

Electrothermally And Electrostatically Tunable Bridge Resonators

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Abstract—This paper demonstrates experimentally, theoretically, and numerically, for the first time, a wide-range tunability of an in-plane clamped-clamped microbeam, bridge, resonator actuated electrothermally and electrostatically. Using both actuation methods, we demonstrate that a single resonator can be operated at a wide range of frequencies. The microbeam is actuated electrothermally, by passing a DC current through it, and electrostatically by applying a DC polarization voltage between the microbeam and the stationary electrode. We show that when increasing the electrothermal voltage, the compressive stress inside the microbeam increases, which leads eventually to its buckling. Before buckling, the fundamental frequency decreases until it drops to very low values, almost to zero. After buckling, the fundamental frequency increases, which is shown to be as high as twice the original resonance frequency. Adding a DC bias changes the qualitative nature of the tunability both before and after buckling, which adds another independent way of tuning. This reduces the dip before buckling, and can eliminate it if desired, and further increases the fundamental frequency after buckling. Analytical results based on the Galerkin discretization of the Euler Bernoulli beam theory are generated and compared to the experimental data and to simulation results of a multi-physics finite-element model. A good agreement is found among all the results.

Keywords—tunability, resonator, electrothermal actuation, electrostatic actuation

I. Introduction

Bistable microelectromechanical (MEMS) structures have been drawing significant attention recently for their interesting advantages in application, such as energy harvesting [1], sensors [2], actuators [3], and MEMS/NEMS based memory elements [4]. Bistable microstructures are characterized by a double-well potential, and hence at least two stable states, and commonly a third one encircles the two local wells. The motion resulting from this third state is large compared with the other two in-well motion. A well-known example of this is the snap-through motion in buckled beams. This is highly desirable feature for many application including micro-mirrors [5], micro-switching [6], and micro-actuators [7]. Bistable microstructure can be realized in many configurations, such as beams sandwiched between two magnets, shallow arches, imperfect microbeams, and buckled beams. This paper is concerned with the third category. The buckled beam is mainly realized by an axial compressive load that can be induced by several methods, such as applying a direct axial load [8] or by using thermal actuation [9]. In this work, we study the generation of axial-compressive load using thermal expansion induced by Joule heating.

Joule heating is a common mechanism of actuation in MEMS thanks to its easy implementation. Joule heating is the conversion of the electrical current energy flowing through a structure into heat. Therefore, applying a current through a doubly clamped microbeam raises its internal temperature, which tends to elongate the microbeam due to thermal expansion. Nonetheless, the elongation is prevented by the presence of the fixed anchors of the microbeam, which induces a compressive force. This compressive load can lead to buckling of the microbeam. This phenomenon can be analyzed from two aspects: electrothermal problem; describing the conversion of the electrical power into heat; and thermo-elastic problem; describing the conversion of the heat power into compressive stress.

Electrothermal actuation has been mainly used as a mechanism to achieve static buckling in microbeams. Chioa and Lin [10] studied theoretically and experimentally the critical current for a fixed-fixed microbeam to buckle. Wang et al [11] developed an electrothermally actuated lateral contact microrelay for RF applications. They studied the required voltage for the microrelay utilizing the parallel six-beam. Chen et al [12] reported a theoretical and experimental investigation of the post-buckling behavior of electrothermally actuated beams. Mastropaolo and Cheung [13] investigated the behavior of SiC clamped-clamped bridge resonators electrothermally actuated with U-shaped aluminum electrodes on top as a function of electrode length, width, and spacing.

Among the few works that utilize electrothermal actuation for dynamic excitation, Sibgatullin et al [14] used the electrothermal excitation as a way to excite the parametric resonance of a clamped-clamped microbeam.

