# Threshold Accepting Approach for Image Registration

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*Abstract*— Image registration is the process of determining the point-to-point correspondence between two images of a scene. It is a very computationally intensive process. Heuristics can be applied to reduce the time involved. In this paper, a modified simulated annealing approach called threshold accepting is applied to image registration. This method provides fast and accurate results compared to the former.

*Keywords*—image registration, normalized correlation, heuristic methods, simulated annealing, threshold accepting

## I. Introduction

Image registration is the process of determining the pointby-point correspondence between two images (the reference and sensed images) of a scene. It transforms different sets of images (two or more) of the same scene taken at different times, from different viewpoints, and/or by different sensors into one coordinate system. It is used in areas like remote sensing, change detection, image mosaicing, creation of superresolution images, medicine, maps creation and computer vision. The goal of image registration is to obtain a spatial transformation which best aligns two or more images.

Four basic steps of image registration procedure are feature detection, feature matching, transform model estimation, and image transformation and resampling. Fig. 1 shows the block diagram of image registration process.

- *Feature detection*: Detects distinctive objects such as corners, lines, curves, templates, regions and patches in both reference and sensed images.
- *Feature matching*: Establishes the correspondence between the features in the reference and sensed image.
- *Transform model estimation*: Estimates the type and parameters of the transformation functions.
- *Image resampling and transformation*: The sensed image is transformed by means of the mapping functions.

Estimating the transformation is highly computation intensive. Heuristics can be applied to find the best transformation with minimum number of steps. The main disadvantage of heuristics is that the desired maximum corresponding to optimal transformation may not be the global maximum of the search space. Even though simulated annealing is proved to converge to the global maximum, its time complexity is very high. Threshold accepting approach overcomes the demerits of simulated annealing. It provides much better solution compared to simulated annealing after same number of steps.



Figure 1. Steps in image registration process.

#### II. RELATED WORKS

Some of the important existing research in this topic are the survey papers by Lisa G. B. [1], and Medha V. Wyawahare, Dr. Pradeep M. Patil, and Hemant K. Abhyankar [2], and the works by Flávio Luiz Seixas, Luiz Satoru Ochi, Aura Conci, and D'ebora C. M. Saade [3] employing genetic algorithms, the technique based on Particle swarm optimization proposed by Mark P. Wachowiak, Renata Smol'ýkov'a, Yufeng Zheng, Jacek M. Zurada, and Adel S. Elmaghraby [4], adaptive simulated annealing proposed by Maryam Zibaeifard, and Mohammad Rahmati [5] and the evaluation of evolutionary methods for medical Image registration by Sergio Damas, Oscar Cordon, and Jose Santamaria [6]. These are briefly reviewed in the following:

Detailed overviews of various image registration techniques/approaches are given in [1, 2]. Classical and modern approaches to image registration and their advantages and disadvantages are discussed in these works. Issues in image registration and the scope for future research are also discussed.

Flávio Luiz Seixas, Luiz Satoru Ochi, Aura Conci, and D'ebora C. M. Saade [3] address the image registration

Proc. of the Intl. Conf. on Advances in Computer, Electronics and Electrical Engineering Editor In Chief Dr. R. K. Singh. Copyright © 2012 Universal Association of Computer and Electronics Engineers ITAL LIBRARY All rights reserved. ISBN: 978-981-07-1847-3 problem using genetic algorithms. The point matching problem was addressed employing a method based on nearestneighbor. The mapping was handled by affine transformations. The main advantage of the genetic approach is that a prealignment between views is not necessary to guarantee good results. However, accuracy depends on the initial population, which is usually initialized with random entries.

A Biomedical Image Registration technique based on Particle swarm optimization is proposed in [4]. It can be adapted for single-slice 3-D-to-3-D biomedical image registration incorporating initial user guidance. In many cases, the hybrid particle swarm technique produced more accurate registrations than the evolutionary strategies with comparable convergence. Hybridization with the crossover operator improves accuracy. However, in few cases this prevented particles from moving toward the global optimum.

Another notable work [5] by Maryam Zibaeifard, and Mohammad Rahmati proposes an adaptive simulated annealing technique for multimodal image registration based on the maximization of their mutual information. The proposed method uses a non-stochastic optimizer in primary stages and simulated annealing in the final stage.

One of the recent works by Sergio Damas, Oscar Cordon, and Jose Santamaria [6] benchmarks various 3D medical image registration methods. The optimization approaches based on both Meta-heuristics (MH) and Evolutionary Computation (EC) are more important. They are the extension of basic heuristics with their inclusion in an iterative process of improvement. EC is one of the most addressed approaches within Meta-heuristics. EC is mainly based on genetic algorithms which are simple and easy to implement. It is independent of the initial solution and the solution representation and has capability to escape from local optima. However, it needs an initial manual tuning of control parameters. Also, the estimation of termination criterion is complex. Even with the shortcomings of EC, it faired over the other traditional methods in providing more accurate results.

The literature survey carried out above reveals that a major issue in image registration is the need for an efficient optimization algorithm for determining the parameters of registration. Exhaustive/ brute force approaches are computationally impractical. Genetic algorithm, particle swarm optimization and simulated annealing are popular search algorithms employing heuristics to arrive at faster solutions. However, there is a need for faster and more accurate solutions for the image registration problem. The proposed work suggests a modified simulated annealing called Threshold Accepting (TA) approach for the image registration problem.

The rest of the paper is organized as follows. Some of the image registration methods are given in Section III and Simulated Annealing in described in Section IV. The proposed approach, Threshold Accepting, is presented in Section V. Finally the experimental results are presented in Section VI followed by conclusion in Section VII.

