

Structural Parameters Uncertainties Effect on Structural Responses

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Abstract—Smart structures are adopting the sensorization and leading the structural systems into a non-trivial situation. Hence those structures required complex monitoring, controlling and updating techniques. In order to understand the changes over time of such systems it is essential to understand the effect of uncertainties. There are several known and unknown sources of uncertainties such as environmental variation which may alter the frequencies of the structures as a results they might encounter severe problems or even full collapse. In order to get the best out of the implemented technologies it is necessary to understand the uncertainties issues related to structural properties i.e., stiffness, damping and mass. Additionally, the structural systems required proper treatment as they will essentially handle extreme unknown dynamic loads. For adapting with the aforementioned changes system parameters need to be identified and updated/adjusted throughout their life time. As dynamic loads are always serious concern, hence, herein the El Centro 1940 earthquake is employed and the response is evaluated. The response is evaluated for several cases for understanding the uncertainty effects of stiffness, damping and mass of the system. In a nutshell, in this study, the dynamical states and system parameters both are assumed to be unknown simultaneously which forms a nonlinear identification problem. To this end, a nonlinear filter so-called the unscented Kalman filter (UKF) is employed. The numerically obtained results show potential which requires further investigation by combining with the different control techniques.

Keywords—Dynamic Loads, Unscented Kalman Filter, Uncertainties, System Identification, Smart Structures.

I. Introduction

The civil structures are prone to natural disasters as well as many uncertainties (e.g. temperature variation, properties changes). Over the past few decades, many research attempt have made to tackle the associated uncertainties of civil structures [14]. In order to resolve the aforementioned issue, the smart materials and structures are getting serious attention in the area of structural engineering. However, complex methodology (i.e., control strategy)

is essential for controlling and modelling those smart structures [16]. In addition to the complex control strategy model updating techniques are necessary. The associated problem is that the real-time model updating issue led to a nonlinear problem due to the bilinear product of the unknown state and parameters [5], [10]. Moreover, the advancement of previously indicated issues are not well established yet. In a recent study, Miah *et al.* (2015) has performed the experimental validation of a newly developed vibration mitigation technique namely the LQR-UKF. Where the stiffness matrix is considered to be uncertain and updated in real-time. And it is reported that the experimental validation is quite critical (even only for stiffness) due to several issues as well as measurement tools. Therefore, further studies are necessary to understand this problem in depth, for example, what happen when damping or mass matrices are unknown. Among many the source of uncertainties might be induced via the unknown extreme force (e.g. earthquake, gale load) as well as by the system properties itself. Hence, updating the model properties via online identification methods are essential for better control.

It is mentioned earlier that uncertainties are unlimited which could be induced either by the internal or external changes. The internal properties changes such as stiffness, damping and mass as well as external ones such as weather changes needs to be studied. Initially, the concept of using observed noisy measurements to update and predict the system response was surfaced by Kalman (1960) [8]. The foregoing filtering technique is widely accepted for linear system where all of the degree-of-freedom responses are not measurable [2],[6],[9],[12][13]. An updated version of nonlinear filter so-called the unscented Kalman filter (UKF) was introduced by Julier & Uhlmann (1997) [7] and extended by Merwe & Wan (2004) [13]. The UKF uses a deterministic sampling technique so-called the unscented transformation [7] which selects some deterministic random points around the mean. In several studies [14] [15], [20] have verified the performance of the UKF for solving the nonlinear joint state and parameter estimation (JSPE) problems. In [4] a JSPE problem has been investigated and a comparison among available alternatives are presented and the superior performance of the UKF is reported. A robust active control approach with system uncertainties is studies in [19].

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In a nutshell, both the parameter identification and model updating issues are a recent topic in the area of structural engineering. The aforementioned process is taking serious attention for better vibration mitigation and control purpose. In existing literature, even though many studies have been performed for the control problems with system uncertainties (i.e., stiffness matrix is unknown). However, still the issue regarding uncertainty is not well understood as the sources are unlimited. Therefore, in this study the effect of uncertainties of different structural parameters such as stiffness, damping and mass are investigated.