Electrothermal actuation is also widely used to excite and to tune compliant resonators. Tunability of Micro/Nano-electromechanical resonators is highly desirable feature for various applications, including communications [15], filtering [16], gyroscopes [17], energy harvesting [18], signal processing [19], and ultrasensitive detection [20]. At the Micro-scale, electrothermal actuation has been utilized to tune the resonance frequency of resonators, however for a very limited range of frequency. Remtema and Lin [21] showed experimentally and theoretically that the resonance frequency of a resistively heated microbeam could be reduced by 6.5%. Goktas and Zaghoul [22] studied the tunability of CMOS-MEMS fixed-fixed beam resonators using embedded heaters to create axial stress inside the resonator. They showed that the frequency could be decreased by 42.6%. Sviličić et al [23] presented design, fabrication, and electrical testing of MEMS resonators actuated electrothermally and including a piezo-electric sensor to detect the resonance frequency of these resonators. They demonstrated that with the increase of the electrothermal actuation voltage a tuning range of 17 kHz could be realized for a device resonating at 1.766 MHz. At

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the Nano-scale, the resonance frequency of electrothermally actuated nanomechanical resonators have been tuned for lower values [24], as demonstrated at the Micro-scale.

The use of electrostatic actuation to tune resonators is well known since the early work of Nathanson et al [25]. However, it does not offer wide range of frequency, and is mainly used to decrease the resonance frequency of resonators through the softening effect [26]; except for few studies that demonstrate that the resonance frequency could be increased marginally as tuning the electrostatic force for specific geometric conditions [27]. Nevertheless, at the Nano-scale recent experimental evidences demonstrated that the resonance frequency of Carbon Nano Tube CNT resonators increases considerably when increasing the DC gate voltage [28].

The combination of both electrothermal and electrostatic actuation has been rarely studied before, for example for MEMS cantilever switches [29]. No studies have been presented so far that exploit the full potential of these actuation mechanisms in achieving large range of tunability for resonators. This will be demonstrated in this work. The resonator under consideration is composed of an elastic clamped-clamped straight microbeam hanging above a stationary electrode; actuated electrothermally by passing a DC current through it and electrostatically by applying a DC voltage between it and the stationary electrode. We aim to investigate theoretically, numerically, and experimentally the tunability, decreasing, and increasing the frequency of this bridge resonator as varying the electrothermal voltage and for several fixed DC polarization voltages.

The rest of the paper is organized as follows. The nonlinear Euler-Bernoulli beam equation combined with the heat conduction equation is solved for the pre-buckled and post-buckled behavior in Section II. Numerically, a multi-physics nonlinear finite element model, which takes into account the structural, electrothermal, and electrostatic domains, is described in Section III. The experimental setup is presented in Section IV. A discussion of the obtained measurements as compared to simulations is reported in Section V. Finally, the main conclusions are summarized in Section VI.

II. Problem Formulation

The device under consideration, Fig. 1, consists of an in-plane clamped-clamped microbeam actuated electrothermally by a DC voltage V_{Th} and electrostatically by a DC polarization voltage V_{DC} , and is subjected to a viscous damping of coefficient \hat{c} . This linearly elastic microbeam, with Young's modulus E and material density ρ , is of length l , width b , and thickness h . It is assumed to have a rectangular cross section area $A = bh$ and a moment of inertia $I = bh^3/12$. The microbeam is separated from a stationary electrode with a gap width d and with a dielectric constant of the medium ϵ . The electrothermal voltage V_{Th} is applied between the anchors of the microbeam inducing a current I_{Th} passing through the microbeam that heats up it and controls its internally induced axial stress.

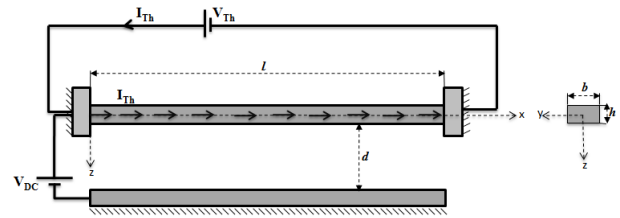


Figure 1. Schematic of an electrothermally and electrostatically actuated clamped-clamped microbeam.

