A ROM-Less Direct Digital Frequency Synthesizer Based on Bezier Curve Approximation

Ravinder Singh, Kathika Roy and C Bhattacharya

Defence Institute of Advanced Technology

Pune, India

Abstract—**This paper describes the design and implementation of a ROM-Less Direct Digital Frequency Synthesizer (DDS) using Bezier curve approximation. With Bezier curve approximation, phase values between 0 to π are mapped to sine amplitudes. Then, half wave symmetry of sine wave is exploited to construct full sine wave. The proposed approximation introduces maximum error of 7.9×10^{-6}, which is equivalent to quantized sinusoid with 16-bit amplitude resolution. This yields very high spectrally pure sine wave output. Based on the approximation, DDS circuit is designed and validated in Matlab-Simulink.**

Keywords—**Direct Digital Frequency Synthesizer, ROM-Less DDS, Bezier curve approximation.**

I. INTRODUCTION

Direct Digital Frequency Synthesizer (DDS) plays a significant role in modern digital communication and electronic instruments. DDS is preferred because of its significant advantages over Phase Locked Loop (PLL) such as sub-hertz frequency resolution, fast switching time, low phase noise, continuous-phase frequency switching and unparalleled matching and control of I and Q synthesized outputs [1]. A typical DDS consists of a phase accumulator (PA) and a sine/cosine generator (phase to sine conversion) as shown in Fig. 1.
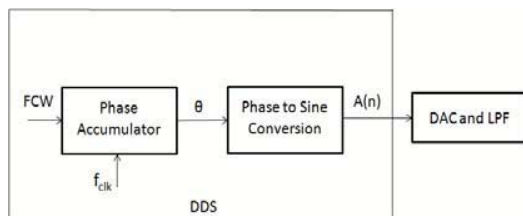

Figure 1.   Simplified block diagram of a DDS.

Phase accumulator is N-bit register. An N-bit Frequency Control Word (FCW), M and clock frequency, f_{clk} control the phase accumulator output. Phase accumulator output, n is N-bit digital sweep whose value ranges from 0 to 2^N-1 and its slope depends upon FCW. These 2^N (0 to 2^N-1) output values of phase accumulator are mapped to phase values, θ as

$$\theta = 2\pi(n/2^N). \qquad (1)$$

Phase value, θ varies from 0 to 2π. The phase accumulator output is fed to sine generator which gives digital samples of sine wave as

$$A(n) = \sin(2\pi(n/2^N)). \qquad (2)$$

These digital samples of sine wave are presented to Digital to Analog Converter (DAC). DAC generates an analog waveform in response to the digital samples of sine wave. Generally, a Low Pass Filter (LPF) is placed after DAC to attenuate the higher frequency components. The fundamental frequency, f_{out} of output sine wave is [2]

$$f_{out} = f_{clk}.(FCW/2^N). \qquad (3)$$

Phase to sine amplitude conversion in sine generator is realized by either ROM (look up table) methods or ROM-Less computational methods [3]. Using larger ROM leads to higher power consumption and lower speed, which is undesirable feature for any electronic circuitry. Due to this reason, there is ever increasing demand of keeping memory size to minimum. To achieve it, different ROM compression techniques are employed [4]. The Sunderland technique based upon trigonometric identities, offers a ROM compression ratio of 59:1. Nicholas architecture uses least square approximation to split the ROM into two smaller memories: a coarse one that computes the sine function with high error and a fine one to interpolate the values of coarse ROM to exact value. It achieves a compression ratio of 128:1. Taylor series approximation [5] offers compression ratio of 128.8:1.

An alternative approach, which employs no ROM (look up table) but computes sine samples from phase values is called ROM-Less or computational method. Various techniques fall in this category are CORDIC, Taylor series, parabolic approximation and polynomial approximations. As this approach avoids use of ROM, so guarantees high speed and low power consumption. In this approach, phase values and mapped to amplitudes of sine wave by a sine function. The error in computation of sine amplitude by these different methods is restricted to maximum of 1.89×10^{-5} [6].

In this paper, a new and more accurate, Bezier curve approximation based design for DDS is introduced. The phase values from phase accumulator are mapped to sine amplitudes by Bezier curve approximation. Next section describes the idea of Bezier curve approximation for generation of sine function. Further on, digital

implementation of approximation and its validation is given in detail.

II. Bezier Curve Approximation

A Bezier curve is parametric curve described by polynomial based on control points [7]. A general Bezier equation is written as

$$B(t) = \sum_{i=0}^{n} b_{i,n}(t)P_i, \ t \in [0,1] \qquad (4)$$

where the polynomials

$$b_{i,n}(t) = C_i^n \, t^i \, (1-t)^{n-i}, \qquad (5)$$

$$C_i^n = \frac{n!}{i!(n-i)!} \qquad (6)$$

and points P_i are called control points.

