On the Extension of a Star Based Parallel Interconnection Network Topology

[Nibedita Adhikari]

Abstract—**Due to architectural innovations design of parallel interconnection network topology is always an area of research. The Star graph is very popular among existing topologies. The present attempts modify star graphs with the application of extension technique. The different topological parameters of the proposed topology are derived. The comparison of topological and performance parameters is carried out to establish the suitability of the proposed network as a better candidate for large scale parallel system.**

Keywords—interconnection network, fault tolerance, parameters, reliability, Isoefficieney

I.   Introduction

The interconnection networks are the backbone of any parallel computing system [1]. Major parallel interconnection networks (IN) are Hypercube, Crossed cube, Star graphs [2, 3, 4, and 5]. With architectural innovations the basic structures are modified. The modification techniques resulted in various new topologies like Folded hypercube [6], Extended hypercue [7], Extended Varietal hypercube (EVH) [8] and Metacube [9] etc.

The EVH is a recursive and fault-tolerant interconnection topology called extended varietal hypercube (EVH) is introduced in [8]. It has reduced diameter, average distance and constant degree of nodes. The Extended crossed cube (ECQ) [10] possess better diameter than the Crossed cube and Hypercube. Also the mean distance between vertices is shorter than that of the hypercube.

The Star graph is a popular alternative to the Hypercube with reduced diameter and constant degree of nodes [5]. The n-Star is a node-symmetric and edge-symmetric graph consisting of n! number of nodes and n! (n - 1)/2 number of edges. In spite of its admitted superiority, the n-Star has a major disadvantage that is; it grows to its next higher dimension by a very large value. For example, the 4-Star contains 24 nodes where as, for a 5-Star network 5! = 120 nodes are needed. The considerable gap between the two consecutive sizes of the n-Star is considered to be a major drawback and therefore needs further attention for its improvement. For this reason a new alternative called Extended Star (ES) was proposed [11].

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The details like construction, topological parameters and routing algorithms are discussed. The current paper is an extension of the same to discuss about the performance parameters like cost effectiveness, terminal reliability, packing density and scalability etc., in order to find the suitability of ES network as a candidate for large scale parallel system. The scalability of a parallel system is a measure of its capacity to increase speedup in proportion to the number of Processing Elements (PE) [12]. It is measured in terms of the Isoefficiency function which is the ratio of speedup to the total number of processors in the system. Packing density is the total number of PEs in the system.

The section two presents briefly the network and its parameters. The performance parameters are discussed in section 3. The Section 4 discusses the results of comparison of ES network against the parent networks. Section 5 concludes the paper.

II.   Extended Star Topology

The Extended star is a variation of the n-Star graph [5]. The details of the proposed ES architecture are described in earlier work [11]. For convenience brief explanation about structure and basic topological parameters are presented below.

A. Construction

Basically the Extended star (ES) is a hierarchical topology with basic module as n-Star graph and one network controller which helps the processing elements to solely concentrate on computational work. An ES consisting of an n-Star and one NC forms the basic module ES(n,1) and is shown in Fig.1.

B. Addressing

The address-labeling scheme is a basic step to construct the multicomputer system as it helps in deciding the shortest diameter. The basic building block for an ES is an n-star. containing n! number of nodes. The node addresses are permutations of 1, 2, 3, ...n. For example in 3-Star there are six nodes and the node addresses are 123, 213, 312, 132, 231, 321 respectively. Currently the node addresses are given in decimal notations that is 0, 1, 2, 3, 4 ... (n!-1) for simplicity. So in ES(3,1), the NC will have address 0. The six children nodes will have node address 00, 01, 02, 03, 04, 05 as shown in Fig 1(a). The address of NC precedes the address of the node. The ES(3,2) is shown in Fig. 1(b) with addresses of the NC's only.
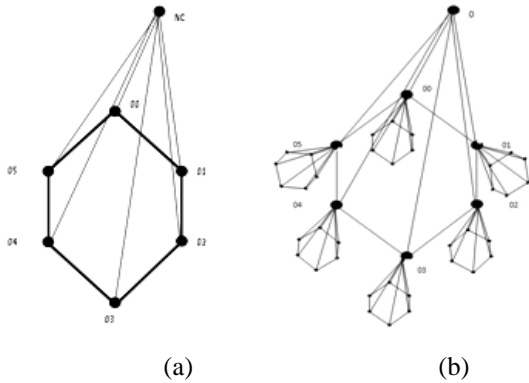
(a)   (b)

