Study and Analysis of the performance of capacity of Ad-hoc Wireless Networks

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Abstract — **The total capacity obtained from a network is an essential performance metric. In wireless ad-hoc networks, the dynamic topology and stochastic variations in channel characteristics makes it difficult to analytically determine this metric. However an analytical closed form expression for the network capacity is developed. The cumulative distribution function of the signal to Interference Power Ratio at each receiver is derived analytically. Then the capacity of network is studied and closed form expressions are determined. By examining the effect of the outage threshold, less simultaneous communications, each with a higher rate results in higher total capacity**.

I.   INTRODUCTION

A wireless ad-hoc network is a class of wireless networks in which in the absence of an infrastructure, all nodes have the same responsibilities and any two nodes wishing to communicate do it directly or use nodes between them to route their packets. The inherent capability to rapid deployment, makes this type of networks an attractive choice for many applications. The lack of any predetermined topology and dynamic changes in wireless channel creates many challenges in this area [1], [2].

A challenging performance analysis issue which is addressed is to determine the capacity of ad-hoc wireless networks. A common approach is to determine the total capacity that can be obtained from the network [3]-[7].

In this paper, a random ad-hoc network is considered and a *closed form expressions* for the total capacity is derived [8], [9]. To do so, an analytical model to examine network capacity is proposed. In the next chapter, a general scenario for the network is introduced and related definition of its capacity is presented. Then in third chapter, the statistics of the received signal to interference power ratio is investigated and the cumulative distribution function is derived analytically. In fourth chapter, after justifying the accuracy of the analytical results, the capacity of an ad-hoc network is studied. Finally, fifth chapter contains some concluding results.

II.   PROBLEM DEFINITIONS

A. Ad-hoc Wireless Network Scenario

It is assumed that the capacity of Ad-hoc network is static. It means that nodes do not move during transmission and reception.

It is assumed that the nodes are randomly located in the network coverage area with uniform distribution. Network coverage area is considered to be a planar circular disk with radius *R*. The Cumulative Distribution Function of the distance of the location of each node from the center of the network r,

$$F_r(r) = Prob.\{r \le R\} = \begin{cases} 0, & r \le 0 \\ \frac{\pi r^2}{\pi R^2}, & 0 \le r \le R \\ 1, & r \ge R \end{cases} \qquad (1)$$

Where *Prob{A}* is the probability of event *A*.
Therefore, the polar coordinates of each node,
(r, θ), will have the following probability density Function (PDF),

$$\begin{cases} f_r(r) = \frac{dF_r(r)}{r} = \frac{2r}{R^2}, & 0 \le r \le R \\ f_\theta(\theta) = \frac{1}{2\pi}, & 0 \le \theta \le 2\pi \end{cases} \qquad (2)$$

Where r is the distance from origin and θ is the ngle between the positive x-axis and the ray from origin to the location.

It is also assumed that every transmitter uses an omnidirectional antenna and randomly chooses its receiver from nodes no closer from than *ε* and no farther than *D* with equal probability

It is assumed that *D<<R* and thus the effect of nodes in the edge of coverage area is negligible. Therefore the probability that a node W chooses a destination within *ρ≥ε* distance away is proportional to the number of nodes in the disk centered at W with radius from *ε* to *ρ*. Since the nodes are uniformly distributed in the coverage area,
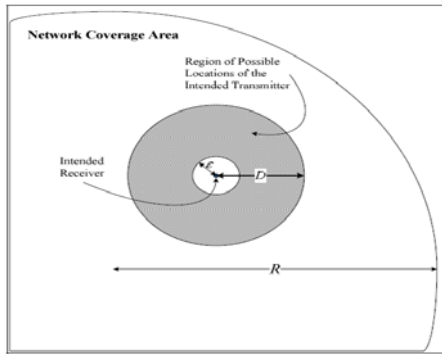
Figure 1. Network coverage area and its dimension and related distances: *R* is the radius of the coverage area, is the minimum distance between a transmitter-receiver pair and *D* is the maximum distance of it.

