Application of Multivariate EWMA for performance monitoring of RMC

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Abstract— Statistical process control is applied in order to monitor and control a process. Statistical process control procedures can be widely applied to understand and improve the production process of construction materials. In performance monitoring of Ready Mix Concrete plant, there are always more than one quality characteristics which affect the performance of concrete and are usually correlated. In this paper, Multivariate exponentially weighted moving average control chart (MEWMA) is applied to monitor the performance of concrete in Ready Mix Concrete plant. The different parameters considered are strength and density at different age of concrete. One difficulty found with multivariate control charts is the interpretation of out-of-control points. The univariate control charts are used to obtain a rough estimate of the sources of Multivariate out-of-control points. EWMA and MEWMA methods are compared for performance monitoring of concrete. The result shows that MEWMA charts are more relevant than EWMA charts.

Keywords— Statistical Process Control, Multivariate EWMA, Univariate EWMA, Ready mix Concrete

I. Introduction

Ready-mixed concrete (RMC) was patented in Germany in 1903, but it was only after the development of the modern truck mixer in the 1950s that the supply of RMC became commercially viable [1].

There is tremendous growth in the use of ready mixed concrete (RMC) for construction in the developing countries. The RMC industry is not regulated or monitored properly, as a result it has contributed to a general disregard for the basics of good quality of RMC [2]. To rectify the situation and to improve the performance of RMC producers, systematic monitoring and inspection is required.

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The SPC tools are very useful in identifying the defects in the process and developing the solutions for eliminating them to improve the quality of the products. The standardization of the system will definitely improve the quality of the products manufactured as in this case it is concrete.

Statistical process control consists of a number of powerful tools for problem solving and improvement of quality control by reducing variability in manufacturing processes. At present, quality control charts for monitoring a continuous quality characteristic is done using Shewhart, Cumulative Sums (CUSUM), exponentially weighted moving average (EWMA), etc. In some cases it may also be necessary to control simultaneously two or more related quality characteristics, which can be done with the multivariate SPC using multivariate charts such as Hotelling's T^2 control charts, multivariate EWMA (MEWMA), etc. [3]

Most of the models developed were dealing primarily with only one quality parameter or characteristic which is mainly strength of the grade of concrete monitored. But higher degree of quality monitoring can be carried out if multiple quality characteristics can be monitored simultaneously. Thereby, Sarkar and Bhattacharjee (2014) made an attempt to develop a multivariate CUSUM methodology (MVCUSUM) where the other parameters like slump, density and temperature of fresh concrete which affects the quality of RMC can be monitored. Later Sarkar and Panchal (2017) did an attempt to explore the potentiality of application of Quality Function Development and particularly House of Quality which primarily focuses on customer satisfaction, for performance monitoring of RMC.

To improve efficiency in the processes with small changes, univariate EWMA and multivariate EWMA (MEWMA) control charts were developed. Their advantage is that they take into account the present and the past information of the process. Therefore they are more powerful to detect small changes than Hotelling's T^2 control charts [4].

These control charts be used as daily or weekly monitoring tool, which can improve the efficiency of the production process [5]. Thus this paper is an attempt to explore the MEWMA control chart as a tool for the process monitoring of more than one variable which can affect the quality of the RMC.

II. Monitoring Multivariate Processes using Control Chart

The multivariate charts can be separated into two categories, i.e., the directionally invariant schemes and the direction specific schemes. Typically, most symmetric two-sided univariate control charts are directionally invariant, whereas multiple univariate schemes used for a multivariate

process are not directionally invariant. These types of multivariate chart are generally used for detection of shifts in the process parameters along their respective axes, and must be aimed in a particular direction [6]. So, in Multivariate EWMA chart there is only Upper Control Limit (UCL) and no lower control limit (LCL).

Exponentially Weighted Moving Average (EWMA) chart

The EWMA chart was developed by Roberts in the late fifties and later developed further by other author Hunter [8]. The chart is effective for detecting small to moderate shifts. It is proposed for applications to the process such as chemical industries, financial and management control systems particularly when the sample size is one. The chart can be used to control: an individual sample, the average of a sample, a ratio or a proportion of mortality in the sample.

The EWMA chart is very insensitive to normality assumptions. In this chart, the older the observation the less weight it conveys. The plotted statistics are exponentially weighted moving average defined by[8]:

$$z_i = \lambda x_i + (1 - \lambda)z_{i-1}, \; 0 < \lambda \leq 1, \; z_0 = \mu_0 \qquad (1)$$

Where λ is a constant and the starting value (required with first sample at i=1) is the process target, in fact z_i is a weighted average of all previous sample means,

$$z_i = \lambda \sum_{j=0}^{i-1}(1 - \lambda)^j x_{i-j} + (1 - \lambda)^i z_0 \qquad (2)$$

And the variance z_i is:

$$\sigma^2_{zi} = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)[1 - (1 - \lambda)^{2i}] \qquad (3)$$