The assumed geometrical, mechanical, thermal and electrical parameters of the studied microbeam are shown in Table 1.

TABLE 1. GEOMETRICAL, MECHANICAL, THERMAL AND ELECTRICAL PROPERTIES OF THE MICROBEAM

Symbol	Quantity	Value
l	Length	600 μm
h	Thickness	2 μm
b	Width	25 μm
d	Gap	7 μm
E	Young's Modulus	120 GPa
ρ	Density	2332 kg/m^3
α	Coefficient of Thermal Expansion	2.6 10^{-6} K^{-1}
k	Thermal Conductivity	165 W/(m K)
σ_e	Electrical Conductivity	0.78 10^4 S/m

A. Electrothermal Problem

In the studied device of this work, applying a potential voltage across the anchors of the microbeam generates a heat flux of density $E = J^2/\sigma_e$ per volume, where J represents the current density defined by I_{Th}/A and σ_e represents the electrical conductivity of the microbeam material. The current density is assumed to be uniformly distributed along the microbeam. In our study the convection and the thermal radiation of the microbeam are assumed negligible. Also, the deformation of the microbeam arising from the thermo-elastic coupling induced by the electric current is neglected. For simplification, thermal conductivity and electrical conductivity are assumed to be independent of temperature. Under all the above assumptions and referring to the heat equation; Fourier's law; the equation governing the average temperature across the section of the microbeam induced by the current I_{Th} is given as below:

$$-k \frac{d^2 T}{dx^2} = \frac{J^2}{\sigma_e} \quad (1)$$

where k is the thermal conductivity of the microbeam material. The current density can be written as a function of

the DC electrothermal voltage $J = \frac{\sigma_e V_{Th}}{l}$. Therefore, the heat equation can be written as

$$-k \frac{d^2 T}{d\hat{x}^2} = \frac{\sigma_e V_{Th}^2}{l^2} \quad (2)$$

Solving (2), assuming that the temperature at the ends of the microbeam is equal to the ambient temperature T_a , gives a close form solution of the distribution of the temperature along the microbeam, which has a parabolic shape and given by the below equation:

$$T[\hat{x}] = \frac{\sigma_e V_{Th}^2}{2k} \left(\frac{\hat{x}}{l} - \frac{\hat{x}^2}{l^2} \right) + T_a \quad (3)$$

The variation of the temperature along the microbeam induces the thermal stress given by

$$\hat{S}_{Th} = \alpha EA \int_0^l (T[\hat{x}] - T_a) d\hat{x} \quad (4)$$

where α is the coefficient of thermal expansion, which is assumed here to be independent of temperature.

B. Equation of Motion

The governing equation of motion of the microbeam under consideration, Fig. 1, describing its transverse deflection $\hat{w}(\hat{x}, \hat{t})$ is written as follows [30]:

$$\begin{aligned} \rho b h \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} + \hat{c} \frac{\partial \hat{w}}{\partial \hat{t}} + EI \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} = \\ \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} \left[\hat{N} + \frac{EA}{2l} \int_0^l \left(\frac{\partial \hat{w}}{\partial \hat{x}} \right)^2 d\hat{x} \right] \\ + \frac{1}{2} \epsilon b \frac{(V_{DC} + V_{AC} \cos(\hat{\Omega} \hat{t}))^2}{(d - \hat{w})^2} \end{aligned} \quad (5)$$

The microbeam is subjected to the following boundary conditions:

$$\begin{aligned} \hat{w}(0, \hat{t}) = \hat{w}(l, \hat{t}) = 0 \\ \left. \frac{d\hat{w}}{d\hat{x}} \right|_{(0, \hat{t})} = \left. \frac{d\hat{w}}{d\hat{x}} \right|_{(l, \hat{t})} = 0 \end{aligned} \quad (6)$$

where \hat{x} is the position along the microbeam and \hat{t} is time. The term $\hat{N} = \hat{N}_0 - \hat{S}_{Th}$ represents the axial load due to the residual axial load, where \hat{N}_0 arising from the fabrication process and the compressive axial load and \hat{S}_{Th} originated from the thermal stress induced by the electrical current I_{Th} given by (4).

