

# Influence of the stall torque on a vibro-impact system with non-ideal excitation

Dzanko Hajradinovic, Miodrag Zukovic, Ivana Kovacic

**Abstract**— A vibro-impact system driven by an DC electric motor is analyzed. The vibro-impact system is consisting of a crank-slider mechanism and one oscillator attached to it. Oscillator can impact in a fixed wall. The impact model has been assumed as inelastic, causing the change of velocity in sign and intensity by the restitution factor. Mathematical model which describes this system is a coupled system of second order nonlinear differential equations. The aim is to determine the influence of the stall torque on the motion of the oscillator. The system of differential equations is solved numerically. The results of the analysis are summarized in diagrams which are showing the relationship between the stall torque and the maximum amplitude of the oscillator movement, and the average value of excitation frequency of the electric motor. Frequency response diagram is also formed. Based on the diagrams regions of impact and non-impact solutions are distinguished, and regions with multiple solutions. For four different values of the stall torque displacement diagrams and the excitation versus time diagrams are shown.

**Keywords**—*Vibro-impact systems; Stall torque; Impact solutions; Non-ideal excitation; Limited power supply.*

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## I. Introduction

Systems driven by electric motors appear in many engineering applications. One of the applications where it can be found are vibro-impact systems. If the system can affect the behavior of the electric motor as the source of excitation such systems are in literature described as systems with non-ideal excitation. The vibro-impact system, which is analyzed in this paper, is driven by a non-ideal power source. The characteristic of the electric motor is assumed to be linear. The power of the electric motor is influenced by the stall torque. Influence of the stall torque on the vibro-impact system is analyzed in this paper. It will be shown that for small values of the stall torque the system behaves more as a system with non-ideal excitation, where for higher values it behaves almost like a system with ideal excitation. A system with a motion barrier can work as an vibro-impact system or a system that oscillates but for some values of parameter it can go in a vibro-impact regime. One regime of work is related to impact solutions and the other to non-impact solutions, only the right value of the stall torque can

give the desired motion of the system. Extensive research has been conducted in the field of vibro-impact systems where the excitation of the system is constant as well as in the field of dynamical non-impact systems with non-ideal excitation [1-14]. However, only few papers have been published related to vibro-impact systems with non-ideal excitation [15-17] where the influence of the stall torque on the system is analyzed. The use of a vibro-impact attachment as a dynamic absorber in a non-impact system with non-ideal excitation is analyzed in [15]. In [16], a vibro-impact system with non-ideal excitation with three degrees of freedom is investigated, where the impact is modelled via a stiff spring where the influence on the motion of the system by electric motor parameters is not analyzed. The motion observed is a relative motion between two bodies impacting each other. Numerical analysis is used in overall research because of the strong nonlinearity of the problem. Possible use of vibro-impact systems as an attachment in chaos control is analyzed in [17] for non-impact systems with non-ideal excitation. The necessity to carry out a run up and close down numerical simulations for obtaining all possible results is also pointed out in [1-6].

The physical model is a crank-slider mechanism containing a vibro-impact system having the form of a spring-mass oscillator that can hit a base. The aim is to determine the interaction between a driving torque and the motion of the oscillator from the viewpoint of the influence of the stall torque on the regions of impact solution. The examination is based on the mathematical model of the driving torque of the electric motor. The parameters in the model of the electric motor is the no-load angular velocity and the stall torque. The no-load angular velocity will be held at constant values, where the value of the stall torque is changed. For small values of the stall torque the power of the electric motor will be small which results that the system behaves as a system with non-ideal excitation. As the magnitude of the stall torque is increased to some large value this will affect the excitation to behave very close to a system with ideal excitation.

The paper is organized in four sections. Section II contains the description of a mechanical model of the system under consideration and the derived differential equations of motion are shown. Section III presents the results and discussion of the numerical analysis. The paper ends up with the section of conclusions summarized in Section IV.

## II. Mechanical model and equations of motion

A vibro-impact system under consideration (Figure 1.) consists of a crank-slider mechanism and a mass-spring system (oscillator) attached to it. The crank-slider mechanism consists of an eccentric drive that is assumed to

be symmetrical or balanced. The system parameters are given in Figure 1.a): the lengths of the rods are  $OA = a$ ,  $AB = l$ ; mass of rod AB is neglected; the moment of inertia of the rod OA with respect to the axis through O is  $J$ ; the mass of the slider is  $m_B$  and of the oscillator  $m_C$ ; the stiffness of the spring is  $c$  and the length of an underformed spring is  $l_0$ . The mass  $m_C$  can collide with a stop (base) in certain regimes. Friction between the oscillator C and the contact surface is introduced as viscous friction force. It will be assumed that the driving torque  $T(\dot{\varphi})$  depends on the angular velocity of the eccentric drive.