Figure 1. Extended Star network ES(3,l) and ES(3,2)

C. Parallel Paths

In the basic module ES(3,1), there are three parallel paths between any two nodes as shown in Fig. 2. For example, the parallel paths between 01 and 03 are given by

a) 01-02-03
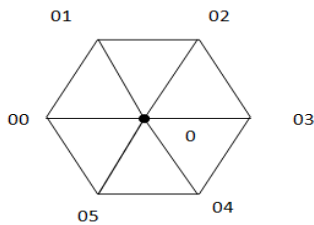b) 01-00-05-04-03
c) 01-0-03

Figure 2. Parallel Paths in ES(3,1)

D. Spanning Tree

The procedure for generating the spanning tree of ES network is as follows:

a) The NC becomes the root.
b) Two farthest nodes become the children of the root.
c) Next the remaining odd/even nodes are placed as grand children of even/odd node.

The spanning tree for the basic module is shown in Fig. 3(a). For next higher level the spanning tree is constructed by appending the spanning tree of the basic module at all the nodes as shown in Fig. 3 (b).

E. Topological Parameters of ES

In this section the various topological parameters of the Extended Star graph are discussed briefly using graph theoretical notations.

Degree:

In an ES(n, k) each PE is directly connected to (n-1) neighbouring PEs of the same Star and to a NC at the next higher level. So the degree of a PE is n. The degree of a NC other than the top most NC is (n!+(n-1)+1) or (n!+n).

Diameter

Theorem 1: The diameter of Extended Star ES(n,k) is

$$D(G) = \left\lceil \frac{3}{2}(n-1) \right\rceil + 2(k-1).$$
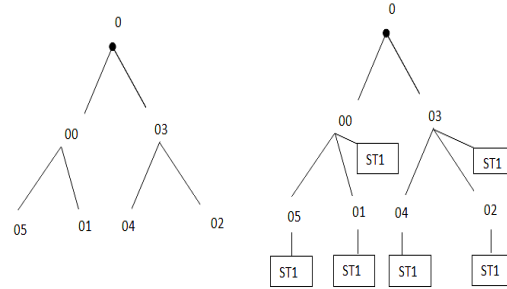

(a)   (b)

Figure 3 (a,b). Spanning Tree of ES(3,1) and ES(3,2)

Average Distance:

For two types of communications: local and global in ES there are two average distances, namely, local and global average distance.

(i) Local Average Distance:

Theorem 2: The local average distance of the ES(n , k) is $n - 4 + \frac{2}{n} + \sum_{i=1}^{n} \frac{1}{i}$.

(ii) Global Average Distance:

Theorem 3: The expression for the global average distance in an ES (n,k) is given by $\bar{d} = \frac{\sum d N_d}{p+M}$, where, N_d is the number of processors at a distance d from the source node. p is the total number of PEs and M is the total number of NCs.

Nodes:

Theorem 4: The total number of nodes in an ES (n, k) denoted by p is given by $p = n!^k + \frac{n!^{k}-1}{n!-1}$

Links:

Theorem 5: The total number of links E in an ES (n, k) is

$$E = \frac{1}{2}(n+1)! \times \frac{(n!)^k - 1}{n! - 1}$$

Cost:

Theorem 6: The cost of the ES(n,k) network is given by

$$\xi = (\left\lceil \frac{3}{2}(n-1) \right\rceil + 2(k-1)) \times \frac{n \times (n!)^k + (n!+n) \times ((n!)^k - 1)/(n!-1))}{(n!)^k + (n!+n) \times ((n!)^k - 1))}.$$

Message Traffic Density:

The performance is very important in handling the message traffic. Assuming that each node is sending a message to a node at a distance d, on an average, the message traffic density is defined by, $\rho = \frac{\bar{d} N_t}{E}$, where E is the total number of communication links, N_t, is total

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number of nodes $Nt = (p + M)$ and $\bar{d}$ is the average message distance.