For convenience, we introduce the nondimensional variables as below:

$$w = \frac{\hat{w}}{d}; x = \frac{\hat{x}}{l} \text{ and } t = \frac{\hat{t}}{T} \quad (7)$$

where $T = \sqrt{\frac{\rho b h l^4}{EI}}$ is a time scale. Substituting (7) into (5)

and (6), we obtain the nondimensional equation of motion of the beam

$$\begin{aligned} \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + \frac{\partial^4 w}{\partial x^4} = \\ \frac{\partial^2 w}{\partial x^2} \left[N_0 - S_{Th} + \alpha_1 \int_0^1 \left(\frac{\partial w}{\partial x} \right)^2 dx \right] + \alpha_2 \frac{(V_{DC} + V_{AC} \cos(\Omega t))^2}{(1-w)^2} \end{aligned} \quad (8)$$

Subjected to the nondimensional boundary conditions

$$\begin{aligned} w(0, t) = w(1, t) = 0 \\ \left. \frac{dw}{dx} \right|_{(0, t)} = \left. \frac{dw}{dx} \right|_{(1, t)} = 0 \end{aligned} \quad (9)$$

The nondimensional (except for α_2) parameters appearing in (8) are defined as below:

$$\begin{aligned} \alpha_1 = 6 \left(\frac{d}{h} \right)^2; \alpha_2 = \frac{6 \epsilon l^4}{E h^3 d^3}; N_0 = \frac{l^2}{EI} \hat{N}_0; \\ S_{Th} = \frac{l^2}{EI} \hat{S}_{Th}; c = \frac{l^4}{EIT} \hat{c} \text{ and } \Omega = T \hat{\Omega} \end{aligned} \quad (10)$$

Since the microbeam is subjected to a compressive load that increases as much as we increase the electrothermal voltage, it is expected that the microbeam encounters a pitchfork bifurcation near a critical load; below which the microbeam remained straight and above which the microbeam buckles. Therefore, next we split the problem into two parts: the pre-buckling behavior and the post-buckling behavior of the microbeam.

C. Pre-Buckling Study

In this part, the microbeam is modeled as a straight microbeam under a compressive axial load and electrostatic force. A reduced-order model is derived to compute the static deflection as well as the variation of the fundamental natural frequency while varying the electrothermal voltage V_{Th} and for fixed values of the electrostatic voltage V_{DC} [30,31].

To determine the static deflection $w_s(x)$ we set the time derivatives and the AC force equal to zero in (8):

$$\begin{aligned} \frac{\partial^4 w_s}{\partial x^4} = \left[N_0 - S_{Th} + \alpha_1 \int_0^1 \left(\frac{\partial w_s}{\partial x} \right)^2 dx \right] \frac{\partial^2 w_s}{\partial x^2} \\ + \alpha_2 \frac{V_{DC}^2}{(1-w_s)^2} \end{aligned} \quad (11)$$

with the associated boundary conditions

$$\begin{aligned} w_s(0) = w_s(1) = 0 \\ \left. \frac{dw_s}{dx} \right|_{x=0} = \left. \frac{dw_s}{dx} \right|_{x=1} = 0 \end{aligned} \quad (12)$$

To solve (11), we refer to the Galerkin procedure in which we use the undamped linear mode shapes of a straight unactuated microbeam as basis functions [30].