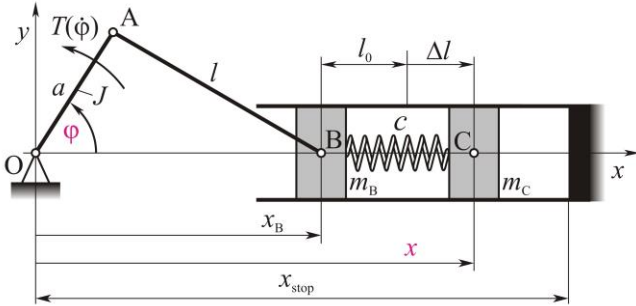


Figure 1. Vibro-impact system - mechanical model of the system under consideration

The system has two degrees of freedom with the generalized coordinates being the displacement of the oscillator  $x$  and the angle of rotation of the eccentric drive  $\varphi$ , as shown in Figure 1.

The driving torque is assumed to be a linear function of the angular velocity of the eccentric drive:

$$T(\dot{\varphi}) = T_0 \left(1 - \frac{\dot{\varphi}}{\Omega_0}\right) \quad (1)$$

Equation (1) describes how the driving torque changes with the angular velocity, and it represents the linear characteristic of the driving DC electric motor. This is presented in Figure 2. The increase of the stall torque  $T_0$ , as shown in Figure 2., rotates clockwise the characteristic of the DC electric motor. For a very large values of  $T_0$ , this characteristic becomes vertical (dashed line), which represents the case of ideal excitation.

The equations of motion can be expressed as:

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = \omega^2 l_0 + \omega^2 u(\varphi)$$

$$\begin{aligned} (m_B u'(\varphi)^2 + J)\ddot{\varphi} + m_B \dot{\varphi}^2 u''(\varphi) \\ = T_0 \left(1 - \frac{\dot{\varphi}}{\Omega_0}\right) + c u'(\varphi)(x - l_0 - u(\varphi)) \end{aligned} \quad (2)$$

where:

$$\omega^2 = \frac{c}{m_C}, 2\delta = \frac{b}{m_C} \quad (3)$$

To define the velocity  $\dot{x}_B$ , a function  $u(\varphi)$  that describes the motion of the slider B, is introduced:

$$\dot{x}_B = u(\varphi) \quad (4)$$

The motion of the slider B,  $u(\varphi)$  Figure 3., and the corresponding derivatives are given by:

$$u(\varphi) = a \cos \varphi + l \sqrt{1 - \left(\frac{a}{l}\right)^2 \sin^2 \varphi} \quad (5)$$

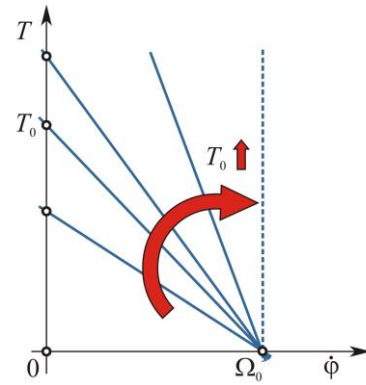


Figure 2. Characteristics of driving torque as a function of the angular velocity  $\dot{\varphi}$  for different values of the parameter  $T_0$

For the sake of simplicity, it is of interest to find the approximation of the function  $u(\varphi)$ . This can be done by developing it into series as follows:

$$u(\varphi) = l + (l \cos \varphi)\epsilon - \left(\frac{l}{4}(1 - \cos 2\varphi)\right)\epsilon^2 + O(\epsilon^3) \quad (6)$$

where the following small parameter  $\epsilon$  is introduced:

$$\epsilon = \frac{a}{l}$$

The first approximation of the function  $u(\varphi)$  is given by:

$$u(\varphi) = l + a \cos \varphi \quad (7)$$

while its derivatives are:

$$u'(\varphi) = -a \sin \varphi, u''(\varphi) = -a \cos \varphi \quad (8)$$

In the case of the first approximation, a simplified model of the system can be formed as shown in Figure 3.