II. Problem Description

In this study, a 2-degrees-of-freedom system subjected to seismic load (\ddot{x}_g) is considered for the investigation presented in Figure 1. In order to evaluate the problem herein it is assumed that the system knowledge is not entirely known *a priori*. For the simulations, only two sensors information are considered more precisely first floor displacement (x_1) and second floor acceleration (\ddot{x}_2).

The parameter estimation/identification is performed by employing the aforementioned measured response. The system response (both the process and measurement) are assumed to be corrupted by adding considerable amount of noise. For the simulation a mathematical problem needs to be formulated. Therefore, as a first task the system has been modeled via the process/system equation described as

$$z_{n+1} = Az_n + Bu_n + w_n \quad (1)$$

where $z = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$ indicates the state vector of the system which contains the displacement and velocity quantities, $A = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ represents the system and input matrices, u is the input vector, w is the process noise vector, typically, the Gaussian distributed with $N(0, Q)$. The added process noise indicates the confidence level to the system, ideally, the process noise needs to be reasonably low. Higher level of process noise not only leads to a poor model but also may create complexity for the solution. There is no certain guideline for the noise selection that how much noise one should consider. However, typically, it is decided by the designer, conventionally, selected by trial and error. The system equation (1) contains all of the information (damping, stiffness, mass) related to the system depicted in Fig. 1.

In order to complete the problem formulation along with the process equation (1) a measurement or observation equation is essential which is given as,

$$y_n = Cz_n + Du_n + v_n \quad (2)$$

where y represents the measured/output vector, $C = \begin{bmatrix} 1 & 0 \\ -m_2^{-1}k_2 & -m_2^{-1}c_2 \end{bmatrix} 0_{2 \times 2}$ is the output matrix while $D = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ indicates the feed-forward matrix, and v is the measurement noise vector distributed with $N(0, R)$. The goal of the measurement noise is to represents the sensors noise. And typically, the measurement noise is much higher than the process noise. The observation equation (2) contains all the measured sensors information including the measurement noise quantities.

In this study, the effect of uncertainties for different parameters (i.e., stiffness, damping and mass) are investigated while the system is partially unknown. A benchmark problem is considered where all of the system properties (i.e., mass, stiffness, damping) are known. Additionally, the following cases are considered to evaluate the response of the system in comparison with the benchmark problem.

- (a). C-01: Stiffness matrix is uncertain
- (b). C-02: Damping matrix is uncertain
- (c). C-03: Mass matrix is uncertain

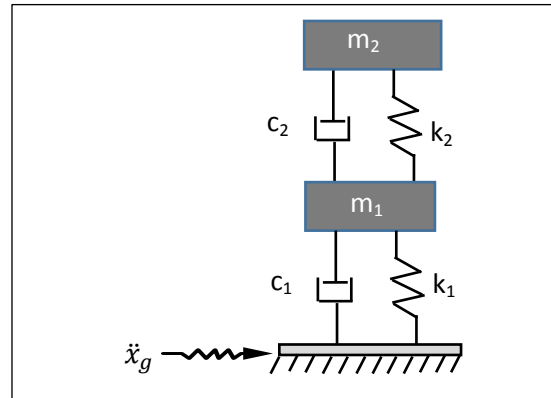


Figure 1. A toy model of 2-degrees-of-freedom system.

For the aforementioned cases (a)-(c), a considerable amount process and measurement noise is added to incorporate the associated uncertainties of the system. Additionally, the system is assumed to be started from an unknown initial guess and then estimate them throughout the simulation period. It is expected that for the Case C-01 & C-03 the resonant frequencies of the system will be highly sensitive compared to Case C-02. This is happening due to the changes of the frequencies resulting from the mass variation throughout the simulation. However, similar situation may also occur due to the stiffness changes over time as frequencies are directly associated to mass and stiffness. Therefore, it necessary to understand the influence of those parameters.