To determine the variation of the natural frequency of the microbeam under the DC polarization voltage and as varying the electrothermal voltage, we solve the eigenvalue

problem obtained by perturbing the deflection around the static configuration. To this end, we resort to the Galerkin discretization to represent the dynamic amplitude $v(x,t)$ as

$$v(x,t) = \sum_{i=0}^n u_i(t) \phi_i(x) \quad (13)$$

where $u_i(t)(i=0,1,2..n)$ denotes the nondimensional modal coordinates and $\phi_i(x)$ presents the undamped mode shapes of a straight unactuated microbeam. Then, the eigenvalue problem of the microbeam under electrothermal and electrostatic voltages is given as follows:

$$\begin{aligned} \ddot{u}_j + u_j \omega_j^2 = & \\ & \left[2\alpha_1 \int_0^1 \left(\phi_j \frac{\partial^2 w_s}{\partial x^2} \right) dx \right] \int_0^1 \left(\sum_{i=0}^n u_i \phi_i \frac{\partial w_s}{\partial x} \right) dx \\ & + \left[\alpha_1 \int_0^1 \left(\frac{\partial w_s}{\partial x} \right)^2 dx \right] \int_0^1 \left(\phi_j \left(\sum_{i=0}^n u_i \phi_i \right) \right) dx \quad (14) \\ & + \int_0^1 \left[\phi_j \frac{2\alpha_2 V_{DC}^2}{(1-w_s)^3} \left(\sum_{i=0}^n u_i \phi_i \right) \right] dx \end{aligned}$$

Using three symmetric mode shapes, three linearized ordinary differential equations are obtained. For a given V_{Th} and V_{DC} , we compute the Jacobin of the system and find the corresponding eigenvalues. Then, by taking the square root of these eigenvalues, we find the natural frequencies of the resonators under V_{Th} and V_{DC} .

D. Post-Buckling Study:

The term ‘‘Post-buckling’’ here refers to the fact that the applied axial force exceeds the critical load of the case without electrostatic force. Essentially, the electrostatic force biases the beam, and hence the pitchfork bifurcation of the buckling instability becomes a perturbed pitchfork bifurcation. Here we use the buckled mode shapes and frequencies in the Galerkin discretization as well as the first buckled configuration, as developed by Nayfeh and coauthors [32, 33]. We study the transverse vibration induced by the electrostatic force around the static buckled configuration. To do so, we split the static deflection induced by both thermal stress and electrostatic force, $w_{s,b}(x)$, into two components as follows

$$w_{s,b}(x) = \psi(x) + \chi(x) \quad (15)$$

where $\psi(x)$ is the buckled configuration given as follows:

$$\psi(x) = \frac{1}{2} b_1 [1 - \cos(2\pi x)] \quad (16)$$

where b_1 is the rise at midpoint of the microbeam of the first buckling mode of the clamped-clamped microbeam and is given by the following expression:

$$b_1 = \sqrt{\frac{2[P - 4\pi^2]}{\alpha_1 \pi^2}} \quad (17)$$

and $\chi(x)$ is deflection induced by the electrostatic force governed by

$$\begin{aligned} \chi^{iv} = & \left[N_0 - S_{Th} + \alpha_1 \int_0^1 \chi'^2 dx + \alpha_1 \int_0^1 \psi'^2 dx + 2\alpha_1 \int_0^1 \psi' \chi' dx \right] \chi'' \\ & + \left[\alpha_1 \int_0^1 \chi'^2 dx + 2\alpha_1 \int_0^1 \psi' \chi' dx \right] \psi'' \quad (18) \\ & + \alpha_2 \frac{V_{DC}^2}{(1-\psi-\chi)^2} \end{aligned}$$

To solve (18), we refer to the Galerkin procedure in which we use the undamped linear mode shapes of a buckled beam given by Nayfeh and coauthors [32,33].

Next, we determine the variation of the natural frequency of the buckled microbeam under the DC polarization voltage and as varying the electrothermal voltage. Toward this, we solve the eigenvalue problem obtained by perturbing the deflection around the static configuration. Thus,

$$w(x,t) = w_{s,b}(x) + y(x,t) \quad (19)$$

The Galerkin discretization is used to represent the dynamic amplitude $y(x,t)$ as

$$y(x,t) = \sum_{i=0}^n z_i(t) \varphi_i(x) \quad (20)$$

where $z_i(t)$ ($i=0,1,2..n$) denotes the nondimensional modal coordinates and $\varphi_i(x)$ is the mode shape of the buckled unactuated beam [32,33].

