

# The Obtaining of Dynamic Equations for Three Degree of Freedom Parallel Robot

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**Abstract**— The purpose of this study is to find dynamic equations for three degree of freedom triglide parallel robot, which is a member of the family of delta parallel robot. The Lagrangian method has been used for this aim. Dynamic equations have been obtained by utilizing abbreviation symbols. In addition to that, mathematical operations have been given for one arm of the robot in this paper.

**Keywords**—triglide, parallel robot, dynamic, lagrangian.

## I. Introduction

Parallel architecture consists of mobile platform connected to a fixed platform with several kinematic chains. Movement of the mobile platform is achieved with spurred extensions of the kinematic chains simultaneously. In the same way, load carried by mobile platform is supported by the various kinematic chains; therefore, this architecture is referred to as parallel architecture. In contrast to serial manipulators having an open kinematic chain structure, parallel manipulators consists of closed kinematic chain. In addition, parallel manipulators exhibit several advantages and disadvantages. The disadvantages of parallel robots are limited workspace, low-skill ability, complexity of forward kinematic solution and singularities. However, parallel robots have advantages such as high stiffness, ease of obtaining inverse kinematic solution, light weight, high accuracy, low inertia of moving parts and high agility. Ease of obtaining inverse kinematics solution provides ability to control readily. Hybrid robots comprise a combination both of parallel and serial robot structures (Sciavicco 1996; Merlet 1996).

Relatively small workspace of parallel robots can reveal an important safety characteristic if designed correctly. Parallel robots move safely near singularity. Forces needed from engines reach high values, when the robot follows a path towards a singular position. The singular positions in serial robots require very high value of joint speeds. This situation reveals a danger for working conditions (Merlet 1992; Bi & Lang 2009; Merlet 1995).

## II. Triglide Parallel Robot

Delta robot, which has been used in wide application fields in recent years, is one of the types of parallel manipulators. In delta robot system, end effector can be made to move to any location  $(x, y, z)$  within workspace via a third motor mounted on a fixed limb. The arm connected to each motor is driven and the arms are fastened to end effector plate by parallelogram mechanisms. Therefore, mobile platform has to be parallel to fixed platform every time. The system, with a small movement of the motors, may reach high accelerations and can be positioned very quickly. Rigidity of the system depends on stiffness of selected arms and joint spaces. The smallest deviations in the manufacturing affect high amount on repeat accuracy of the system.

Triglide robot, a bit more simplified version of the delta robot, is a three degree of freedom parallel robot. Triglide robot has three arms and these arms can be moved by linear motors, seen in Figure 1. The biggest advantage of triglide robot is to allow an infinite range of motion in the  $z$  direction (Mustafa & Misuari 2007; Stan & Manic 2008).

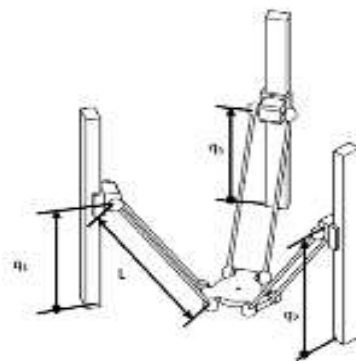


FIGURE 1. Triglide parallel robot (Stan & Manic 2008).

1, 2 and 3 indexes represent the points of contact to the fixed and mobile platforms of each parallelogram. Fixed columns are positioned with  $120^\circ$  angle to each other.

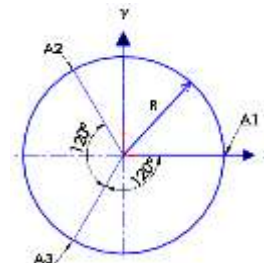


FIGURE 2. Fixed platform.

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A fixed platform shown in “Fig.2” is chosen as the bottom point of fix columns where the linear motor moves. Then, the fixed axis frame is located to the midpoint of the fixed platform as shown in “Fig.2”. Finally, z direction is selected on the direction of gravity.

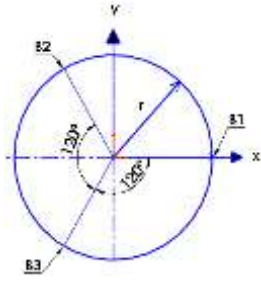


FIGURE 3. Mobile platform.

