

Soliton solutions of Schrödinger Equation with Quadratic Cubic non linearity and its Stability

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ABSTRACT: In this work, We achieved optical soliton and solitary wave solutions of resonant nonlinear schrödinger equation having quadratic-cubic nonlinearity by employing extended simple equation method, which have key applications in engineering and applied physics. We also given the movement of some achieved solutions graphically, which facilitates to recognize the physical phenomena's of this non-linear model.

Keywords: Extended Simple equation method, nonlinear schrödinger equation with quadratic-cubic nonlinearity, Solitons, periodic solutions.

1 Introduction

In many branches of engineering and applied science, non-linear phenomena exist naturally. Several non-linear evolution equations (NLEEs) have been derived to explain these real nonlinear situations. The nonlinear Schrödinger equations in NLEEs have been considered as the models to illustrate some non-linear phenomenon in different areas such as condensed matter physics, fluids, fluid dynamics, quantum mechanics, plasmas, optics, biophysics and particle physics [1–7]. The studies of the analytical solutions in different form for example solitons, solitary wave and so on to different types of nonlinear Schrödinger equations have become a very significant practice by various scholars [8]. The solutions of NLSEs play a very important role in solving the problems of real-life. Soliton for the different NLSEs have been searched to investigation the non-linear phenomena, where the solitons might be whichever dark or bright on the information of the governing NLSEs [9–12], and dark one is a mark with a characteristic phase step crossways it, while the amplitude of bright soliton is peak [13].

In the previous many decades, the non-linear dynamics of optical solitons are structured by the famous NLSE [14] which has been investigated widely. Ahead of 1997, researchers have been focused almost on the optical solitons in restricted non-linear media having refractive index (RI)

at a specific position is only associated to intensity of the beam at same position. The nonlinear nonlocal model was projected by the authors in [15] that permits the RI of a object at a specific point to be associated to the intensity of beam at every other points of objects in the year of 1997. Consequently, the nonlocal solitons attained much concentration, and numerous new solitons are established, such as gray solitons [16], Gaussian and higher-order Gaussian solitons [15], vector Laguerre–Gaussian solitons [17], multipole and dipole solitons [18], surface solitons [19] and so on [20, 21]. The solitons in different reveal various distinctive properties, such as shift of large phase [22], self-induced mode transformation [23], attraction between dark solitons [24], long-range interaction [25]. These days various engineering methods have been build to investigate the non-linearity of the non-linear media [26].

The searching of exact solitary wave and solition solutions of NLSEs have been investigated broadly and deeply in different aspects since it was derived. Several powerful and systematic methods have been developed to achieve the solitary wave and soliton solutions such as modified extended direct algebraic method, semi-inverse variational principle, Jacobi elliptic function method [7], Kudryashov method, Simple equation method, similarity transformations method, Darboux transformation, rational expansion method, expansion method, auxiliary equation method, exp-function method, Binary Bell polynomials, spectral collection method and many more [27–35, 37–42]. The dynamical behavior of the NLSEs were investigated by the authors in [?]. The exact solitary wave solutions of the longitudinal wave motion equation in a non-linear magneto-electro-elastic circular rod were constructed via employing the enhanced (G'/G) -expansion method [43] and more it was utilized to gain analytical solutions of the (2+1)-dimensional Painleve integrable Burgers equation and the Drinfel'd-Sokolov-Wilson equation. In [44], the extension of the (G'/G) -expansion method has been employed to obtain the solitary wave solutions of the modified KdV equation.

In many branches of physics, modulation instability (MI) is fundamental procedure [5, 6, 28], which arises as a outcome of the collaboration between diffraction in the spatial domain or dispersion in the time domain and nonlinearity. The acumen in the media of non-Kerr has verified to be an immensely exciting study field in MI, for scrutinize promulgation of an optical pulse in NLSE of higher order [46]. The significant cause behind is that the modulation instability may be supposed as, to the ancestor of soliton construction.