Then, the eigenvalue problem of the buckled microbeam under electrothermal and electrostatic voltages is given by the below equation [28]:

$$\begin{aligned} \ddot{z}_j + z_j \omega_j^2 = & \\ & \left[2\alpha_1 \int_0^1 \left(\varphi_j \frac{\partial^2 w_s}{\partial x^2} \right) dx \right] \int_0^1 \left(\sum_{i=0}^n u_i \varphi_i \frac{\partial w_s}{\partial x} \right) dx \\ & + \left[\alpha_1 \int_0^1 \left(\frac{\partial w_s}{\partial x} \right)^2 dx \right] \int_0^1 \left(\varphi_j \left(\sum_{i=0}^n u_i \varphi_i \right) \right) dx \quad (21) \\ & + \int_0^1 \left[\varphi_j \frac{2\alpha_2 V_{DC}^2}{(1-w_s)^3} \left(\sum_{i=0}^n u_i \varphi_i \right) \right] dx \end{aligned}$$

III. Experimental Setup

The experimental investigation was conducted on an in-plane 600 μm long clamped-clamped microbeam fabricated from SOI wafers with highly conductive 25 μm Si device layer by MEMSCAP [34]. Micro System Analyzer with in-plane microstructure vibration and motion analysis using stroboscopic video microscopy from Polytec [35], Fig. 2, is used to determine the deflection as well as the resonance frequency of the microbeam.

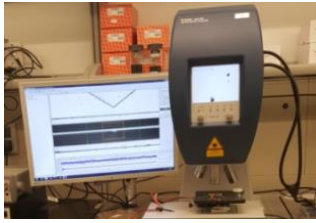


Figure 2. Experimental setup.

At the beginning, the microbeam under consideration is actuated electrothermally by passing a DC current through it. A topographical characterization is used to determine the static deflection of the mid-point of the microbeam as varying the electrothermal voltage. The resonance frequencies of the microbeam are measured, as varying the DC electrothermal voltage, using ring down measurement and getting the fast Fourier transform (FFT). The FFT for a zero electrothermal voltage of the microbeam under consideration is depicted in Fig. 3. The first resonance frequency of the unactuated microbeam is found at 57 kHz.

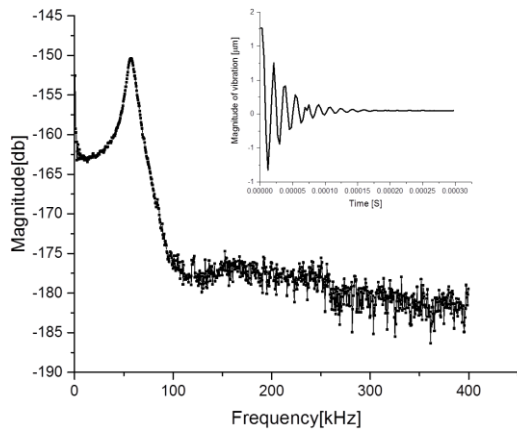


Figure 3. FFT of the unactuated in-plane clamped-clamped microbeam.

Then, the microbeam under consideration is actuated in addition to the electrothermal actuation electrostatically by adding an external circuit. We applied a constant DC bias voltage to actuate the microbeam electrostatically and as varying the DC electrothermal voltage we measured the resonance frequency using the ring down measurement.

IV. Results and Discussion

In this study, to further verify the analytical findings, we refer to the 3D multi-physics finite-element simulation that takes into account the structural, electrothermal, and electrostatic domains. The analysis is done using the commercial finite element software COMSOL [34].

The microbeam is first actuated electrothermally without including the electrostatic force. The electrothermal voltage V_{Th} is increased from small values, and therefore the compressive stress inside the microbeam increases. The exact solution of the static deflection due to the electrothermal actuation is given by the first buckling configuration (16). Fig. 4 shows the measured and the simulated mid-point deflection of the microbeam due to

electrothermal actuation without the electrostatic force demonstrating the critical buckling limit. After this limit, the microbeam is no longer straight and it is buckled. Fig. 4 demonstrates that the microbeam encounters a pitchfork bifurcation at the critical buckling limit that describes the required voltage to buckle the microbeam. A good agreement is shown among the analytical, finite element, and experimental results.