As shown in “Fig.2” and “Fig.3”, the fixed and mobile platforms are circular, and their radius are R and r respectively. Touching places to the fix columns of the fixed platform are  $A_1, A_2$  and  $A_3$ . The points connected to the parallelogram of mobile platform are  $B_1, B_2$  and  $B_3$ .

Dynamic simulation works about Triglide is provided by Rat & Neagoe (2011). In literature, many studies are labored as to the development, optimal design, and the inverse kinematics of the triglide robot, and also to the virtual reality interface and control via SimMechanics model (Stan & Manic 2008; Budde & Last 2007; Budde & Rose 2008; Verdeş & Stan 2009).

In this study, dynamic equations are analytically obtained by utilizing abbreviation symbols for Triglide robot. The Lagrangian method are used for calculatings.

### III. Dynamic Calculations for Triglide Parallel Robot

Forward kinematics solutions are needed for dynamic calculations of Triglide robot. These solutions are obtained by Aydin & Alli (2012) and they have been given below.

$$z_p = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (1)$$

$$x_p = \frac{2q_1^2 - q_2^2 - q_3^2 - (4q_1 - 2q_2 - 2q_3)z_p}{6(R-r)} \quad (2)$$

$$y_p = \frac{q_2^2 - q_3^2 + 2(q_3 - q_2)z_p}{2\sqrt{3}(R-r)} \quad (3)$$

a, b and c in equation (2.1) are expressed respectively as follows.

$$a = 1 + \frac{(4q_1 - 2q_2 - 2q_3)^2}{36(R-r)^2} + \frac{(q_3 - q_2)^2}{3(R-r)^2} \quad (4)$$

$$b = \frac{(-2q_1 - 2q_2 - 2q_3)}{3} - \frac{(2q_1^2 - q_2^2 - q_3^2)(4q_1 - 2q_2 - 2q_3)}{18(R-r)^2} + \frac{(q_2^2 - q_3^2)(q_3 - q_2)}{3(R-r)^2} \quad (5)$$

$$c = -L^2 + (R-r)^2 + \frac{(q_1^2 + q_2^2 + q_3^2)}{3} + \frac{(2q_1^2 - q_2^2 - q_3^2)^2}{36(R-r)^2} + \frac{(q_2^2 - q_3^2)^2}{12(R-r)^2} \quad (6)$$

TABLE 1. The coordinates of the center of gravity of Triglide paralel robot.

First engine	$(R, 0, q_1)$
Second engine	$\left(-\frac{R}{2}, \frac{R\sqrt{3}}{2}, q_2\right)$
Third engine	$\left(-\frac{R}{2}, -\frac{R\sqrt{3}}{2}, q_3\right)$
First parallelogram	$\left(\frac{(x_p + r + R)}{2}, \frac{y_p}{2}, \frac{(z_p + q_1)}{2}\right)$
Second parallelogram	$\left(\frac{(2x_p - R - r)}{4}, \frac{(2y_p + R\sqrt{3} + r\sqrt{3})}{4}, \frac{(z_p + q_2)}{2}\right)$
Third parallelogram	$\left(\frac{(2x_p - R - r)}{4}, \frac{(2y_p - R\sqrt{3} - r\sqrt{3})}{4}, \frac{(z_p + q_3)}{2}\right)$
Mobile platform	$(x_p, y_p, z_p)$

The coordinates of the center of gravity of Triglide paralel robot are given in Table 1. Dynamics of the system are obtained by using the coordinates of the center of gravity of elements by the Lagrangian method. The frictions are ignored.

The total kinetic energy of the system can be written by taking into considerations of engines, parallelograms and mobile platform and it is shown in equation (2.7).

$$T = \frac{1}{8}m_p(3(\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2) + 2\dot{z}_p(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) + \dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) + \frac{1}{2}m_{motor}(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) + \frac{1}{2}m_{tabla}(\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2) \quad (7)$$

In the same way, the total potantial energy are calculated in equation (2.8).

$$V = m_{motor}g(q_1 + q_2 + q_3) + m_p g \left( \frac{q_1 + q_2 + q_3}{2} + \frac{3z_p}{2} \right) + m_{tabla}gz_p \quad (8)$$

Consequently, the Lagrangian function has been expressed in equation (2.9).

$$L = T - V \quad (9)$$

Equation of motion for  $q_1$  (first arm) can be found from equation (2.10).