In the current article, the exact solitons and solitary wave solutions in different form of resonant NLSE, for quadratic-cubic nonlinearity are constructed by the described extended simple equation scheme. The achieved analytical solutions are novel and more generalized.

The rest of this article is structured as follows: The theoretical model is illustrated in Section 2. The key steps of modified F-expansion scheme are presented in Section 3. In Section 4, the appliance of the described scheme is presented. In Section 5, the MI analysis is used to discuss the stability of model. The results and discussion of achieved exact solutions are discussed in Section 6. In Last section, the conclusion is revealed.

2 Theoretical Model

The observing resonant non-linear Schrödinger equation with Quadratic Cubic (QC) nonlinearity is written in its dimensionless form as

$$iq_t + a_1 q_{xx} + a_2 \frac{(|q|)_{xx}}{|q|} q + (a_3 |q| + a_4 |q|^2) q = i\lambda (|q|^{2m} q)_x + i\alpha q_x + i\nu (|q|^{2m})_x q + i\theta |q|^{2m} q_x + \sigma \frac{(q^*)_{xx}}{|q|^2} q^2 = 0, \quad (1)$$

where the function $q(x, t)$ is dependent that shows the complex valued wave profile with $i = \sqrt{-1}$. The dependent variables x and t are the variables of temporal and spatial correspondingly. Furthermore the parameter a_1 and a_2 are the group velocity dispersion and Boham potential for chiral solitons with quantum hall cause respectively. The terms a_3 and a_4 are QC non-linear together. The α is signifies inter-model dispersion, also λ accounts for self-steepening having short pulses while ν and θ are associated to non-linear dispersion. Lastly γ provides the relativistic effect in Plasmas. The parameter m is the full nonlinearity.

3 Description of Method

In this part, we will describe the algorithm of extended simple equation for finding the exact solutions of NLEEs. In generally, assume NLESE having independent variable x and t as

$$F(q, q_t, q_x, q_{xx}, q_{xxx}, \dots) = 0, \quad (2)$$

where the function $q(x, t)$ is unknown and polynomial function is F having some specified functions or variables, which have linear and non-linear of highest order derivative terms of $q(x, t)$ and be able to reduce into a polynomial function

via using transformation in which the independent variables can be combine into a complex variable. The main steps of this technique are as:

Step I: Considering travelling wave transformation to alter independent variables into single variable as

$$q(t, x) = V(\xi), \quad \xi = \gamma x + \tau t \quad (3)$$

where γ and τ are wave length and frequency. through utilizing transformation (3), the equation (2) is shrink into ODE as

$$P(V, V', V'', V''', \dots) = 0, \quad (4)$$

where prime(') shows derivative of V with respect to ξ and P is polynomial of $V(\xi)$.

Step II: Consider the solution of equation (4) in the following form as:

$$V(\xi) = \sum_{i=-N}^N b_i (\phi(\xi))^i, \quad (5)$$

where the real constants are b_i and n . $\phi(\xi)$ gratifies the giving below ansatz equation

$$\phi'(\xi) = c_0 + c_1 \phi(\xi) + c_2 \phi^2(\xi) + c_3 \phi^3(\xi), \quad (6)$$

where the arbitrary constants are c_0, c_1, c_2 and c_3 .

Step III: The positive integer N is generally obtained via utilizing the homogeneous balancing principle on equation (4), and the coefficients $b_{-N}, b_{-N+1}, \dots, b_0, b_1, \dots, b_N, \gamma, \tau$ are parameters can be obtained.

Step IV: putting equation (5) and equation (6) into equation (4) and putting the coefficients of dissimilar powers of $\phi^i(\xi)$ to zero, gives way a system of algebraic equations. Mathematica software is utilized to solve this system of equations, then the solution of parameters can be obtained.

Step V: substituting the parameters value achieving in previous step into equation (5) then solutions of equation (4) can be attained.