$$F_d(\rho) = Prob\{d \le \rho\} = \begin{cases} 0, & \rho \le \varepsilon \\ \frac{\pi \rho^2 - \pi \varepsilon^2}{\pi D^2 - \pi \varepsilon^2}, & \varepsilon \le \rho \le D \\ 1, & \rho \ge D \end{cases} \quad (3)$$

Where $F_d(\rho)$ is the probability distribution function (CDF) of the distance between a transmitter to its intended receiver. The distinction between transmitter-receiver pair and source-destination pair should be emphasized. A node as a source may randomly select any other node as a destination of its data, but depending to its distance to the selected destination, a number of intermediate nodes are needed to realize a chain of transmitter-receiver pairs that starts by source node and ends at the destination node to pass on the data in a multi hop fashion.

B. Capacity Definition

The outage event is defined as the event when the signal-to-interference ratio (SIR) at the receiver's output drops below a threshold [15]. Then if the network condition is such that at most *M* simultaneous communications take place with a predetermined rate of *k* bits per second, while ensuring that a certain target signal-to-interference ratio is maintained at each receiver, the total capacity of the network would be *Mk* bits per second [3],[9],[11].
The outage probability,

$$Pr_{outage} = Prob.\{SIR \le \beta\} = F_\gamma(\beta) \quad (4)$$

Where $F_\gamma(\beta)$ is the probability distribution function (CDF) of the received SIR and β the outage threshold.
SIR of each link is a stochastic process and by ignoring the edge effects, it is assumed that "*probability distribution of SIR*" is the same for all nodes in the network due to the symmetry of ad-hoc network.
According to (4), the probability that the communication can take place while the minimum required SIR is met at the receiver equals to (1-Pr_{outage}) or equivalently (1- $F_\gamma(\beta)$).If there are *N simultaneously active* transmitter-receiver pairs in the network, the number of successful communications, n, follows a binomial probability distribution and assuming that each communication occurs at the rate of k bits per second,

Total capacity=

$$\sum_{n=0}^{N} \left\{ \binom{N}{n}(1 - F_\gamma(\beta))^n Pr_{outage}^{N-n} \times nk[bps] \right\} = (1 - F_\gamma(\beta)).Nk[bps]. \quad (5)$$

III. SIGNAL TO INTERFERENCE RATIO (SIR) ANALYSIS:

According to the definitions of the network capacity, cumulative distribution function (CDF) of SIR at the receiver location can calculate the capacity.
It is assumed that there are *N* pairs of transmitter-receiver nodes in the network.

$$SIR = \frac{P_t/d^\alpha}{\sum_{i=2}^{N} P_t/r_i^\alpha} = \frac{1/d^\alpha}{\sum_{i=2}^{N} 1/r_i^\alpha}, \quad (6)$$

Where P_t is the common power of each transmitter, *d* is the distance between the intended transmitter (the first transmitter) and the intended receiver and r_i *(2≤i≤N)* is distance from the ith transmitter to the intended receiver. Also α is the path loss exponent.
Then CDF of SIR [12],

$$F_{SIR}(\gamma) = Prob. \left\{ \frac{1/d^\alpha}{\sum_{i=2}^{N} 1/r_i^\alpha} \le \gamma \right\} \quad (7)$$

Where *Prob{A}* is the probability of event *A*.
Interfering power at the receiver,

$$I = \frac{\sum_{i=2}^{N} 1/r_i^\alpha}{} \quad (8)$$

When the network radius *R* is large, the edge effects will be negligible and the location of the intended receiver can be considered as the centre of the network [13]. With this assumption, interferes are uniformly distributed around the center and their distances to it, vary from a minimum value ε to a maximum value *R,* where ε is the minimum distance between every two nodes in the network and *R* is the network radius. In such case, the distance from the ith interfering node *(2≤i≤N)* to the intended receiver r_i has the following distribution function:

$$F_{r_i}(r) = Prob.\{r_i \le r\} = \begin{cases} 0, & r \le \varepsilon \\ \frac{\pi r^2 - \pi \varepsilon^2}{\pi R^2 - \pi \varepsilon^2}, & \varepsilon \le r \le R, \\ 1, & r \ge R \end{cases} \quad (9)$$