A curve is formed by connecting the Bezier points with lines, starting with P_0, normally not passes through any of other control points and terminating at P_n is called Bezier curve. In general, every curve can be described by Bezier curves with some minimal error. A Bezier curve with n = 6 is used to approximate sine function. Equation (7) gives the half cycle of sine wave as

$$A(t) = (1-t)^6 P_0 + 6t(1-t)^5 P_1 + 15t^2(1-t)^4 P_2 + 20t^3(1-t)^3 P_3 + 15t^4(1-t)^2 P_4 + 6t^5(1-t)P_5 + t^6 P_6 \qquad (7)$$

where t ranges from 0 to 1.

To generate the sine wave having unit amplitude, values of control points P_0 to P_6 are calculated and are shown in Table I. P_0 and P_6 are kept equal to zero to reduce the terms of Bezier equation. $P_1 = P_5$ and $P_2 = P_4$ are due to fact that sine wave is symmetrical about $\pi/2$.

TABLE I.   Values Of Control Points

Control Points	Value
P_0	0
P_1	0.5237
P_2	1.0464
P_3	1.3162
P_4	1.0464
P_5	0.5237
P_6	0

As parameter 't' in (7) varies from 0 to 1 and phase angle 'θ' varies from 0 to π for first half of sine wave, so replacing 't' with 'θ/π' and substituting values of control points in (7) leads to

$$A(t) = Sin(\theta) = 3.14222(\theta/\pi)(1 - (\theta/\pi))^5 + 15.69544(\theta/\pi)^2 (1 - (\theta/\pi))^4 + 26.32417(\theta/\pi)^3(1 - (\theta/\pi))^3 + 15.69544(\theta/\pi)^4 (1 - (\theta/\pi))^2 + 3.14222(\theta/\pi)^5(1 - (\theta/\pi)). \qquad (8)$$

Simplification of (8) gives

$$Sin(\theta) = 3.14222(\theta/\pi) - 0.015669(\theta/\pi)^2 - 5.03539(\theta/\pi)^3 - 0.52663(\theta/\pi)^4 + 3.65319(\theta/\pi)^5 - 1.217743(\theta/\pi)^5. \qquad (9)$$

For phase value, θ between 0 to π, maximum error between real sine wave and proposed approximation is limited to maximum of 7.9×10^{-6}. Fig. 2 shows the approximation error between real sine wave and sine wave generated by proposed approximation.
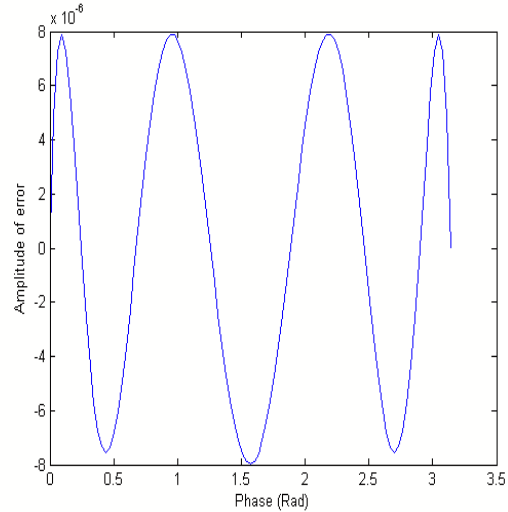

Figure 2.   Approximation errors for Bezier curve approximation.

Table II. summarizes the comparison of various sine function approximation techniques along with the proposed approximation. The maximum approximation error in sine amplitude computation by the proposed technique is restricted to maximum of 7.9×10^{-6}. Employing this approximation in a DDS as sine computation technique, makes it possible to have a ROM-Less DDS architecture with 16-bit amplitude resolution ($7.9\times10^{-6} < 2^{-16}$).