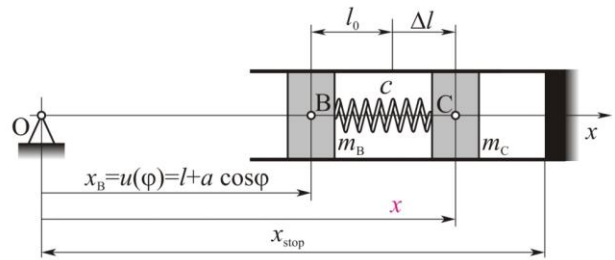


Figure 3. Simplified mechanical model of the system under consideration

The corresponding equations of motion in the first approximation are:

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = \omega^2(l_0 + l) + \omega^2 a \cos \varphi$$

$$\begin{aligned} (m_B a^2 \sin^2 \varphi + J)\ddot{\varphi} + m_B \dot{\varphi}^2 a^2 \sin \varphi \cos \varphi \\ = T_0 \left(1 - \frac{\dot{\varphi}}{\Omega_0}\right) - ca \sin \varphi (x - l_0 - l - a \cos \varphi) \end{aligned} \quad (9)$$

Given the form of Eq. (9) and the existing coupling between the generalized coordinates  $\varphi$  and  $x$ , the angular velocity will not be constant, but it will change in time. The system of differential equations (9) represents a system of coupled nonlinear differential equations of motion. They cannot be solved analytically in exact form, but their solutions can be found by carrying out numerical simulations or utilizing approximate analytical methods.

As noted in Figure 1.b) and Figure 3., the oscillator can hit the base, where the coordinate marked by  $x_{\text{stop}}$  represents the position of a fixed wall at which the oscillator can impact. If the impact occurs periodically during the

oscillating motion of the attachment, the system corresponds to a vibro-impact system.

The impact condition, based on the stop position can be described as follows:

$$\begin{aligned} x(t^-) &= x(t^+) = \Delta \\ \dot{x}(t^+) &= -k \dot{x}(t^-) \end{aligned} \quad (10)$$

By involving the impact condition, additional nonlinearity is involved in the system described by the coupled system of nonlinear differential equations (9).

The system parameters used to in these considerations are given in Table I.

TABLE I. SYSTEM PARAMETERS

$m_C$	0.35	[kg]	$l$	0.12	[m]
$m_B$	0.2	[kg]	$l_0$	0.36	[m]
$c$	$9\pi^2$	[N/m]	$k$	0.5	[-]
$b$	0.5	[Ns/m]	$T_0$	0-5	[Nm]
$a$	0.04	[m]	$\omega$	$\sqrt{c/m_C}$	[rad/s]
$J$	0.06	[kg m <sup>2</sup> ]	$\Omega_0$	$1.5\omega$	[rad/s]

### III. Results and discussion

The analysis of the influence caused by the variation of the stall torque on the vibro-impact system is done using numerical analysis. The main goal is to show that for one specific values of the no-load angular velocity  $\Omega_0$  of the driving torque and different values of the stall torque various solution can be obtained. As mentioned in the introduction and by [1-6] run up and close down simulations are conducted. These simulations are done in the matter that for the initial value of  $T_0$  the initial conditions are  $x_C = l + l_0$ ,  $\dot{x}_C = 0$ ,  $\varphi = 0$  and  $\dot{\varphi} = 0$  and for every next step the initial condition are taken to be the last values at which the previous simulation stopped. The run up simulation is marked on the diagrams that follows with (black) circles, where the close down simulation is marked with (magenta) x. All analysis is related to steady state motion which will occur after dissipation of the energy by friction force.

Through numerical analysis the relationship between the stall torque and the values of the frequency ratio  $\Omega_{avg}/\omega$  is obtained and the results are shown in Figure 4. In this figure the average value of the excitation frequency is used. The no-load angular velocity of the driving torque is chosen to be

$\Omega_0 = 1.5\omega$ . For small values of the stall torque the average value of the excitation frequency is below the no-load angular velocity. This region represents the region where the system behaves as a system with non-ideal excitation. Also in this region for the simulation of run up and close down double solution are obtained. When the average value of excitation frequency comes closer to the value of no-load angular velocity the system behaves like a system with ideal excitation. The transition point is close to the value  $T_0 = 4$ . After this value the average value of the excitation frequency stays the same for every value of the stall torque that follows.