Each random element of equation (6) has been statistically characterized, now the interfering power at the receiver *I* defined in equation (8) is analyzed. Using the law of large numbers, *I* is replaced by (N-1) $E\{1-r^\alpha\}$ where $E\{1-r^\alpha\}$ is expected value.
For small value of α (around 1 or less), *I* approaches (N-1) $E\{1-r^\alpha\}$ as *N* is increased. However, for most common

wireless channels, path loss exponent α is larger than 2(actually $2 \leq \alpha \leq 4$).

Since the variance of I is too large, it uses logarithm and studies the powers in dBm or Dbw and SIR in dB units. Therefore,

$$I_{db} \triangleq 10 log_{10} I = 10 log_{10} \left(\sum_{i=2}^{N} \frac{1}{\eta_i^{\alpha}} \right), \qquad (10)$$

Estimation of capacity (I_{dB}),

$$I_{db} \cong 10 log_{10} \left(max_i \left\{ \frac{1}{r_i^{\alpha}} \right\} \right) = 10 log_{10} [(min_i \{\eta_i\})^{-\alpha}] \quad (11)$$

Where $max_{i|}a_{ij}$ and $min_{i|}a_{ij}$ are the maximum and minimum of the a_i for $(2 \leq i \leq N)$ respectively. Hence, the CDF of I_{dB} can be estimated as,

$$F_{I_{db}}(x) = prob. \{I_{db} \leq x\}$$
$$\cong prob. \left\{ min_i \{\eta_i\} \geq 10^{-\frac{x}{10\alpha}} \right\}$$
$$= prob. \left\{ \eta_i \geq 10^{-\frac{x}{10\alpha}}, \forall 2 \leq i \leq N \right\}$$
$$= \left[1 - F_r \left(10^{-\frac{x}{10\alpha}} \right) \right]^{(N-1)} \qquad (12)$$

$F_r(r)$ is the CDF of the distance between each interferer and the intended receiver. Replacing $F_r(r)$ from (9) in the above equation, results in the following approximation for $F_{IdB}(x)$:

$$F_{I_{db}}(x) \cong \left[\frac{R^2 - 10^{-\frac{x}{5\alpha}}}{R^2 - \varepsilon^2} \right]^{(N-1)}$$

$$-10 log_{10} R \leq x \leq -10 log_{10} \varepsilon \qquad (13)$$

Now, equation (6) is modified to obtain SIR in dB:

$$SIR_{db} = 10 log_{10} \left(\frac{1/d^{\alpha}}{\sum_{i=2}^{N} 1/\eta_i^{\alpha}} \right)$$

$$= 10 log_{10} \left(\frac{1}{d^{\alpha}} \right) - I_{db}, \qquad (14)$$

Then, CDF of SIR_{dB}, $F_{\gamma}(\gamma)$,
$$F_{\gamma}(\gamma) = prob. \{SIR_{db} \leq \gamma\}$$
$$= prob. \left\{ 10 log_{10} \left(\frac{1}{d^{\alpha}} \right) - I_{db} \leq \gamma \right\}$$
$$= prob. \left\{ d \geq 10^{-\frac{\gamma + I_{db}}{10\alpha}} \right\}. \qquad (15)$$

In the right side of equation (15), both the distance between the intended transmitter and receiver, d, and the interfering power I_{dB} are random variables, therefore,

$$F_{\gamma}(\gamma) = \int_{-\infty}^{+\infty} prob. \left\{ d \geq 10^{-\frac{\gamma + I_{db}}{10\alpha}} | I_{db} = x \right\} f_{I_{db}}(x) dx$$

$$= \int_{-\infty}^{+\infty} \left(1 - F_d \left(10^{-\frac{\gamma + x}{10\alpha}} \right) \right) f_{I_{db}}(x) dx, \qquad (16)$$