III. Numerical Investigation

It needs to be noted that the process and observation equations are nonlinear due to the bilinear product of two unknowns (i.e., states and parameters). This problem is known as the joint state and parameter estimation (JSPE) [13]-[14]. The UKF algorithm is employed for evaluating the JSPE problem. For the fulfillment of the study UKF algorithm is briefly described below herein. A nonlinear problem formulation is described in this study via the process and observation equations.

$$\begin{aligned} \bar{\mathbf{z}}_{n+1} &= \mathbf{f}(\bar{\mathbf{z}}_n, \mathbf{u}_n, \mathbf{w}_n) \\ \mathbf{y}_n &= \mathbf{h}(\bar{\mathbf{z}}_n, \mathbf{u}_n, \mathbf{v}_n) \end{aligned} \quad (3)$$

where both the process equation \mathbf{f} and the observation equation \mathbf{h} can be nonlinear functions. The state variable $\bar{\mathbf{z}}_n = [\mathbf{z} \ \boldsymbol{\alpha}]^T$ contains the state \mathbf{z} and unknown parameters $\boldsymbol{\alpha}$ of the system. In this study, $\boldsymbol{\alpha}$ symbolized the unknown parameters such as stiffness, damping and mass for the considered cases (a-c).

Briefly, the UKF algorithm requires the initialization at time, t_0 , $\bar{\mathbf{z}}_0 = E[\bar{\mathbf{z}}_0]$, by using the initial states initial covariance P_0 are estimated. And then the next time-steps are predicted and propagated through the nonlinear functions as a result the posteriors are calculated. Herein due to the space limitation detail formulation of the UKF is ignored. However, interested reader may obtain detail formulation from [4], [7], [10], [11], [14].

A. Simulations and Results

The simulations are performed via the use of MATLAB/SIMULINK[®]. The El Centro 1940 earthquake data is considered as input disturbance. The summary of the identified and real parameters are concise in Table I. All of the cases are explained in the above section. The stiffness are considered to be unknown for case 1 while the damping coefficients are unknown for the case 2. Additionally, another case is considered to evaluate the effect of mass changes over time. For the complete application all of the considered cases might take places either as singly or simultaneously manner.

TABLE I. SUMMARY OF THE IDENTIFIED PAPPARAMETERS.

| Cases | Unknown Parameters | Real | Identified |
|-------|--------------------|--------|------------|
| C-01 | k_1 (N/m) | 4.00 | 3.9692 |
| | k_2 (N/m) | 2.00 | 2.0824 |
| C-02 | c_1 (N-s/m) | 0.2828 | 0.3040 |
| | c_2 (N-s/m) | 0.2236 | 0.224 |
| C-03 | m_1 (kg) | 2.00 | 1.9615 |
| | m_2 (kg) | 1.00 | 0.8846 |

It can be observed from Table I that the stiffness and damping parameters are estimated quite accurately. And most importantly almost all of the parameters remain stable throughout the simulation period. However, in comparison to the previous cases (a-b), the floor masses are not estimated efficiently at the beginning of the simulations. In other words, mass took longer initial training period to converge to the true data depicted in Fig. 7.

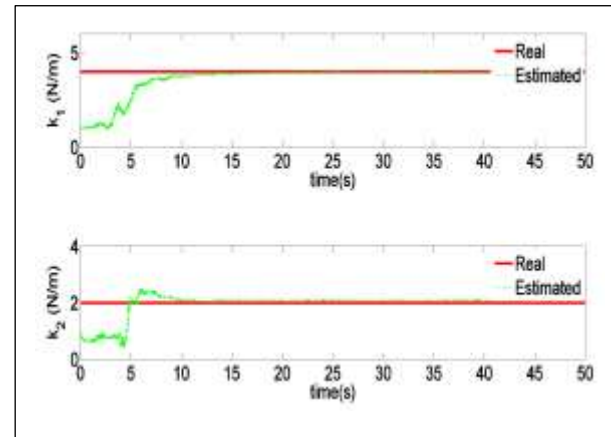


Figure 2. Comparison of the estimated versus real stiffness.