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_1} \right) - \frac{\partial L}{\partial q_1} = F_1 \quad (10)$$

When  $\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_1} \right)$  and  $\frac{\partial L}{\partial q_1}$  expressions are written instead in equation (2.10), equation of motion is obtained as below.

$$\begin{aligned}
 & m_{motor}\ddot{q}_1 + \frac{3}{4}m_p\ddot{x}_p \frac{\partial \dot{x}_p}{\partial \dot{q}_1} + \frac{3}{4}m_p\ddot{y}_p \frac{\partial \dot{y}_p}{\partial \dot{q}_1} + \frac{3}{4}m_p\ddot{z}_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \\
 & + \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} (\dot{q}_1 + \dot{q}_2 + \dot{q}_3) + \frac{1}{4}m_p\ddot{x}_p + \frac{1}{4}m_p\ddot{q}_1 + m_{tabla}\ddot{x}_p \frac{\partial \dot{x}_p}{\partial \dot{q}_1} \\
 & + m_{tabla}\ddot{y}_p \frac{\partial \dot{y}_p}{\partial \dot{q}_1} + m_{tabla}\ddot{z}_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} + A_1 = F_1 \quad (11)
 \end{aligned}$$

$m_{motor}$ ,  $m_p$  ve  $m_{tabla}$  in equation (11) are constant values and they are masses of engine, parallelogram and mobile platform respectively. Also,  $F_1$  is torque of the first engine. The equivalence of expression  $A_1$  is given in equation (12).

$$\begin{aligned}
 A_1 = & \frac{3}{4}m_p \left( \dot{x}_p \frac{d}{dt} \left( \frac{\partial \dot{x}_p}{\partial \dot{q}_1} \right) + \dot{y}_p \frac{d}{dt} \left( \frac{\partial \dot{y}_p}{\partial \dot{q}_1} \right) + \dot{z}_p \frac{d}{dt} \left( \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) \right) + \frac{1}{4}m_p \left( \frac{d}{dt} \left( \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) (\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \right) \\
 & + \frac{1}{4}m_p \left( \frac{d}{dt} \left( \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) (\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \right) + m_{tabla} \left( \dot{x}_p \frac{d}{dt} \left( \frac{\partial \dot{x}_p}{\partial \dot{q}_1} \right) + \dot{y}_p \frac{d}{dt} \left( \frac{\partial \dot{y}_p}{\partial \dot{q}_1} \right) + \dot{z}_p \frac{d}{dt} \left( \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) \right) \\
 & - m_{tabla} \left( \dot{x}_p \frac{\partial \dot{x}_p}{\partial \dot{q}_1} + \dot{y}_p \frac{\partial \dot{y}_p}{\partial \dot{q}_1} + \dot{z}_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) - \frac{1}{4}m_p \left( 3 \left( \dot{x}_p \frac{\partial \dot{x}_p}{\partial \dot{q}_1} + \dot{y}_p \frac{\partial \dot{y}_p}{\partial \dot{q}_1} + \dot{z}_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) + \frac{\partial \dot{z}_p}{\partial \dot{q}_1} (\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \right) \\
 & + m_{motor}g + m_p g \left( \frac{1}{2} + \frac{3}{2} \frac{\partial z_p}{\partial q_1} \right) + m_{tabla}g \frac{\partial z_p}{\partial q_1} \quad (12)
 \end{aligned}$$

If expressions  $\ddot{q}_1$ ,  $\ddot{q}_2$  ve  $\ddot{q}_3$  in equation (11) are gathered in common bracket, the equation of motion for  $q_1$  is expressed as follows by using abbreviations.

$$\begin{aligned}
 & \left( m_{motor} + \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} + \frac{1}{4}m_p \right) \ddot{q}_1 + \left( \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) \ddot{q}_2 + \left( \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) \ddot{q}_3 \\
 & + B_1\ddot{x}_p + C_1\ddot{y}_p + D_1\ddot{z}_p + A_1 = F_1 \quad (13)
 \end{aligned}$$

In the above equation, expressions  $B_1$ ,  $C_1$  and  $D_1$  are given in equations (14), (15) and (16).