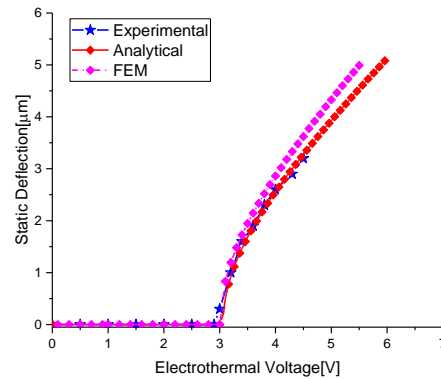


Figure 4. The mid-point deflection of the microbeam under electrothermal actuation.

The variation of the first resonance frequency of the studied resonator as varying the electrothermal actuation voltage without including the electrostatic force is shown in Fig. 5. The pre-buckling eigenvalue is determining by solving the eigenvalue problem of the straight microbeam under axial load presented in (14). For the buckling problem, the eigenvalue problem of buckled beam presented in the appendix is solved. As shown in Fig. 5, the frequency decreases from the initial resonance frequency to almost zero before buckling. After buckling, the fundamental frequency increases from zero to higher values, which can be as high as double the original resonance frequency. In that case, the behavior of the beam is transformed from straight to buckle beam. A good agreement is shown among the analytical, finite element, and experimental results.

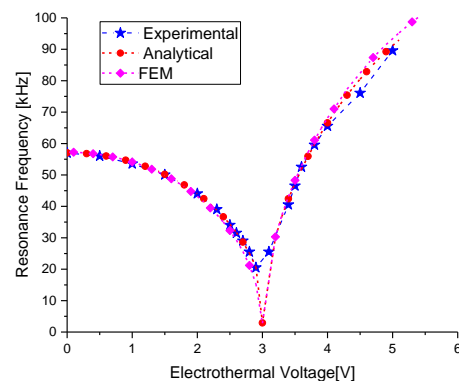


Figure 5. The first resonance frequency of the microbeam under electrothermal actuation with no DC electrostatic bias voltage.

Next, we combined the electrothermal actuation with the electrostatic force. We applied a constant DC polarization voltage, far from the pull-in voltage, between the microbeam and the stationary electrode, and then we varied the

electrothermal voltage. The analytical results of the static deflection of the mid-point of the microbeam as varying the electrothermal voltage for a fixed DC polarization voltage is shown in Fig. 6. The static deflection is obtained by solving (11), before the critical load, and (15), after exceeding the critical load. Fig. 6 shows that the microbeam encounters a perturbed pitchfork bifurcation, where the discontinuity in the deflection curve observed in Fig. 4 disappears.

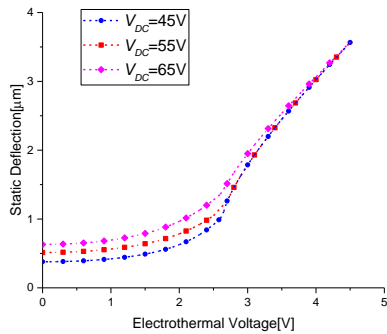
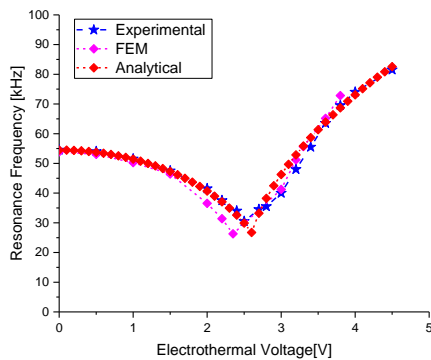
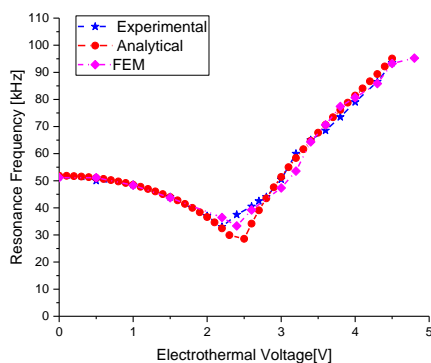


Figure 6. The static deflection versus electrothermal actuation voltage for a constant DC electrostatic bias voltage.

