization of this method, we propose an algorithm following the fixed point method to approach a less expensive resolution.

The discretization of the global system allowed us to build a numerical method of resolution of the differential system obtained at the end of the modeling of each subsystem. The models developed here in general, for the rest of the work, is implemented in the case of the new railway bridge on the Wouri river [10, 17].



IV. Numerical Simulation

A. Data Preparation

The numerical model of the bridge deck, as well as that of the train, takes into account data based on a parameterized definition of its geometrical and mechanical characteristics. The characteristic data of the vehicle are defined according to the European railway standard namely UIC 71 and SW loading systems [17, 18], in order to execute computations that are in conformity to the existing actual project of the second railway bridge on the Wouri river. Thus, data adopted in this work are retrieved from characteristics of the Wouri river bridge project as shown in figure 3.

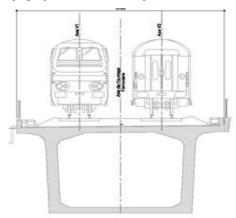


Figure 3. Cross section of the railway bridge system model of the Wouri river Bridge Project.

The first bridge on the Wouri river was built in 1954 with sixteen simply-supported pre-stressed concrete (PC) girders of 45 m length/span, 14.60 m width, and carries one railroad between two single-line highway, each with a sidewalk. The second Bridge, consisting of two-railway line (box girder of 10.1 m wide and 742.4 m of total span) and highway (cell box girder of 25.5 m wide and 752 m of total span) bridges, was commissioned on December 2017 after 4 years of construction. The railway bridge has two simple supported spans of 21.2m, one at each end of the bridge, and a six-span continuous bridge at its middle spanning $91\text{m}+4\times129.5+91\text{m}$. For an academic comparison purpose, we perform the simulation considering two working lengths (90 m and 130 m) and four vehicle speed values (50 km/h, 70 km/h, 100 km/h and 120 km/h).

The bridge deck the following numerical values are adopted as: the Young modulus $E_d = 34.10^9 \ Pa$, the shear modulus $G_d = 13.10^9 \ Pa$, the density $\rho_d = 2400 \ Kg/m^3$, the inertia modulus $I_{d,y} = 4.07 \ m^4$, the cross section $A_d = 7.52 \ m^2$. The rail properties are the rigidity $K_{pd,i} = 1.2.10^8 \ N/m$ and the damping constant $C_{pd,i} = 1.24.10^5 \ N.s/m$. The wood sleepers properties are the mass $M_{Sl,j} = 237 \ Kg$ and their spacing $L_{Sl} = 0.6m$. The ballast material properties are the mass $M_{b,j} = 683 \ Kg$, the rigidity $K_{b,j} = 2.4.10^8 \ N/m$ and the damping constant $C_{b,j} = 5.88.10^4 N.s/m$. The railway vehicle

properties described above are the total, secondary and primary masses $F_{Total} = -6.8.10^5~N$, $K_{su,1} = 3.28.10^6 N/m$, $K_{su,2} = 1.31.10^6~N/m$, $C_{su,1} = 9.10^4~N.\frac{s}{m}$, $C_{su,2} = 3.10^4~N.s/m$, the Hertzian constant $K_H = 8.70.10^{10}~N/~m^{3/2}$, the vehicle sewing mass $M_c = 5.35.10^4~Kg$, the boggle mass $M_t = 3260~Kg$, the wheel mass $M_w = 2000~Kg$, the car section moment of inertia $J_c = 2.24.10^6~Kg.m^2$, the boggle section moment of inertia $J_t = 2.45.10^3~Kg.m^2$, the primary and secondary suspensions spacing $L_1 = 1.5~m$ and $L_2~7~m$.

B. Results and Interpretation

In the simulation process, we were interested in the interaction of the two subsystems. It is taken according to the differences of displacement amplitudes of the two subsystems at identical instants. This interaction makes it possible to translate the stability of the overall bridge-train system. With the increase of the speed and the presence of the imperfections of the rails surface, instabilities arise at the level of the interaction of the bridge and the train as seen in figure 4.

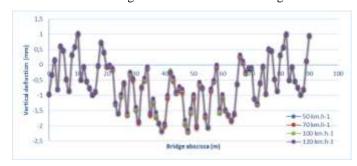


Figure 4. Relative motions of the vehicle on a 90 m long span bridge.