Table 1 shows the simulated result for the values of average node distance $\bar{d}$, N ,M ,E and ρ for ES$(3,k)$ for different values of k keeping n=3.

Extensibility:

The ES is hierarchical in nature and can be built by extension of the number of levels without affecting the structure of the basic module. The most important advantage of this property is that the degree of any node remains the same, independent of the number of nodes in the network and hence allows for further expansion. The different topological parameters of the proposed Extended star graph are compared with those of the Star graph in Table 2. In the next section the performance analysis of the Extended star network is presented.

TABLE I. Message Traffice Density(ρ) of ES(3,k)

k	$\bar{d}$	P	M	E	ρ
1	1.5	6	1	12	0.85
2	3.23	42	7	84	1.884
3	5.23	258	43	516	3.05
4	7.26	1554	259	3108	4.22
5	9.29	9330	1555	18660	5.33

TABLE II. Comparison of basic parameters of Star and Extended star

Parameter	n-Star	Extended star
Size/Nodes	$n!$	$n!^k + \dfrac{n!^k - 1}{n! - 1}$
Links	$n! \times (\dfrac{n-1}{2})$	$\dfrac{1}{2}(n + 1)! \times \dfrac{(n!)^k - 1}{n! - 1}$
Node degree	n-1	$n! + n$
Diameter	$\left\lfloor \dfrac{3}{2}(n - 1) \right\rfloor$	$\left\lfloor \dfrac{3}{2}(n - 1) \right\rfloor + 2(k - 1)$
Average distance	$n - 4 + \dfrac{2}{n} + \sum\limits_{i=1}^{n}\dfrac{1}{i}$	$\bar{d} = \dfrac{\sum dN_d}{p + M}$
Fault Diameter	If n=3 or 4 then original diameter + 2 And if n>4 then original diameter + 1.	$\left\lfloor \dfrac{3}{2}(n - 1) \right\rfloor + 2k - 1$
Cost	$(n - 1)\left\lfloor \dfrac{3}{2}(n - 1) \right\rfloor$	$(n! + n)\left\lfloor \dfrac{3}{2}(n - 1) \right\rfloor + 2(k - 1)$

III. Performance Analysis of Extended Star

A. Fault Tolerance

The ES network is a hierarchical structure of n-star with PEs at lowest level which provides additional path through the network controller there by increasing the fault tolerance property of the network as compared to the star graph. For all the PEs, the node degree is *n*. This means that the network can tolerate up to (n-1) faults.

B. Fault Diameter:

In an IN, if one or more links (nodes) is faulty, then the network gets deformed resulting in a new diameter, called the Fault diameter d_f, where d is defined as the maximum diameter of any graph obtained from G by deleting at most *f* vertices. The fault diameter should be close to the original diameter.

Theorem 7: For ES(n,k) the fault diameter is $d_f = \left\lfloor \dfrac{3}{2}(n - 1) \right\rfloor + 2k - 1$.

Cost Effectiveness Factor:

Theorem 8: The cost effectiveness of ES(n, k) is derived as $CEF(p) = \dfrac{1}{1 + \rho \times \frac{(n+1)!}{2} \times (1 - \frac{(n!)^k}{p})}$

Proof: In general the number of links is a function of the number of nodes that is E= f(p). The total number of processors links in ES is given by

$$p = (n!)^k + \frac{(n!)^k - 1}{n! - 1}, \quad E = \frac{1}{2}(n + 1)! \times \frac{(n!)^k - 1}{n! - 1}$$

$$= \frac{(n+1)!}{2} \times (p - (n!)^k) = f(p)$$

So, $g(p) = \dfrac{f(p)}{p} = \dfrac{\frac{(n+1)!}{2} \times (p - (n!)^k)}{p} = \dfrac{(n+1)!}{2} \times (1 - \dfrac{(n!)^k}{p})$