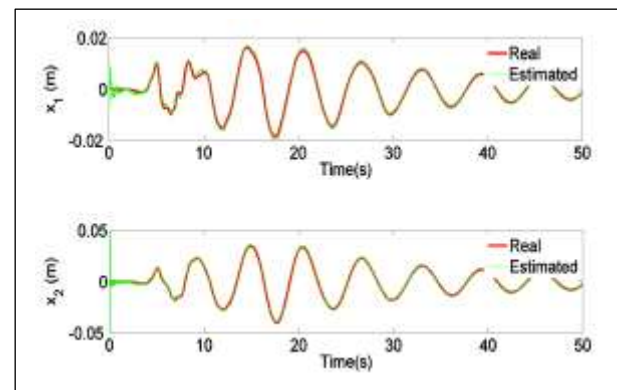


Figure 3. The 1st and 2nd floor displacement with uncertain stiffness quantities.

Along with the Table I, following figures (Fig. 2-7) are presented for the visualization purpose. Firstly, Fig. 2 shows the time history comparison of stiffness between true and estimated data. It is clearly visible that the parameters are converged to the true values quite efficiently. In order to converge to the true data all of the estimated parameters took some initial training time and they remain stable throughout the simulation period. Additionally, Fig. 3 is presented along with the previous figure where the displacement responses of both floors are estimated and compared with the true/benchmark system response. The estimated responses via the UKF are almost identical which confirms the efficacy of the studied method where

significant uncertainties have been handled. The uncertainties are induced by the process and measurement noise, unknown properties and more interestingly, the system starts from a random initial guess. However, initialization of the unknown properties lead to a critical situation as there is no proper guideline. Therefore, it is necessary to perform few initial simulations which leads to a trial-and-error situations. In other words, the starting of the uncertain parameters needs to be set by the designer independently depending on the problems at hand.

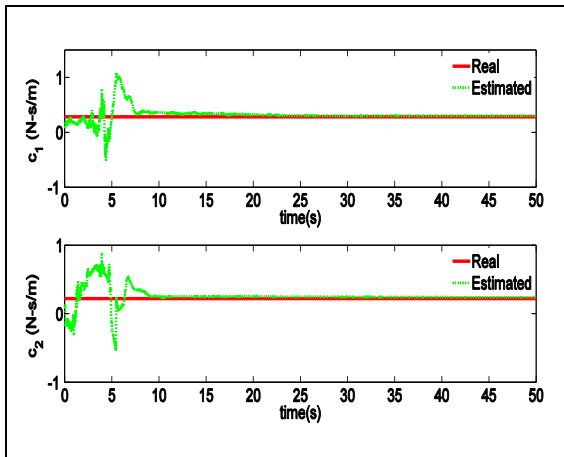


Figure 4. Comparison of the estimated versus real damping.

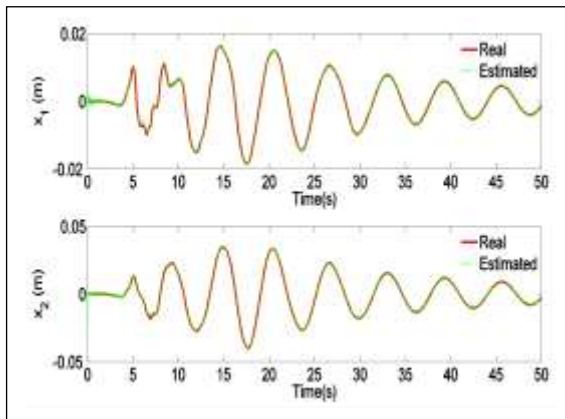


Figure 5. The 1st and 2nd floor displacement with the uncertain viscous damping coefficients.