$$B_1 = \left( \frac{3}{4}m_p \frac{\partial \dot{x}_p}{\partial \dot{q}_1} + m_{tabla} \frac{\partial \dot{x}_p}{\partial \dot{q}_1} \right) \quad (14)$$

$$C_1 = \left( \frac{3}{4}m_p \frac{\partial \dot{y}_p}{\partial \dot{q}_1} + m_{tabla} \frac{\partial \dot{y}_p}{\partial \dot{q}_1} \right) \quad (15)$$

$$D_1 = \left( \frac{3}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} + \frac{1}{4}m_p + m_{tabla} \frac{\partial \dot{z}_p}{\partial \dot{q}_1} \right) \quad (16)$$

$\ddot{x}_p$  can be expressed as in equation (17).

$$\begin{aligned}
 \ddot{x}_p = & E + \left( \frac{4q_1 - 4z_p}{6(R-r)} \right) \ddot{q}_1 - \left( \frac{2q_2 - 2z_p}{6(R-r)} \right) \ddot{q}_2 - \left( \frac{2q_3 - 2z_p}{6(R-r)} \right) \ddot{q}_3 \\
 & - \frac{(4q_1 - 2q_2 - 2q_3)\ddot{z}_p}{6(R-r)} \quad (17)
 \end{aligned}$$

E in the above equation has been shown as follows.

$$E = \frac{4\dot{q}_1^2 - 2\dot{q}_2^2 - 2\dot{q}_3^2 - (8\dot{q}_1 - 4\dot{q}_2 - 4\dot{q}_3)\dot{z}_p}{6(R-r)} \quad (18)$$

$\ddot{y}_p$  can be written as below.

$$\ddot{y}_p = G + \left( \frac{2q_2 - 2z_p}{2\sqrt{3}(R-r)} \right) \ddot{q}_2 - \left( \frac{2q_3 - 2z_p}{2\sqrt{3}(R-r)} \right) \ddot{q}_3 + \frac{2(q_3 - q_2)}{2\sqrt{3}(R-r)} \dot{z}_p \quad (19)$$

G in equation (19) are given in equation (20).

$$G = \frac{2\dot{q}_2^2 - 2\dot{q}_3^2 + 4(\dot{q}_3 - \dot{q}_2)\dot{z}_p}{2\sqrt{3}(R-r)} \quad (20)$$

When  $\ddot{x}_p$  and  $\ddot{y}_p$  statements are located in equation (13), equation (21) is obtained.

$$\begin{aligned}
 & \left( m_{motor} + \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} + \frac{1}{4}m_p + B_1 \left( \frac{4q_1 - 4z_p}{6(R-r)} \right) \right) \ddot{q}_1 \\
 & + \left( \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} - B_1 \left( \frac{2q_2 - 2z_p}{6(R-r)} \right) + C_1 \left( \frac{2q_2 - 2z_p}{2\sqrt{3}(R-r)} \right) \right) \ddot{q}_2 \\
 & + \left( \frac{1}{4}m_p \frac{\partial \dot{z}_p}{\partial \dot{q}_1} - B_1 \left( \frac{2q_3 - 2z_p}{6(R-r)} \right) - C_1 \left( \frac{2q_3 - 2z_p}{2\sqrt{3}(R-r)} \right) \right) \ddot{q}_3 \\
 & + \left( -B_1 \frac{(4q_1 - 2q_2 - 2q_3)}{6(R-r)} + C_1 \frac{2(q_3 - q_2)}{2\sqrt{3}(R-r)} + D_1 \right) \dot{z}_p \\
 & + B_1E + C_1G + A_1 = F_1 \quad (21)
 \end{aligned}$$

$\ddot{z}_p$  can be written as follows

$$\ddot{z}_p = H\ddot{a} + J\ddot{b} + K\ddot{c} + L \quad (22)$$

In the above equation, expressions H, J, K and L are given in equations (23), (24), (25) and (26) respectively.

$$H = \left( \frac{b + \sqrt{b^2 - 4ac}}{2a^2} + \frac{2c}{2a\sqrt{b^2 - 4ac}} \right) \quad (23)$$

$$J = \left( -\frac{1}{2a^2} - \frac{b}{2a\sqrt{b^2 - 4ac}} \right) \quad (24)$$

$$K = \frac{2a}{2a\sqrt{b^2 - 4ac}} \quad (25)$$

$$\begin{aligned}
 L = & \frac{\dot{a}(b^2 - 4ac)^{\frac{1}{2}}(b\dot{b} - 2\dot{a}c - 2\dot{a}c) - 4a\dot{a}(\dot{a}b + \dot{a}\sqrt{b^2 - 4ac} - \dot{a}b)}{2a^2} + \frac{4\dot{a}c - \dot{b}^2}{2a\sqrt{b^2 - 4ac}} \\
 & - \frac{(2\dot{a}\sqrt{b^2 - 4ac} + a(b^2 - 4ac)^{\frac{1}{2}}2(b\dot{b} - 2\dot{a}c - 2\dot{a}c))(2\dot{a}c + 2\dot{a}c - b\dot{b})}{4a^2(b^2 - 4ac)} \quad (26)
 \end{aligned}$$