Where g(p) is the ratio of number of links to the number of processors and ρ is the ratio of link cost to processor cost. Hence,

$$CEF(p) = \frac{1}{1 + \rho g(p)} = \frac{1}{1 + \rho \times \frac{(n+1)!}{2} \times (1 - \frac{(n!)^k}{p})}$$

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Time Cost Effectiveness Factor:

Theorem 9: The time cost effectiveness factor of the ES network is given by

$$TCEF(p, ES) = \frac{1+\sigma}{1+\rho\frac{(n+1)!}{2}\times\left(1-\frac{(n!)^k}{p}\right)+\frac{n!-1}{(n!)^{k+1}-1}}.$$

Proof: In Theorem 8, the $g(p) = \frac{(n+1)!}{2} \times (1 - \frac{(n!)^k}{p})$

For any IN topology, the TCEF is given by

$$TCEF(p, T_p) = \frac{1+\sigma T_1^{\alpha-1}}{1+\rho g(p)+T_1^{\alpha-1}\sigma/p}$$

where T_1 is the time to solve the problem by uniprocessor system with fastest sequential algorithm and T_p is the time of parallel algorithm using a multiprocessor system having p processors. Next ρ = of link cost/ processor cost and σ =cost of penalty/cost of processors. For a linear time penalty in T_p, $\alpha = 1$.

$$p = (n!)^k + \frac{(n!)^k-1}{n!-1} = \frac{(n!)^{k+1}-1}{n!-1}, \text{ So } \frac{1}{p} = \frac{n!-1}{(n!)^{k+1}-1}$$

Then $TCEF(p, ES) = \dfrac{1+\sigma}{1+\rho\frac{(n+1)!}{2}\times\left(1-\frac{(n!)^k}{p}\right)+\frac{n!-1}{(n!)^{k+1}-1}}$

Hence the result.

Comparison of *Isoefficiency* for the Proposed Network

The Isoefficiency determines the ease with which the parallel system can maintain the constant efficiency and hence can achieve scalability. The Isoefficiency (f_e) is defined as follows:
Generally the parallel run time (T_p) for 'p' processing elements and input size 'w' is defined as [12]

$$T_p = \frac{w+T_o(w,p)}{p} \qquad (1)$$

where T_o is the overhead of the system due to idling of the processor. The speedup (S) is defined as

$$S = \frac{W}{T_p} \qquad (2)$$

The *Isoefficiency* $f_e(p)$ is defined as

$$f_e(p) = \frac{S}{p} = \frac{1}{1+T_o(w,p)/w} \qquad (3)$$

If a system can maintain constant *Isoefficiency* for different problem sizes then it is scalable. The value lies between 0 and 1. Using Eq. 3, the *Isoefficiency* metric for all the networks is evaluated. For evaluation the input size 'w' is specified and is stated in Table 3.

TABLE III. Problem size specification

Size(w)	Scale
50-100	Small scale
100-500	Medium scale
500-1000	Big scale
1000-10000	Large scale

IV. Results and Discussions

Next the different parameters of the Extended star (ES) network are evaluated and compared with the contemporary networks.

The comparison of the diameter of ES is shown in Fig. 4. The diameter of the ES network has been compared with that of the EH, EVH, Star graph, HC and the Crossed cube. The result reveals that at lower dimensions the ES network is having the lowest value. When increase in dimension the total number of nodes in the ES network increases than that of the parent networks. Hence, the diameter also increase as the system is scaled up. Next in Fig. 5, the cost versus dimension of the $ES(k,l)$ network is compared with that of the EH, EVH and ECC with fixed k and varying l. The result shows that the ES network possesses the lowest value and slower growth rate with higher packing density at same dimension. At lower n the cost of ES is higher than that of ECC as it contains more nodes. The variations of the cost effectiveness and time cost effectiveness factors for the ES network are shown in Fig. 6 and 7 respectively. The TCEF attains its maximum at dimension 12 for ES network with 3-star as the base.