Then, solving the system of algebraic equations, (14), for the pre-buckling behavior and the system of algebraic equations, (21) for the post-buckling behavior, we compute the first natural frequency of the microbeam under constant V_{DC} and as varying V_{Th} . Figures 7.a, 7.b display the first resonance frequency of the microbeam versus the electrothermal actuation for a DC electrostatic bias voltage equal to 45V and 65V, respectively. A good agreement is shown among the analytical, finite element, and experimental results.



(a)



(b)

Figure 7. The first resonance frequency of the microbeam versus electrothermal actuation voltage for a constant DC electrostatic bias voltage. (a) $V_{DC} = 45V$. (b) $V_{DC} = 65V$.

One can note that adding a DC electrostatic bias changes the qualitative nature of the tunability both before and after buckling, which adds another independent way of tuning. Fig. 8 further clarifies this aspect. The figure shows the resonance frequency, computed analytically, while varying the electrothermal voltage for different values of DC polarization voltages. Adding a DC electrostatic bias reduces the dip in the resonance frequency before buckling, and can eliminate it if desired with increasing the DC polarization voltage, and further enhances the increase in the resonance frequency after buckling. Additionally, Fig. 8 displays that the initial natural frequency, at zero electrothermal voltage, increases as the DC voltage exceeds 65V. This fact is due to the dominating effect of mid-plane stretching over electrostatic force for large gaps, as shown in previous works [27, 37].

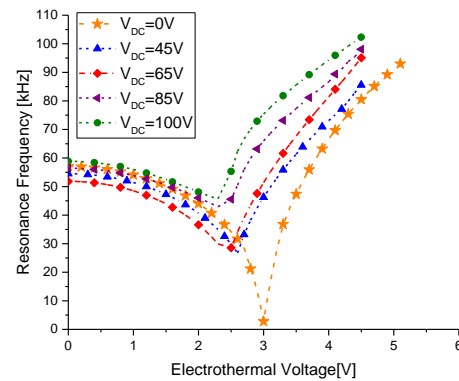


Figure 8. The first resonance frequency of the microbeam versus electrothermal actuation voltage for different constant DC electrostatic bias voltages.

v. Conclusions:

In this paper, we investigated experimentally, analytically, and numerically, using a finite-element method, the tunability of in-plane clamped-clamped microbeam electrothermally and electrostatically actuated. We compared these results to both the experimental and the simulated data. A good agreement is shown among analytical, experimental, and numerical results. At first we studied the static deflection and the variation of the first natural frequency of the microbeam under only electrothermal actuation. We showed that as increasing the DC electrothermal voltage the microbeam buckles after a certain critical electrothermal voltage and it is no longer straight beam. Before buckling, the fundamental frequency decreases until the resonance frequency drops to very low values (almost zero). After buckling, the resonance frequency increases to high values, which can reach the double of the original resonance frequency. Then, we added a DC electrostatic bias in addition to the electrothermal actuation. The microbeam encounters a perturbed pitchfork bifurcation due to DC polarization voltage. We showed that the dip in the resonance frequency before buckling is reduced and the resonance frequency after buckling is increased as increasing the DC polarization voltage. In conclusion, we

demonstrate that a single resonator electrothermally and electrostatically actuated can be operated at a wide range of resonance frequency, as low as almost zero frequency to as high as twice of its unactuated resonance frequency by only controlling the electrothermal and the electrostatic voltages.

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