4 Soliton Formation of resonant NLSE

As the equation (1) is complex, so we suppose the solution in the form traveling wave of equation (1) as

$$q(x, t) = V(\xi)e^{i\mu}, \quad \mu = kx + \beta t + \epsilon, \quad (7)$$

where the amplitude component of the wave profiles is $V(\xi)$, phase factor is μ and k, ϵ, β symbolize the solitons frequency, the phase constant, the wave number respectively. Putting equation (7) into equation (1) and splitting into parts, give way as

$$(a_1 \gamma^2 + a_2 \gamma^2 - \gamma^2 \sigma) V'' + (k\alpha + a_1 k^2 - \beta + k^2 \sigma) V + a_3 V^2 + a_4 V^3 + (k\theta + k\lambda) V^{1+2m} = 0. \quad (8)$$

$$\gamma(\theta + \lambda + 2m\lambda + 2m\nu) V^{2m} - 2a_1 k\gamma + \alpha\gamma - 2\gamma k\sigma - \tau = 0. \quad (9)$$

Substitute equation (9) into equation (8), we have

$$(a_1\gamma^2 + a_2\gamma^2 - \gamma^2\sigma) V'' + V (a_1k^2 - \beta + k\alpha + \sigma k^2 + (k\theta + k\lambda) \left(\frac{2a_1k\gamma - \alpha\gamma + 2k\gamma\sigma + \tau}{\gamma\theta + \gamma\lambda + 2m\gamma\nu + 2m\lambda\gamma} \right)) + a_3V^2 + a_4V^3 = 0. \quad (10)$$

Utilizing homogeneous balancing principle on equation (10) and consider the solution of equation (10) is in the form as

$$V(\xi) = \frac{b_{-2}}{\phi^2(\xi)} + \frac{b_{-1}}{\phi(\xi)} + b_0 + b_1\phi(\xi) + b_2\phi^2(\xi). \quad (11)$$

Substituting equation (11) together with equation (6) into equation (10) and taking the coefficients of dissimilar powers of $\phi^i(\xi)$ to zero, we attained a algebraic system of Equations in parameters $b_{-2}, b_{-1}, b_0, b_1, b_2, c_0, c_1, c_2, c_3, k, \nu, \omega, \theta, m$ and β . Mathematica 9.0.1 is utilized to solve this algebraic system. The following cases of solutions are achieved as

Case 1: $c_0 = c_3 = 0$,

$$\tau = \left(\sqrt{2}a_3 (9a_4 (a_1k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu)) - 2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu)) / (27a_4k(\theta + \lambda) \sqrt{a_4c_1^2(\sigma - a_1 - \alpha_2)} \right), \gamma = -\frac{\sqrt{2}a_3}{3\sqrt{a_4c_1^2(-a_1 - a_2 + \sigma)}},$$

$$b_{-2} = b_{-1} = 0, b_0 = -\frac{2a_3}{3a_4}, b_1 = -\frac{2a_3c_2}{3a_4c_1}, b_2 = 0. \quad (12)$$

$$\tau = \left(\sqrt{2}a_3 (2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu) - 9a_4 (a_1k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu))) / (27a_4k(\theta + \lambda) \sqrt{a_4c_1^2(\sigma - a_1 - a_2)} \right), \gamma = \frac{\sqrt{2}a_3}{3\sqrt{a_4c_1^2(-a_1 - a_2 + \sigma)}},$$

$$b_{-2} = b_{-1} = b_2 = 0, b_0 = 0, b_1 = \frac{2a_3c_2}{3a_4c_1}. \quad (13)$$

The following soliton solutions of equation (1) are obtained from solution (12) as

$$q_{11}(x, t) = \frac{(2a_3)}{3a_4 (c_2 e^{c_1(d+\xi)} - 1)} e^{i(kx+\beta t+\epsilon)}, c_1 > 0. \quad (14)$$

$$q_{12}(x, t) = -\frac{2a_3}{3a_4 (c_2 e^{c_1(d+\xi)} + 1)} e^{i(kx+\beta t+\epsilon)}, c_1 < 0 \quad (15)$$