The comparison of the two terminal reliability for ES network is shown in Figs 8 and 9. The Fig. 8 shows that the ES network is highly reliable than the EVH for all n. The graph is plotted against the dimension with constant link and processor failure rate at mission time 1000Hrs. Next the reliability of both ES and ECC is compared with respect to mission time. Here the basic building blocks are CC_3 and 3-Star. The Extended Star network exhibits better reliability values up to a mission time of 5000Hrs. Beyond that both the networks become equally reliable as shown in Fig. 9. The *Isoefficiency* graph in Fig. 10
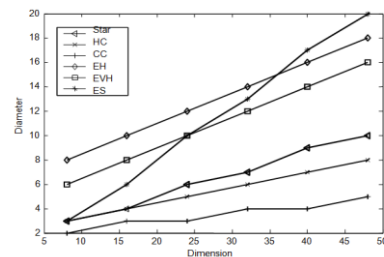

Figure 4. Comparison of diameter of ES network

shows the value of the ES network never crosses 1. The EH and ECC network posses same number of nodes and thus both graphs completely match. The up and down nature of the graph for ECC and EH network indicates inferior usability. But the ES network exhibits better scalability for small scale networks (50 to 200) and is constantly scalable for medium as well as big scale problems (1000 to 10000).

V. Conclusion

The current work discussed about a cost effective and reliable parallel interconnection network called the Extended star. A through approach for constructing the proposed network using the principle of extension is

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discussed. It also presented an in depth analysis on its various topological parameters through illustrations. Due
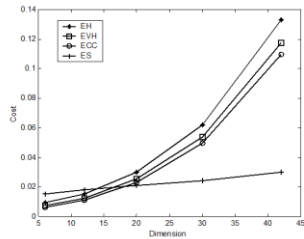
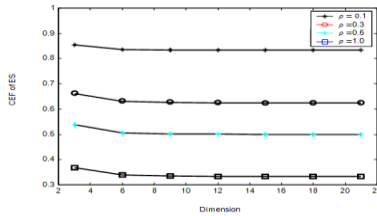

Figure 5. Comparison of cost of ES network

Figure 6. Cost-effectiveness factor of extended star network

to extension the network's performance is improved in terms of fault tolerance and reliability. The ES topology is found to be maximally fault tolerant and scalable. The detailed analysis of results reveals that ES network is better than its parent networks: EH, CC, Star and the EVH and also ECC network.

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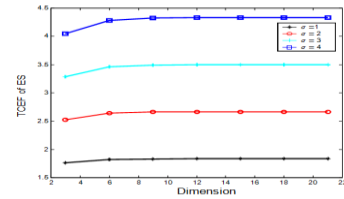
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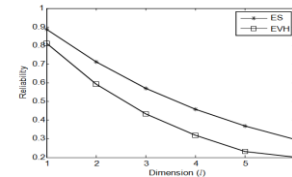Figure 7. TCEF Vs dimension of Extended star network

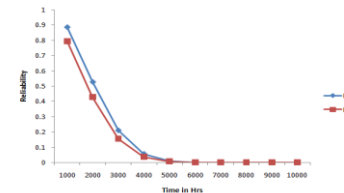Figure 8. Two terminal reliability evaluation of ES(3,l) network

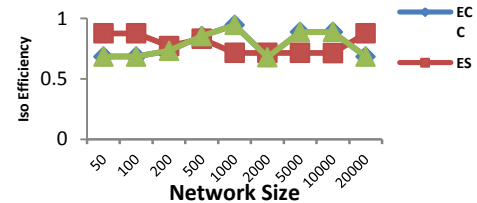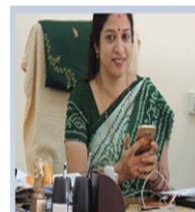Figure 9. Comparison of reliability of ES and ECC

Figure 10. Comparison of Isoefficiency of the networks

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