#### III. METHODS OF REGISTRATION

The image registration algorithms are based on the similarity measures employed. Some of the approaches are Landmark, correlation and mutual information. The exact method chosen is based on the applications for which it is used.

Landmark registration is the simplest and accurate if reliable and consistent landmarks are available. However, it is difficult to get reliable landmarks except in CT brains. Correlation works satisfactorily if the images are similar, like the cases where both the images are of CT or PET. Mutual information is a robust method that does not rely on the two images being of the same kind.

Normalized Cross-correlation and mutual information methods are briefly described in the following.

#### A. Normalized Cross-Correlation

Normalized cross-correlation [1] gives a measure of the degree of similarity between an image I and a template T, where T is small compared to I. It is best suited to search a template T in the image I. The equation for normalized cross-correlation is given as

$$C(u,v) = \frac{\sum_{x} \sum_{y} T(x,y) I(x-u,y-v)}{\sqrt{\sum_{x} \sum_{y} I^{2}(x-u,y-v)}}$$
(1)

#### **B.** Mutual Information

Mutual Information (MI) [7, 8] measures the statistical dependence between two variables or the amount of information that one variable contains about the other. If A and B are two variables with marginal probability distribution,  $p_A(a)$  and  $p_B(b)$  and joint probability distribution  $p_{AB}(a,b)$ , MI measures the degree of dependence of A and B.

$$MI(A, B) = H(A) + H(B) - H(A, B)$$
 (2)

H(A) and H(B) are the entropy of A and B, respectively. H(A,B) is their joint entropy.

$$H(A) = -\sum_{a} p_{A}(a) log p_{A}(a)$$

$$H(B) = -\sum_{b} p_{B}(b) log p_{B}(b)$$

$$H(A, B) = -\sum_{a,b} p_{AB}(b) log p_{AB}(a, b)$$
(3)

### **IV. SIMULATED ANNEALING**

Simple hill climbing starts with a random initial solution and then explores its neighborhood for a better solution. The algorithm stops whenever there is no better solution available



in the immediate neighborhood. Simulated Annealing and Threshold accepting overcomes this problem of stopping in local minima by accepting new solutions with lower similarity.

Simulated Annealing (SA) [5] is a probabilistic metaheuristic used to solve optimization problems. It has been proven to deliver a globally optimal solution, and can be applied to a wide range of problems. Its main strength is the ability to escape local minima during the search process. It is mathematically proven [9] to converge to global optimum.

The concept of simulated annealing comes from annealing in metallurgy. It involves heating and controlled cooling of metals to reduce their defects. The heat causes the atoms to wander randomly and the slow cooling helps in reaching configurations with lower internal energy than the initial one.

The flowchart of the optimization process in simulated annealing is given in Fig. 2. The algorithm starts from an initial solution  $s_0$  with an initial temperature, T. In each iteration of the algorithm, a new random solution is selected which is near the current solution. Each step of the algorithm attempts to replace the current solution by a random solution. The new solution may then be accepted with a probability that depends both on the difference between solutions and the current temperature. The temperature T is decreased gradually during the process until the temperature limit (set based on application) is reached.

When T is large, the chance of selecting a worst solution is high. However, the algorithm selects only the better solutions as T goes to zero. The acceptance of worst solutions helps the method to escape from local optima.



Figure 2. Flowchart of Simulated Annealing.

#### v. THRESHOLD ACCEPTING

Threshold accepting [10, 11] is similar to simulated annealing which differs only in the selection criteria of solutions. Also they share similar convergence properties. It does not accept worse solutions beyond a certain threshold. However simulated annealing accepts worst solutions with a low probability.

The flowchart of Threshold accepting approach is given in Fig. 3. The algorithm randomly chooses an initial solution with an initial high threshold T. In each iteration, a new random solution is selected from solution space depending on the current threshold. Each step of the algorithm attempts to replace the current solution by a random solution. The new solution is accepted only if it falls inside the current threshold. The threshold T is decreased gradually during the process until the threshold limit is reached.

TA always accepts a solution with higher similarity, but deteriorations are accepted only if they are not worse than a particular threshold. Over time, the threshold decreases to zero, thus TA turns into a Simple hill climbing algorithm.



Figure 3. Flowchart of Threshold Accepting.



### **VI. EXPERIMENTAL RESULTS**

The experiments were performed in MATLAB v7.10. The registration process was tested on 512x512px images. Fig. 4 shows the sample images used in the experiments. Simulated annealing and Threshold accepting methods were applied on the images separately using Normalized Cross-Correlation as the similarity measure.

Table I shows the results of the experiments. The best solution was obtained using exhaustive search. However, it has a time complexity of  $O(n^2)$  which is infeasible in real world applications. Simulated Annealing provided solutions which were comparable to exhaustive search but with very less time complexity.

Threshold accepting algorithm produced better results compared to simulated annealing after equal number of iterations.



Figure 4. Left— Reference image. Right— Source image to be registered.

TABLE I. COMPARISON RESULTS OF VARIOUS ALGORITHMS

Algorithm	Iterations performed	Best Solution
Exhaustive Search	262144	7829.96
Simulated Annealing	2106	7477.43
Threshold Accepting	1620	7609.79

## VII. CONCLUSION AND FUTURE WORK

A modified version of simulated annealing called threshold accepting is applied to image registration optimization problem. It provided better results compared to simulated annealing method. The future work will be to improve on the threshold accepting method and further reduce the time complexity.

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