More soliton solutions of equation (1) are obtained from solution (13) as

$$q_{13}(x, t) = -\frac{2a_3c_2e^{c_1(d+\xi)}}{3a_4 (c_2 e^{c_1(d+\xi)} - 1)} e^{i(kx+\beta t+\epsilon)}, c_1 > 0. \quad (16)$$

$$q_{14}(x, t) = -\frac{2a_3c_2e^{c_1(d+\xi)}}{3a_4 (c_2 e^{c_1(d+\xi)} + 1)} e^{i(kx+\beta t+\epsilon)}, c_1 < 0. \quad (17)$$

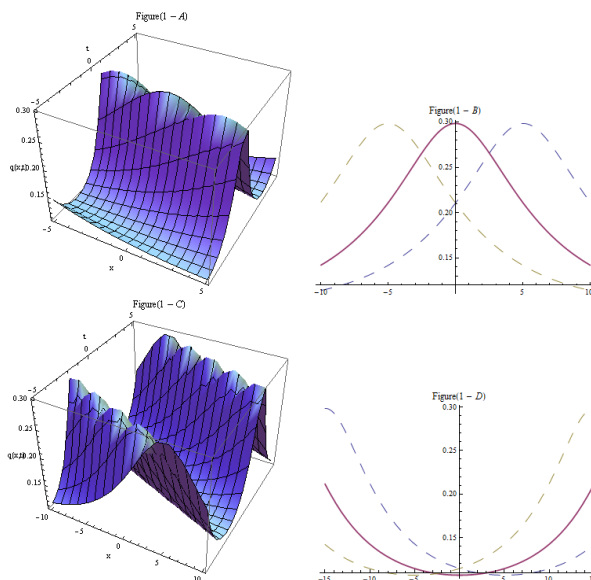


Figure 1: Solitons are drawn in different forms of Case 1 solutions.

Case 2: $c_1 = c_3 = 0$,

$$\tau = (a_3 (9a_4 (a_1k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu)) - 2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu)) / (27\sqrt{2}a_4k(\theta + \lambda) \sqrt{a_4c_0c_2(a_1 + a_2 - \sigma)}),$$

$$b_{-2} = b_{-1} = 0, b_0 = -\frac{a_3\sqrt{c_0}}{3a_4}, b_1 = -\frac{ia_3\sqrt{c_2}}{3a_4\sqrt{c_0}},$$

$$b_2 = 0, \gamma = -\frac{a_3}{3\sqrt{2}a_4c_0c_2(a_1 + a_2 - \sigma)}. \quad (18)$$

$$b_{-2} = b_{-1} = 0, b_0 = \frac{a_3\sqrt{c_0}}{3a_4}, b_1 = \frac{ia_3\sqrt{c_2}}{3a_4\sqrt{c_0}},$$

$$b_2 = 0, \gamma = -\frac{a_3}{3\sqrt{2}\sqrt{-a_4c_0c_2(-a_1 - \alpha_2 + \sigma)}},$$

$$\tau = (a_3 (9a_4 (a_1k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu)) - 2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu)) / (27\sqrt{2}a_4k(\theta + \lambda) \sqrt{-a_4c_0c_2(-a_1 - \alpha_2 + \sigma)}). \quad (19)$$

The following solitary wave solutions of Eq.(1) are obtained from solution (18) as:

$$q_{21}(x, t) = -\frac{a_3 (1 + i \tan(\sqrt{c_0c_2}(d + \xi)))}{3a_4} e^{i(kx+\beta t+\epsilon)}, c_0c_2 > 0. \quad (20)$$

$$q_{22}(x, t) = -\frac{ia_3 (\tan(\sqrt{c_0c_2}(d + \xi)) - i)}{3a_4} e^{i(kx+\beta t+\epsilon)}, c_0c_2 < 0. \quad (21)$$