Where, $F_d(d)$ is the distribution function of d which is given in equation (3).also, $f_{IdB}(x)$ is the probability density function (PDF) of I_{dB} which is by definition the derivative of the CDF given in (13) :

$$f_{I_{db}}(x) = \frac{dF_{I_{db}}(x)}{dx}, \qquad (17)$$

Replacing equations (17) and (3) in equation (16) and maintaining boundary values of $F_d(d)$ and $f_{IdB}(x)$, give the CDF of SIR_{dB}, $F_{\gamma}(\gamma)$, After several mathematical calculations, $F_{\gamma}(\gamma)$ will be obtained as in equation (18). In this equation, ε is the minimum distance between every two nodes in the network and R is the network radius. D is the maximum distance between a transmitter-receiver pair and N is the total number of active transmitter-receiver pairs in the networks. Also α is the path loss exponent.

$$F_{\gamma}(\gamma) = \begin{cases} 0, & for: \gamma \leq 10\alpha log_{10}\left(\frac{\varepsilon}{D}\right) \\ \frac{D^2}{D^2 - \varepsilon^2}\left[H\left(D^2 10^{\frac{\gamma}{5\alpha}}\right) - H(\varepsilon^2)\right] - \frac{10^{-\frac{\gamma}{5\alpha}}}{D^2 - \varepsilon^2}\left[G\left(D^2 10^{\frac{\gamma}{5\alpha}}\right) - G(\varepsilon^2)\right] & for: 10\alpha log_{10}\left(\frac{\varepsilon}{D}\right) \leq \gamma \leq 0 \\ \left[H\left(D^2 10^{\frac{\gamma}{5\alpha}}\right) - H(\varepsilon^2)\right] + \frac{D^2}{D^2 - \varepsilon^2}\left[H\left(min\left\{R^2, D^2 10^{\frac{\gamma}{5\alpha}}\right\}\right) - H\left(\varepsilon^2 10^{\frac{\gamma}{5\alpha}}\right)\right] & \\ \quad - \frac{10^{-\frac{\gamma}{5\alpha}}}{D^2 - \varepsilon^2}\left[G\left(min\left\{R^2, D^2 10^{\frac{\gamma}{5\alpha}}\right\}\right) - G\left(\varepsilon^2 10^{\frac{\gamma}{5\alpha}}\right)\right] & for: 0 \leq \gamma \leq 10\alpha log_{10}\left(\frac{R}{\varepsilon}\right) \\ 1, & for: \gamma \geq 10\alpha log_{10}\left(\frac{R}{\varepsilon}\right) \end{cases} \qquad (18)$$

$$H(x) \triangleq \int (N-1)\frac{(R^2 - x)^{(N-2)}}{(R^2 - \varepsilon^2)^{(N-1)}} dx = -\frac{(R^2 - x)^{(N-1)}}{(R^2 - \varepsilon^2)^{(N-1)}} \qquad (19a)$$

$$G(x) \triangleq \int x(N-1)\frac{(R^2 - x)^{(N-2)}}{(R^2 - \varepsilon^2)^{(N-1)}} dx = -\frac{(N-1)}{(\varepsilon^2 - R^2)^{(N-1)}}\left[\frac{(x - R^2)^N}{N} + \frac{R^2(x - R^2)^{(N-1)}}{N-1}\right] \qquad (19b)$$

IV. CAPACITY OF AD-HOC WIRELESS NETWORK ANALYSIS:

Before proceeding to examine the capacity of ad-hoc wireless network, the accuracy of approximation is verified. After having the required random locations, the distances between every interfering transmitter node and the selected receiver, is calculated. Subsequently, using equation (6) Signal to Interference power ratio and then SIR_{dB} [10] is computed.