Diagram shown in Figure 5, shows how the displacement is changing with varying the value of the stall torque. The upper full (green) line represents the position of the stop which has the value:

$$x = x_{stop} = l + l_0 + \Delta \quad (11)$$

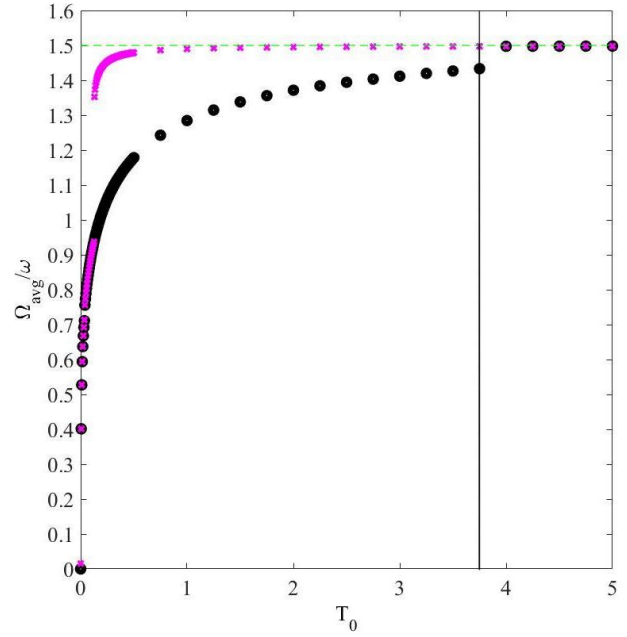


Figure 4. Relation between frequency ratio  $\Omega_{avg}/\omega$  and the stall torque  $T_0$

Two different regions can be distinguished from the diagram shown in Figure 5.

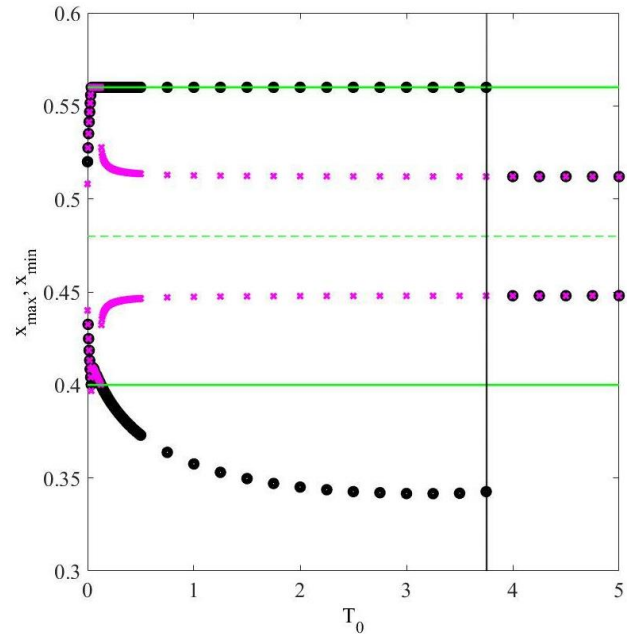


Figure 5. Dependence of the oscillator extreme motion values  $x_{max}, x_{min}$  and the stall torque  $T_0$

Region where two solutions exists, and the region where only one solution is existing for the run up and close down simulation. The region with two solutions is related to small values of the stall torque. As mentioned early, this region is the region where the system behaves like a system with non-ideal excitation. In this region for the same value of  $T_0$  the oscillator can hit the stop or the maximum movement of it is lower than  $x_{stop}$ . This means that one solution represents an impact solution and the other a non-impact solution. This possibility can be adjusted by choosing the right initial conditions of the system. For higher values of the stall

torque only non-impact solutions exists. This information is very important because of the desired work regime of the system. If the designer of the system wants the system to operate in an impact regime of work, the parameter of stall torque must be chosen in the first region. Additionally, he must choose the right initial conditions also.

From the two previous diagrams where the relation of the average excitation frequency and the extreme values of the displacement are shown in reference to the stall torque a frequency response diagram is also formed. This diagram is shown in Figure 6. The run up and close down simulation show that at certain point jumps on this diagram occurs. Jumps in the frequency response diagrams are specific for all nonlinear systems, but in the quantitative change of the displacement value. For this system the jump is qualitative in the sense that change both in the value of the displacement magnitude and the excitation frequency occurs. Multiple solutions are occurring in a short region above the value  $\Omega_{avg}/\omega = 1.3$  and there can be impact or non-impact solutions. For the value higher than  $\Omega_{avg}/\omega = 0.7$  till almost to  $\Omega_{avg}/\omega = 1.5$  impact solutions are existing. This diagram, based on the average values of the excitation frequency shown in Figure 4. is formed from the region till the value  $T_0 = 5$ , or the region for every value of the stall torque where the system behaves like a system with non-ideal excitation. At the value close to  $\Omega_{avg}/\omega = 1.5$  solutions for every value  $T_0 > 5$  is contained.