More solutions in the form of solitary wave solutions are constructed of equation (1) from solution (19) as

$$q_{23}(x, t) = \frac{ia_3 (\tan(\sqrt{c_0}\sqrt{c_2}(d + \xi)) + i)}{3a_4} e^{i(kx + \beta t + \epsilon)}, \quad c_0 c_2 > 0. \quad (22)$$

$$q_{24}(x, t) = \frac{ia_3 (\tan(\sqrt{c_0}\sqrt{c_2}(d + \xi)) + i)}{3a_4} e^{i(kx + \beta t + \epsilon)}, \quad c_0 c_2 < 0. \quad (23)$$

We achieve the following exact solution of equation (1) from solutions (24) and (25) as

$$q_{31}(\xi) = \left(3\gamma\sqrt{2a_4}(\sigma - a_1 - a_2) \left(\sqrt{4c_0c_2 - c_1^2} \tan\left(\frac{\sqrt{4c_0c_2 - c_1^2}}{2}(d + \xi)\right) - 2c_1 \right) - 2a_3 \right) / (6a_4) e^{i\mu}, \quad 4c_0c_2 > c_1^2. \quad (26)$$

$$q_{32}(\xi) = \left(\frac{\gamma\sqrt{4c_0c_2 - c_1^2}\sqrt{\sigma - a_1 - a_2} \tan\left(\frac{\sqrt{4c_0c_2 - c_1^2}}{2}(d + \xi)\right)}{\sqrt{2}\sqrt{a_4}} - \frac{a_3}{3a_4} \right) e^{i(kx + \beta t + \epsilon)}, \quad 4c_0c_2 > c_1^2. \quad (27)$$

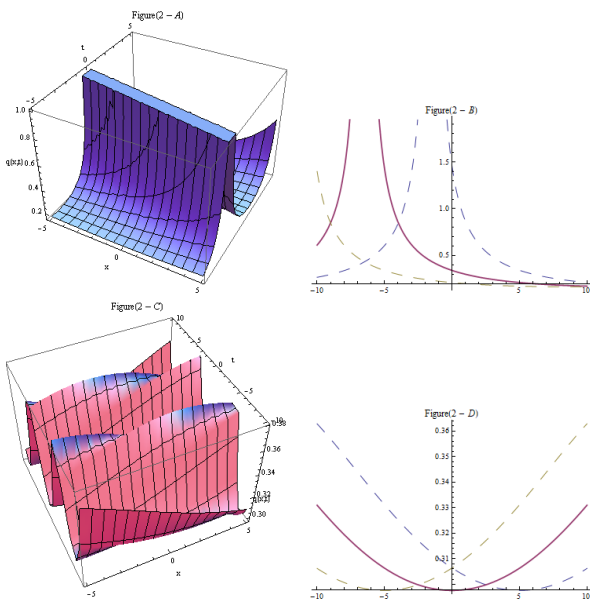


Figure 2: Solitary wave and soliton in various forms are plotted of Case 2 solutions.

Case 3: $c_3 = 0$,

$$b_{-2} = b_{-1} = 0, b_0 = -\frac{3c_1\gamma\sqrt{2a_4}\sqrt{-a_1 - a_2 + \sigma} + 2a_3}{6a_4},$$

$$b_1 = -\frac{\sqrt{2}\gamma c_2\sqrt{\sigma - a_1 - a_2}}{\sqrt{a_4}}, c_0 = \frac{9a_4\gamma^2 c_1^2 (\sigma - a_1 - a_2) - 2a_3^2}{36a_4\gamma^2 c_2 (\sigma - a_1 - a_2)},$$

$$b_2 = 0, \tau = (\gamma (2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu) - 9a_4 (a_1 k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu)))) / (9a_4 k(\theta + \lambda)). \quad (24)$$