When increases: $(1 - \lambda)^{2i} \to 0$, therefore

$$\sigma^2_{zi} = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right) \qquad (4)$$

The control limits for this chart are as follows:

$$\begin{cases} UCL = \mu_0 + L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1 - (1 - \lambda)^{2i}]} \\ \qquad Centre \; line = \mu_0 \\ LCL = \mu_0 - L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1 - (1 - \lambda)^{2i}]} \end{cases} \qquad (5)$$

Multivariate Exponentially Weighted Moving Average control chart (MEWMA)

To simultaneously monitoring p correlated quality characteristics, the multivariate EWMA (MEWMA) chart can be applied. Ignoring correlation between variables is the main weakness in the present practice of using independent univariate charts to track each of the p quality characteristics individually. The MEWMA introduced by Lowry et al. (1992) [4] which is a logical extension of the univariate EWMA and is defined as follows:

$$z_i = \lambda x_i + (1 - \lambda)z_{i-1}, z_0 = 0 \qquad (6)$$

Where $\lambda = diag(\lambda_1, \lambda_2, \lambda_{3,\dots}\lambda_p)$ and $0 \leq \lambda \leq 1$, and 1 is the identity matrix.

The quantity plotted on the control chart is

$$T_i^{\,2} = Z_i' \Sigma_{z_i}^{\,-1} Z_i$$

Where the covariance matrix is:

$$\Sigma_{z_i} = \left(\frac{\lambda}{2 - \lambda}\right)[1 - (1 - \lambda)^{2i}]$$

III. Identifying the out-of-control points

Your process may be out of control (OOC) if one or more of the following occurs [7]:

1. One or more points beyond 3 sigma from center line
2. 9 points in a row on same side of center line
3. 6 points in a row, all increasing or all decreasing
4. 14 points in a row, alternating up and down
5. 2 out of 3 consecutive points beyond 2 sigma from center line (same side)
6. 4 out of 5 consecutive points beyond 1 sigma from center line (same side)
7. 15 points in a row within 1 sigma of center line (either side)
8. 8 points in a row beyond 1 sigma from center line (either side)

IV. Statistical Analysis

For the performance monitoring of RMC there are many factors affecting the process. The factors considered in the study are 7 days compressive strength, 28 days compressive strength and density of a single sample with 3 subgroups. This 3 variables are correlated with each other. Primary Data has been collected from a commercial RMC plant in Bhuj-Kutch, region of Gujarat, India.

The MEWMA chart is deployed to monitor the performance of each sample based on the three characteristics that defines the performance of concrete. The upper control limit in this paper is produced by the statistical package MINITAB 17 based on ARL=200. Analogous to the situation in the univariate case, the MEWMA chart is equivalent to T^2 (or chi-square) chart if $\lambda = 1$. It is possible to place the control limits at a desired level which is called the accepted fluctuation level for a particular quality characteristic.

Thus, a large value of λ (near to 1) gives more weight to recent data and less weight to older data; a small value of λ (near to 0) gives more weight to older data. The value of λ is usually set between 0.2 and 0.5. So λ value of 0.4 is taken for the analysis.

Table 1: Correlation Matrix of 3 Variables

	7 days strength	28 days strength
28 days strength	0.566	
Density	0.166	0.25

Table 1 indicates the correlation matrix of quality characteristics of the samples. For instance, the 7 days compressive strength and 28 days compressive strength are correlated with each other (r = 0.566). The r value shows the correlation between both characteristics and so on as shown in the Table 1.
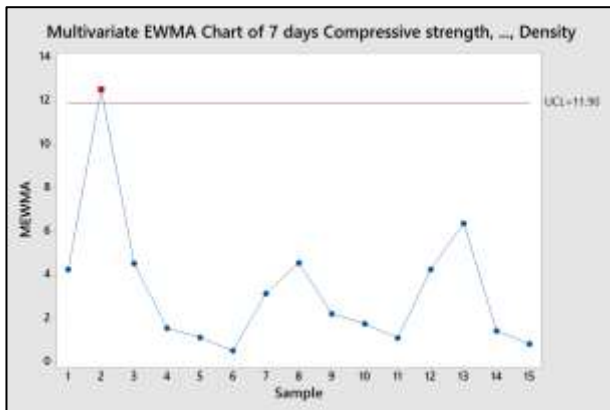

Figure 1: Multivariate EWMA chart for 7 days, 28 days compressive strength and Density

Figure 1 shows the MEWMA chart of these samples with λ=0.4. According to the plot, if all the three characteristics are monitored at the same time, the sample is out of control only at reading time 2 (red point).