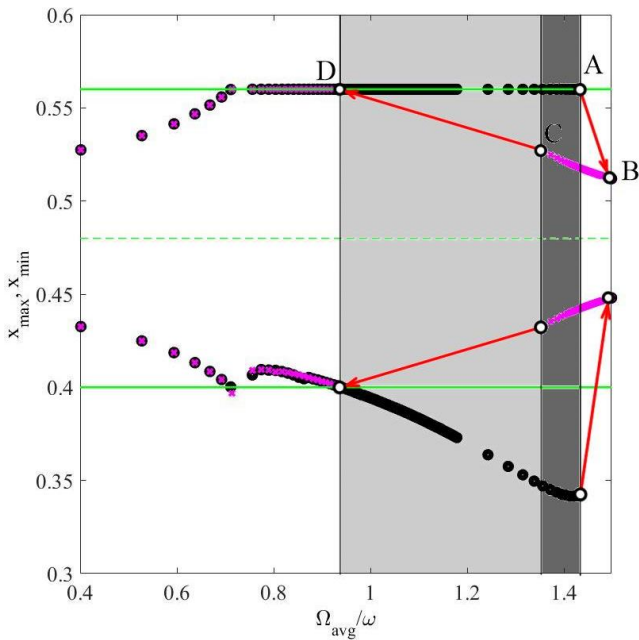


Figure 6. Frequency response diagram

The empty space on Figure 6. Is related to the step in numerical analysis, if the step is smaller than the space would be filled but the analysis would take much longer and no new qualitative data would be gathered.

Based on the previous diagrams impact and non-impact solutions can occur for different values of stall torque. To investigate this in a deeper manner diagrams of the displacement and excitation frequency in time for specific values of the stall torque are shown in Figure 7. Left diagrams represent the displacement diagrams in time, where on the right diagrams  $\dot{\varphi}/\omega$  vs  $t$  are shown. In Figure 7.a) for the value  $T_0 = 0.02$  non-impact solutions are

obtained and the excitation frequency is smaller than the no-load angular velocity value.

For the higher value of the stall torque  $T_0 = 2$ , shown in Figure 7.b) impact solution is occurring and the excitation frequency comes very close to the value of the no-load angular frequency. The response in time of the excitation frequency is much faster.