$$b_{-2} = b_{-1} = 0, b_0 = \frac{3\sqrt{2a_4}\gamma c_1\sqrt{\sigma - a_1 - a_2} - 2a_3}{6a_4},$$

$$b_1 = \frac{\sqrt{2}\gamma c_2\sqrt{\sigma - a_1 - a_2}}{\sqrt{a_4}}, c_0 = \frac{9a_4\gamma^2 c_1^2 (\sigma - a_1 - a_2) - 2a_3^2}{36a_4\gamma^2 c_2 (\sigma - a_1 - a_2)},$$

$$b_2 = 0, \tau = (\gamma (2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu) - 9a_4 (a_1 k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu)))) / (9a_4 k(\theta + \lambda)). \quad (25)$$

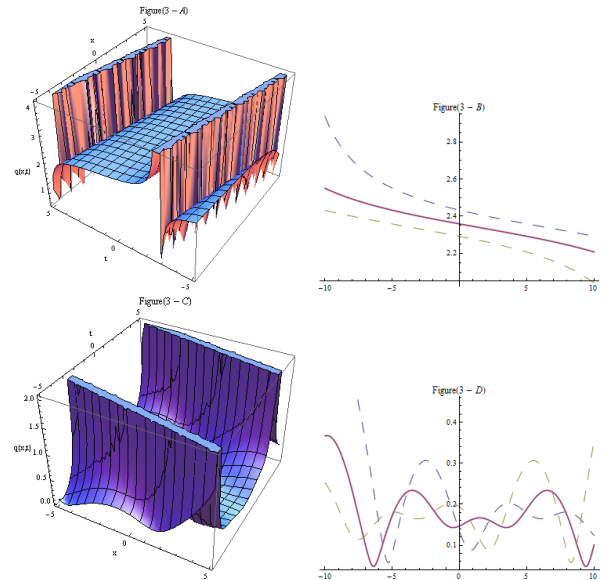


Figure 3: Different shapes of solitary wave solutions are drawn of Case 3.

Case 4: $c_0 = c_2 = 0$,

$$b_{-2} = b_{-1} = 0, b_0 = -\frac{2a_3}{3a_4}, b_1 = 0,$$

$$b_2 = -\frac{2a_3 c_3}{3a_4 c_1}, \gamma = -\frac{ia_3}{3\sqrt{2a_4}c_1^2 (a_1 + a_2 - \sigma)},$$

$$\tau = (ia_3 (9a_4 (a_1 k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) + k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda + 2\lambda m + 2m\nu)) - 2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu))) / \left(27\sqrt{2a_4}k(\theta + \lambda)\sqrt{-a_4 c_1^2 (-a_1 - a_2 + \sigma)} \right). \quad (28)$$

$$\begin{aligned}
 b_{-2} = b_{-1} = 0, b_0 &= \frac{2a_3c_3n^2}{3a_4c_1}, b_1 = 0, \\
 b_2 &= \frac{2a_3c_3}{3a_4c_1}, \gamma = -\frac{ia_3}{3\sqrt{2a_4c_1^2(a_1 + \alpha_2 - \sigma)}}, \\
 \tau &= (ia_3(9a_4(a_1k^2(3\theta + 3\lambda + 2\lambda m + 2m\nu) \\
 &+ k(3k\sigma(\theta + \lambda) + 2m(\lambda + \nu)(\alpha + k\sigma)) - \beta(\theta + \lambda \\
 &+ 2\lambda m + 2m\nu)) - 2a_3^2(\theta + \lambda + 2\lambda m + 2m\nu))) \\
 &/ \left(27\sqrt{2a_4k(\theta + \lambda)}\sqrt{-a_4c_1^2(-a_1 - \alpha_2 + \sigma)} \right). \quad (29)
 \end{aligned}$$