In Fig.2, the resultant CDF's from both simulation and analysis are shown for various values of the path loss exponent, α, for 20 active transmitter-receiver pairs in the
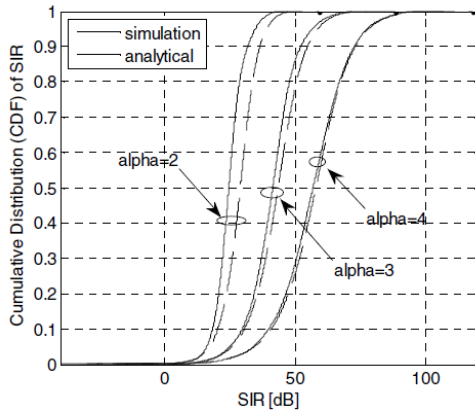

Figure 2. Analytically approximated cumulative distribution of SIR beside the result of the Monte Carlo simulation for various values of path loss exponent, "alpha" for 20 active transmitter-receiver pairs in the network.

its maximum possible value. This value can be obtained as a function of β.

$$k = log_2\left(1 + 10^{\beta/10}\right) \ [bps/Hz] \quad (20)$$

Equation (20) represents Shannon capacity which is the maximum communication rate when the signal-to-noise-power-ratio equals to β in dB units.

With these assumptions, the total capacity of the network versus number of active transmitters, N, is plotted in Fig.3 for several values of the outage threshold β, in dB.

In Fig.3, by setting a higher outage threshold, it results in a high capacity. A higher β increases the probability of outage which has a destructive effect on the network capacity by reducing the first term in the right-side of equation (5). According to equation (20), the constant communication rate of each link, k in equation (5), is an increasing function of β.

Although a higher outage threshold reduces the number of simultaneous communications in the network, it makes it possible that each communication take place with a higher rate and in the end the total network capacity will be improved.

network (N=20). Analytical analysis becomes more accurate by increasing the value of α. Since the interfering power of a transmitter located in the distance of r from receiver is proportional to $r^{-\alpha}$, it decays faster when α is higher. Therefore, in such a case, neglecting the effects of far interferers has less destructive effect in the accuracy of the analysis.

The capacity has been characterized as a function of the CDF of the received SIR in equation (5). In order to examine the capacity of a network, the parameters are set and then for several number of transmitter-receiver pairs, N, the capacity is calculated.

The path loss exponent, α, is set to 4 and the distances ε, D and R are chosen proportional to 1:10:100. Also the constant communication rate of each link, k in equation (5), is set to
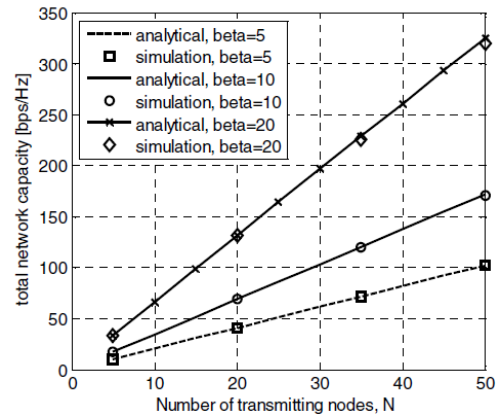

Figure 3. Total capacity of the network as a function of number of active transmitters, N, for various values of the outage threshold, "beta".

V. CONCLUSION

A statistical model to derive a *Closed Form Expression* for the Cumulative Distribution Function (CDF) of Signal-to-Interference Ratio (SIR) is developed in the network. Using this CDF, the capacity of the network is studied. CDF results in closed form solution for total network capacity. For higher values of path loss exponent (α) i.e. 2, 3, 4, analytical approximation is more accurate, especially at α=4. Also for higher value of Outage threshold (β), network has a better performance. In this case, less simultaneous communications, each with a higher rate, are possible in the network and the total capacity of network is increased.

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