TABLE II.   Summary Of Sine Approximations Techniques

Approximations Methods	Maximum Error	Equivalent Amplitude Resolution (bits)
2nd-order parabolic approximation [8]	9.0×10^{-04}	10
3rd-order parabolic approximation [9]	4.8882×10^{-04}	11
5th-order polynomial approximation [10]	2.0×10^{-04}	12
8th-order polynomial approximation [11]	1.89×10^{-05}	15
Proposed Bezier curve approximation	**7.9×10^{-06}**	**16**

III. DIGITAL IMPLEMEMTATION

The phase accumulator output, n is a digital sweep whose value ranges from 0 to 2^N-1. The phase output, n is related to the phase angle, θ as given in (1). Substituting 'θ' with 'n' and multiplying of each term of (9) by $1024/2^{10}$ ($1024/2^{10} = 1$),(9) becomes

$$\text{Sin (n)} = 1/2^{10}(3218(n/2^{N-1}) - 16(n/2^{N-1})^2 - 5156(n/2^{N-1})^3 - 539(n/2^{N-1})^4 + 3741(n/2^{N-1})^5 - 1248(n/2^{N-1})^6). \quad (10)$$

Considering truncated phase bits N=16 (as not all bits of phase accumulator are forward to phase to sine conversion stage, but only few MSBs are forwarded and rest are truncated to reduce the ROM requirement. Assuming N=16 is truncated phase bits to be forwarded for sine function calculation). With N=16, (10) reduces to

$$\text{Sin (n)} = 1/2^{10}(3218(n/2^{15}) - 16(n/2^{15})^2 - 5156(n/2^{15})^3 - 539(n/2^{15})^4 + 3741(n/2^{15})^5 - 1248(n/2^{15})^6) \quad (11)$$

Equation (11) can be rewritten as

$$\text{Sin (n)} = n/2^{25}(3218 - n/2^{15}(16 + n/2^{15}(5156 + n/2^{15}(539 - n/2^{15}(3741 - n/2^{15}(1248)))))) \quad (12)$$

The last term in (12), $n/2^{15}(1248)$ requires a multiplier in its digital implementation, which is not a desirable feature. This term can be replaced with $n/2^{15}(2^{10}+2^8-2^5)$, which is easily implemented by shift-and- add/subtractor. By this, demand for number of multiplier reduces by one. So (12) can be rewritten as

$$\text{Sin (n)} = n/2^{25}(3218 - n/2^{15}(16 + n/2^{15}(5156 + n/2^{15}(539 - n/2^{15}(3741 - n/2^5 - n/2^7 + n/2^{10})))))). \quad (13)$$

Equation (13) can be realized by digital circuitry. The block diagram based upon (13), for the proposed Bezier curve generator is shown in Fig. 3. It consists of five 15×15-bit multipliers, five shift-and-add scalers, and one adder/subtractor.
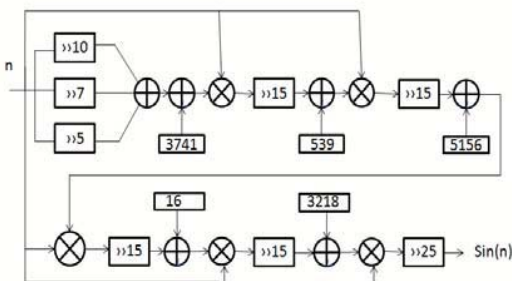

Figure 3. Block diagram of Bezier curve approximation

Fig. 4 shows the proposed architecture of ROM-Less DDS using Bezier curve approximation. The N-bit phase

accumulator generates a digital ramp corresponding to complete cycle of sine wave (0 to 2π). (N−1) LSBs of phase accumulator, corresponding to 0 to π are converted to +ve half cycles of sine wave by Bezier curve approximation generator. The frequency of these half cycles is twice of output frequency. The MSB is used to determine whether the sine amplitude output has to be inverted. Every alternative half cycle is inverted to make complete sine wave.
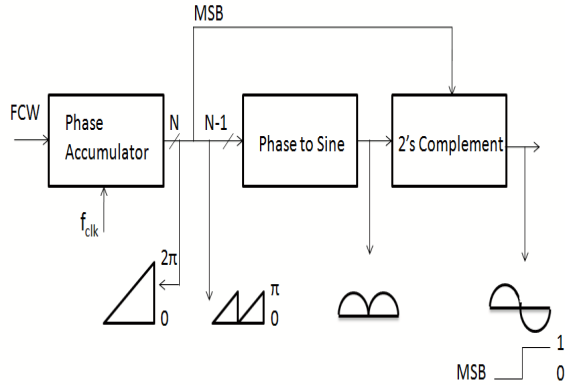

Figure 4. Architecture of Bezier curve approximation based DDS.

IV. RESULT AND ANALYSIS

The Bezier curve approximation function is modelled in Matlab-Simulink for validating the approximation. The Bezier function block was inserted in place of ROM (look up table) block of DDS. The specifications of the proposed DDS simulation are shown in Table III.