The following solutions in the form of solitary wave are constructed from solution (28) of equation (1) as

$$q_{41}(\xi) = \frac{2a_3 \left(\frac{c_3 e^{2c_1^{3/2}\xi}}{c_1(c_3 e^{2c_1\xi} - 1)} - 1 \right)}{3a_4} e^{i\mu}, \quad c_1 > 0. \quad (30)$$

$$q_{42}(\xi) = \frac{2a_3 \left(\frac{-c_3 e^{2(\sqrt{-c_1+1})c_1\xi}}{c_1(e^{2c_1\xi} - c_3)} - 1 \right)}{3a_4} e^{i\mu}, \quad c_1 < 0. \quad (31)$$

In the similar way, more exact solutions of equation (1) are also obtained from solution (29) as

$$q_{43}(x, t) = \frac{2a_3c_3e^{2c_1^{3/2}\xi}}{3a_4c_1 - 3a_4c_1c_3e^{2c_1\xi}} e^{i(kx + \beta t + \epsilon)}. \quad (32)$$

$$q_{44}(x, t) = \frac{2a_3c_3e^{2(\sqrt{-c_1+1})c_1\xi}}{3a_4c_1(e^{2c_1\xi} - c_3)} e^{i(kx + \beta t + \epsilon)}. \quad (33)$$

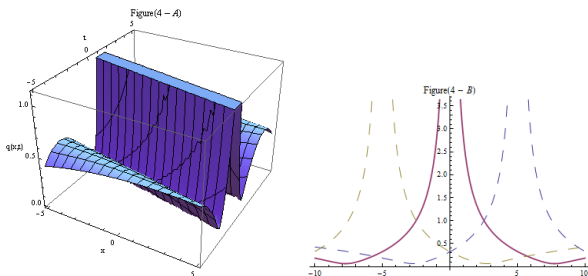


Figure 4: Solitary wave solution in different forms is drawn of Case 4 solution.

5 Results and Discussion

The achieved exact solutions via the proposed scheme are dissimilar from the constructed solutions of different researchers through existing techniques due to the supposed equation (6) of the proposed scheme is different from the existing schemes. Various special form of solutions such as trigonometric, hyperbolic trigonometric and rational functions are achieved of equation (6) by grants special values to parameters. The researchers in [47] employed semi-inverse variational principle to obtain resonant soliton solutions of equation (1). So, our achieved analytical solutions are novel and have not constructed in literature.

The Figure 1 illustrates exact Solitons in different forms are pinched of Case 1. The figure(1-A) and (1-C) demonstrate the bright soliton and periodic aoliton of solutions (14) and (15) respectively, and the figure(1-B) and figure(1-D) evaluate of same solutions in one-dimension at same values of parameters. The Figure 2 indicates Solitary wave and solitons in different forms are plotted drawn. The figure(2-A) and figure(2-C) illustrates the solitary wave and soliton of solutions (20) and (21) respectively, and the Figure(2-B) and Figure(2-D) evaluate of the same solutions in one-dimension at same values of parameters. The Figure 3 indicates exact Solitary waves in different shapes are drawn. The figure(3-A) and figure(3-C) illustrates the periodic solitary waves of solutions (26) and (27) respectively, and the Figure(3-B) and Figure(3-D) evaluate of the same solutions in one-dimension at same values of parameter. The Figure 4 evaluate Solitary wave in various shapes are plotted. The Figures(4-A) show solitary wave of solution (31) and the Figure(4-B) signify same solution in one-dimension at same values of parameter.

6 Conclusion

We have been effectively utilized the modified extended simple equation technique to get optical soliton and solitary wave solution to the resonant NLSE with quadratic-cubic nonlinearity that have key application in engineering, physics and applied Mathematics. we have also shown the some obtained solutions graphically, which facilitates to recognize the physical phenomena of this nonlinear model. All the achieved results are novel and does not exist in literature also. The power of current method shows it can be applied in future to solve many problems arises in engineering and applied physics.

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