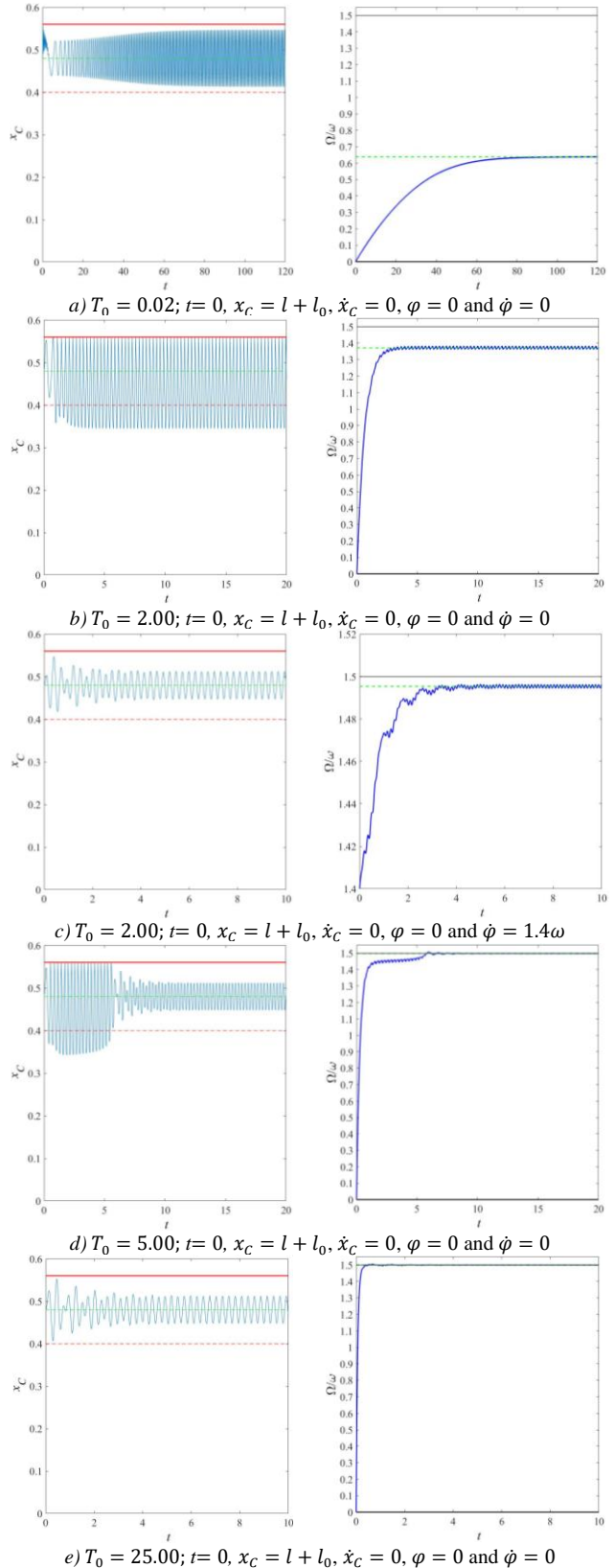


Figure 7. Diagrams  $x - t$  and  $\dot{\varphi}/\omega - t$  fro different values of  $T_0$



Based on Figure 6. for a specific value of the stall torque two solutions can occur which depends on the initial conditions as it was mentioned. In Figure 7.c) in reference to the solution shown in Figure 7.b) for the specific value of the stall torque  $T_0 = 2$  an non-impact solution is shown for a different set of initial conditions.

In Figure 7.d) the oscillator behaves so that for a short time is operating in a vibro-impact manner than changes to non-impact motion. This instant relates also to the excitation frequency where a jump happens from one value to another higher value of excitation frequency. The dashed (green) line represents the average values of the excitation frequency in all right diagrams given in Figure 7.

Figure 7.e) represent the case where the stall torque is high which makes the power of the excitation source high and makes the system to operate very close to the behavior of a system with ideal excitation. The excitation frequency response is almost at an instant. The oscillator operates in a non-impact manner as can be seen on the left diagram in Figure 7.e).

#### iv. Conclusion

The influence of the stall torque on a vibro-impact system with non-ideal excitation is analysed. The vibro-impact system consists of a crank-slider mechanism with a mass-spring oscillator system attached to it. The system is driven using a DC electric motor. The characteristic of the electric motor is presented as a linear function of the excitation frequency with two influencing parameters. One parameter is the no-load angular velocity of the electric motor which is held constant, and the other parameter is the stall torque. The analysis is conducted using numerical simulations. In the simulations the stall torque is the variable parameter for which the behaviour of the oscillator is analysed. The results of the analysis are represented in different diagrams.

Through run up and close down analysis diagrams that are showing the change of the excitation frequency and extreme displacement of the oscillator in reference to the stall torque are formed and shown in the paper. From this two diagrams the two regions are noticed and pointed out. These two regions divide the behaviour of the system where for some range of the stall torque the system behaves like a system with non-ideal excitation. These behaviour is noted for small values of the stall torque. The second region is related to higher values of the stall torque and the system behaves like a system with ideal excitation. In this case the average value of excitation frequency is equal to the value of the no-load angular velocity of the electric motor. In the region of smaller values of the stall torque multiple solutions are obtained for the run up and close down simulations. One solution represents an impact solution, where the other one represents the non-impact solution. The dependence in which the system will operate is related to the initial conditions. For specific values of the initial condition the system will operate in a vibro-impact regime.

Based on the results a frequency response diagram is formed. The frequency response diagram shows the dependence of the extreme displacement values of the oscillator in reference to the excitation frequency values that

are in the region of small values of the stall torque. Or the diagram is related only to the values of the stall torque where the system operates as a system with non-ideal excitation. Qualitative jumps are pointed out for the run up and close down simulation. Jumps occurs so that a change in the excitation frequency and displacement amplitude of the oscillator happens. As it is pointed out by [6] the jump happens, on the frequency response diagram, from a unstable to a stable solution.

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