TABLE III. SPECIFICATION OF THE SIMULATED DDS

Specification	Value
FCW	32-bit
Phase accumulator output	16-bit
Clock frequency	1GHz
Output frequency	10MHz
Spurious Free Dynamic Range (SFDR)	−99.76dBc

The Fig. 5 shows the output of phase accumulator and sine function generator. The output waveform is spectrally very pure. Fig. 6 shows the power spectrum of waveform synthesized by DDS based on Bezier curve approximation. The SFDR evaluated for the proposed method is −99.76dBc.
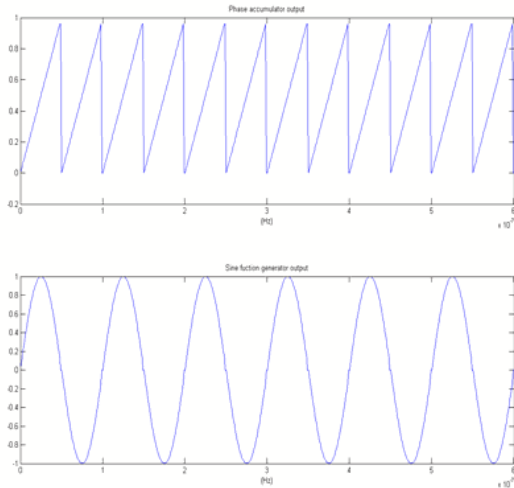
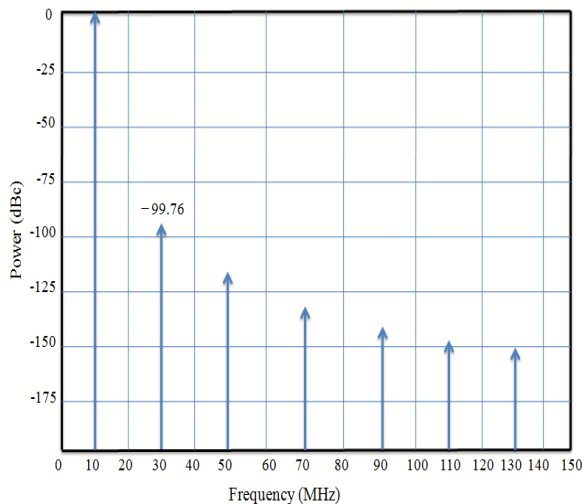Figure 5.   Phase accumulator and sine fuction generator output.

Figure 6.   Power spectrum of simulated sine wave.

V.   CONCLUSION

The DDS using Bezier curve approximation has been proposed and simulated. It avoids use of ROM (look up table) and computes the sine function. The proposed approximation introduces maximum error of 7.9×10^{-6}, which is equivalent to quantized sinusoid with 16-bit amplitude resolution. The SFDR of proposed ROM-Less DDS is evaluated using Matlab-Simulink, which is equal to -99.76dBc. Important advantage lies with this idea of approximation is that it uses full digital circuitry (no memory), gives 16-bit resolution and spectrally very pure sine waveform.

REFERENCES

[1]   V. F. Kroupa, Direct Digital Frequency Synthesizers. New York, US: Wiley-IEEE Press, 1999.

[2]   A Technical Tutorial on Digital Signal Synthesis, Analog Devices, 2011.

[3]   X. Li, L. Lai, A. Lei, and Z. Lai, "A direct digital frequency synthesizer based on two segment fourth-order parabolic approximation," IEEE Transaction on Consumer Electronics, vol. 55, pp. 322-326, May 2009.

[4]   J. Vankka, "Methods of mapping from phase to sine amplitude in direct digital Synthesis," IEEE International Frequency Control Symposium, pp. 942-950, 1996.

[5]   M. M. E. Said and M. I. Elmasry, "An improved ROM compression technique for direct digital frequency synthesizers," IEEE International Symposium on Circuits and Systems, vol. 5 pp. 437-440, 2002.

[6]   Y. H.Chen and Y. A. Chau, "A direct digital frequency synthesizer based on a new form of polynomial approximations," IEEE Transaction on Consumer Electronics, vol. 56, pp. 436-440, May 2010.

[7]   H. Pottmann and J. Wallner, Computational Line Geometry. New York, US: Springer, 2010.

[8]   M. Sodagar and G.R.Lahiji, "A novel architechture for ROM-Less sine-output direct digital frequency synthesizers by using the 2nd-order parabolic approximation," IEEE International Frequency Control Symposium, pp. 284-289, 2000.

[9]   A. Mohammad and A. M. Sodagar, "ROM-Less direct digital frequency synthesis using the 3rd-order parabolic approximation" IET Frequency and Time Forum, pp. 325-328, 2004.

[10]   C. Meenakam and A. Thanachayanont, "A ROM-Less direct digital frequency synthesiser using a polynomial approximation," IEEE International Symposium on Circuits and Systems, vol. 1, pp. 625-628, 2003.