It is a common practice in statistical quality control to use the individual charts to *identify the characteristics* responsible for the multivariate out-of-control signals. For this reason, the univariate EWMA chart is made for each ind*ividual quality characteristic s*hown in Figure 2, 3 and 4.
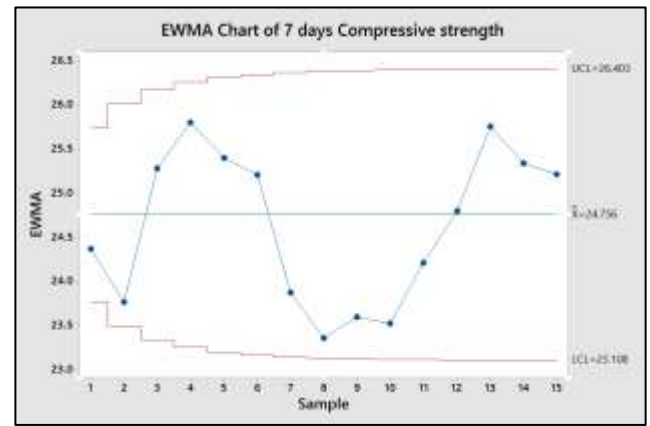

Figure 2: EWMA chart for 7 days compressive strength

In Figure 2 there are no out of control points in the EWMA chart, which shows that the process is in control for 7 days compressive strength.

In Figure 3, four points are shown out of control i.e. point 1, 2, 12 and 13. Also in Figure 4, point 2 is out of control. All the out of control points are marked in red.

Based on all the 3 individual plots it can be said that the main area of concern is the sample set 2 according to the MEWMA plot shown in Figure 1. In Figure 3, the points 12 and 13 are outlier in individual charts but they show "False alarm" as they are no threat to the process according to Figure 1.

So it can be concluded that the MEWMA chart is more reliable then the univariate EWMA chart as it does not consider the correlation between the characteristics.

Based on the plots, once the characteristics are monitored individually, the control charts tend to produce fewer out-of-control signals. However, there are many situations in which the simultaneous monitoring of two or more correlated quality characteristics is necessary.
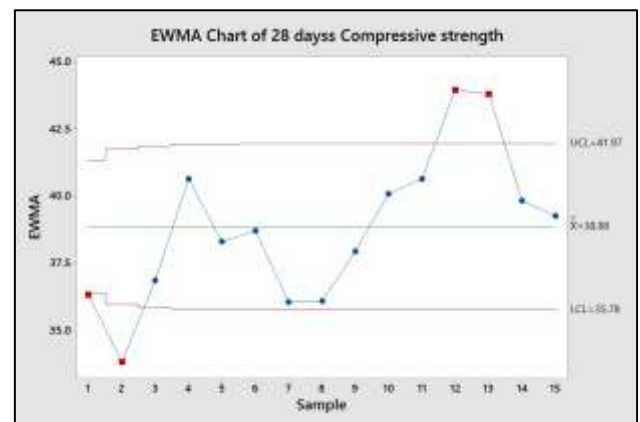

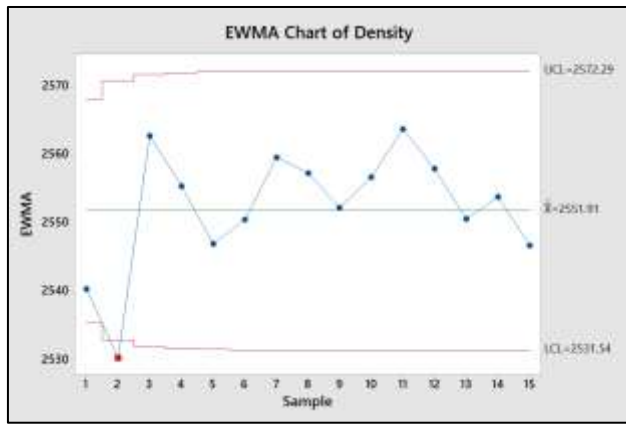Figure 3: EWMA chart for 28 days compressive strength

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Figure 4: EWMA chart for Density

Because there is a significant correlation between these characteristics based on Table 1, the multivariate EWMA chart will produce the out-of-control signals more accurately than the univariate EWMA charts.

On the other hand, it is possible that MEWMA chart detects an in-control signal, while univariate EWMA charts show out of control process. For instance, MEWMA chart (Figure 1) identifies an out-of-control signal only at sample set 2, but in case of univariate charts (Figure 4-7) it shows an alarm at several points. The reason would be the correlations between the proposed characteristics (see Table 1) that are not taken into account in univariate charts.

V. Conclusion

In this paper, a basic analysis has been presented for improving achievable quality in the multivariate monitoring of Ready mix concrete plant, by applying the multivariate control chart in place of the univariate approach. It has been demonstrated that univariate charts, in some cases, can give a misleading indication in RMC process, while the multivariate approach can properly indicate the problem. It is also shown that when there are three quality characteristics, which describes the performance of the concrete, the individually monitoring would lead to false conclusions. However, it is observed in some cases where univariate charts gives out of control signals, while the multivariate chart produces no out of control signals. This may lead to unnecessary cost of quality for concrete creating a "false alarm". The out of control points are identified at the end and by root cause analysis the cause for the same is identified. The control measures that can be taken for the performance monitoring are change in mix design, change in the quality of material as well as workmanship.

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