The analysis of the graph thus reveals that the system is all the more stable as we approach the neighborhood of the midspan for a significant range and for low speeds. Despite the bending of the structure, and low vibration of the vehicle, we note that the system tends to stabilize in the vicinity of the midspan. The interaction is positive because it contributes to the stability of the general system. It is also noted that the low flexibility of the span ends allows to record negative interaction peaks due to times when the sum of the effects of all actions does not contribute to stabilize the system due to the rigidity of the work.

When considering the 130 m span length in figure 5, and looking at vertical displacements history, we notice that the system now has greater instabilities near the mid span. The length of the span enhances vibrations, and might have a considerable impact on the interaction of the overall stability. Indeed, the appearance of the positive peaks at the relative displacement curves is caused by the vibration of the bridge along its relative static deflection curve. This vibration, which is coherent with that of the vehicle at a certain moment causes conjugated effects which result in the instability of the system.



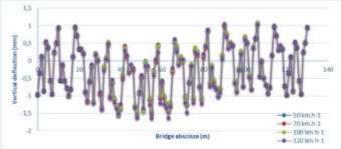


Figure 5. Relative motions of the vehicle on a 130 m long span bridge

The two subsystems have greater stability in the vicinity of mid-span. However, for the 90 m span, the interaction energy deduced from these displacements will be greater than that obtained for a span of 130 m. Thus one can deduce that the stability is inversely proportional to the span range for a fixed speed.

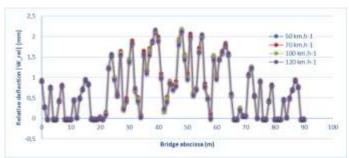


Figure 6. Vehicle-Bridge Interaction on the 90m long span bridge.

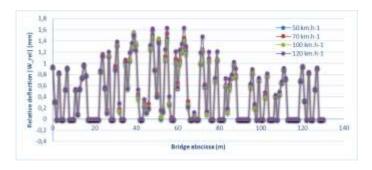


Figure7. Vehicle-Bridge Interaction on the 130m long span bridge

The results obtained allowed us to highlight the vibratory behavior of the bridge-train interaction system observed in figures 6 and 7 for various velocities. Indeed, the velocity of the vehicle influences the frequency of wheel-rail contacts. It is this wheel-rail contact which, under the action of the friction that results, allows the vehicle to move on the rails. It is necessary to emphasize that the friction of the bridge-train interaction system is enhanced by the surface irregularities of the rails.

Increasing the speed of the vehicle causes an increase in the frequency of the friction during wheel-rail contact. This friction causes the excitation of the structure. The increase of the frequency of the friction causes the increase of the frequency of the vibrations of the structure.

v. Conclusions

Increase the awareness of the bridge-vehicle interaction of field civil engineers during the design and construction process is the main purpose achieved in this work through an implementation of simplified theoretical and numerical model subdividing the global system into two independent and interconnected subsystems, each subsystem leading to an appropriate sub model that is in return used to develop the system model. Each subsystem locally has its own specific dynamic characteristics such as the mass, the rigidity and the damping with structural and mechanical properties of the interface behavior being kept during modeling in order to preserve the consistency and the accuracy of the modeling process both locally and globally. The proposed methodology outlines the theoretical formulation and the numerical approximation given at each stage of the subsystem and the system modelization using the two-dimensional Euler-Bernoulli beam theory associated with the Mindlin's plate theory considering irregularities on the wheel-rail interface, and resulting in higher order differential dynamic equations that are solved using higher-order and fixed-point approximation methods in order to clearly assess the vehiclebridge interaction level.

Numerical analysis done on the proposed railway bridge on the Wouri river for four reference speeds on the chosen spans displays results of the displacement history as well as of the relative deflection during the motion. From the results we can point out these important facts: there is a correlation between the vertical displacement amplitudes and the train position on the span; for each of the span length the global system remain dynamically stable as the train approaches the vicinity of the middle span notwithstanding the importance of wheel-rail interface imperfections; the existence of the critical train position on the deck and its critical occurrence time is an important factor of the train-bridge interaction; the existence of the dynamic friction in the wheel-rail interface enhances the appearance of surface irregularities of the rail and the wheels; increasing the speed of the vehicle causes an increase in the frequency of friction during wheel-rail contact. Even if the proposed methodology can be easily applied by local civil engineers with adequate computational wares, the work can be extended to the determination of the distribution of dynamic stresses in the bridge deck due to the bridge-train interaction in order to clearly define the variation of the critical coordinates of a system subjected dynamic loading.



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