In a second case, the viscous damping coefficients are assumed to be unknown. And both of the floors damping parameters are estimated by solving the JSPE described in the previous section. The estimated results in time series are presented in Fig. 4 and the displacements are presented in Fig. 5. Once again similar results (i.e., almost identical) are obtained where it is observed that the damping parameters are identified efficiently with very low errors.

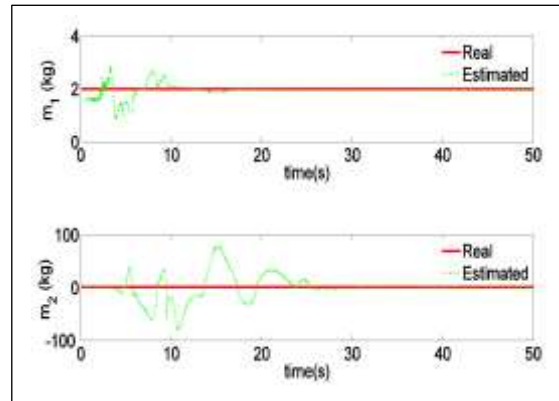


Figure 6. Comparison of the 1st and 2nd floor displacement with the real system response.

Finally, the masses (first and second floor mass) are also assumed to be uncertain. The uncertainty indicates that the mass are not entirely known from the starting of the simulation which will be estimated over the simulation period. It is observed herein that the mass took little more initial training time to converge to the real data which is visible both Fig. 6. The full-time history of the estimated parameters are depicted in Fig. 6. From Fig. 6, it can be seen that after the training period the estimated data converged to the true data and remain stable until the end. Additionally, the response (displacement) of the first and second floor is presented in Fig. 7 and similar results are obtained. It needs to be clear that, the goal for the last case C-03 is to see how sensitive the system is when mass of the system is uncertain. The aforesaid issue is crucial for vibration mitigation and control of any systems. And further study is required to verify the effect of masse with uncertainties.

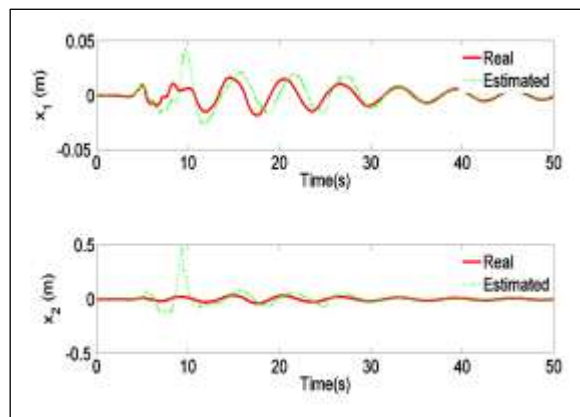


Figure 7. Comparison of the 1st and 2nd floor displacement with the real system response.

iv. Conclusion

This work successfully investigated the effect of uncertainties of structural parameters such as stiffness, damping and mass. To do this end, the JSPE is performed by employing the UKF as a nonlinear observer. The system is considered to be started from wrong assumptions including the process and observation noise quantities. And the aforementioned unknown parameters are estimated during the simulation. The preliminary results show that the uncertainties effect of stiffness and damping parameters can be handled quite efficiently. However, in case of mass it seems little bit complex for instance a good estimation of the floor masses are possible after reasonably longer initial training period. This might create a serious problem or even lead to a permanent damage or collapse. Hence uncertainties related to mass needs to be studied further for deeper understanding. In a nutshell, this study summarizes the effect of uncertainties from different sources and the model is updated throughout the simulation. Further investigation of the studied work is required for the possible real-time vibration control and model updating implementations.

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