$\ddot{a}$  are expressed as in equation (27).

$$\ddot{a} = M\ddot{q}_1 + N\ddot{q}_2 + O\ddot{q}_3 + P \quad (27)$$

M, N, O and P in equation (2.30) are respectively shown below.

$$M = \frac{4(2q_1 - q_2 - q_3)}{9(R-r)^2} \quad (28)$$

$$N = \left( -\frac{4(q_1 - 2q_2 + q_3)}{9(R-r)^2} \right) \quad (29)$$

$$O = \left( -\frac{4(q_1 - q_2 + 2q_3)}{9(R-r)^2} \right) \quad (30)$$

$$P = \frac{4(2\dot{q}_1 - \dot{q}_2 - \dot{q}_3)\dot{q}_1 - 4(\dot{q}_1 - 2\dot{q}_2 + \dot{q}_3)\dot{q}_2 - 4(\dot{q}_1 - \dot{q}_2 + 2\dot{q}_3)\dot{q}_3}{9(R-r)^2} \quad (31)$$

Expression  $\ddot{b}$  is simplified as in equation (32).

$$\ddot{b} = R\ddot{q}_1 + S\ddot{q}_2 + T\ddot{q}_3 + U \quad (32)$$

Expressions R, S, T and U in the above equation are given in equations (33), (34), (35) and (36) orderly.

$$R = \left( -\frac{2}{3} + \frac{(4q_1q_2 + 4q_1q_3 - 12q_1^2 + 2q_2^2 + 2q_3^2)}{9(R-r)^2} \right) \quad (33)$$

$$S = -\frac{2}{3} + \frac{(4q_1q_2 + 4q_2q_3 + 2q_1^2 - 12q_2^2 + 2q_3^2)}{9(R-r)^2} \quad (34)$$

$$T = -\frac{2}{3} + \frac{(4q_1q_3 + 4q_2q_3 + 2q_1^2 + 2q_2^2 - 12q_3^2)}{9(R-r)^2} \quad (35)$$

$$U = \frac{(2\dot{q}_2^2 - 2\dot{q}_3^2)(q_3 - q_2) + 4(\dot{q}_3 - \dot{q}_2)(q_2\dot{q}_2 - q_3\dot{q}_3)}{3(R-r)^2} - \frac{4(2\dot{q}_1^2 - \dot{q}_2^2 - \dot{q}_3^2)(2q_1 - q_2 - q_3)}{18(R-r)^2} - \frac{8(2q_1\dot{q}_1 - q_2\dot{q}_2 - q_3\dot{q}_3)(2\dot{q}_1 - \dot{q}_2 - \dot{q}_3)}{18(R-r)^2} \quad (36)$$

$\ddot{c}$  can be expressed as in equation (37).

$$\ddot{c} = W\ddot{q}_1 + V\ddot{q}_2 + Y\ddot{q}_3 + Z \quad (37)$$

Expressions W, V, Y and Z in the above equation are given in equations (38), (39), (40) and (41) respectively.

$$W = \left( \frac{2q_1(2q_1^2 - q_2^2 - q_3^2)}{9(R-r)^2} + \frac{2q_1}{3} \right) \quad (38)$$

$$V = \left( -\frac{q_2(2q_1^2 - 4q_2^2 - 2q_3^2)}{9(R-r)^2} + \frac{2q_2}{3} \right) \quad (39)$$

$$Y = \left( -\frac{q_3(2q_1^2 + 2q_2^2 - 4q_3^2)}{9(R-r)^2} + \frac{2q_3}{3} \right) \quad (40)$$

$$Z = \frac{2(2q_1\dot{q}_1 - q_2\dot{q}_2 - q_3\dot{q}_3)^2 + (2q_1^2 - q_2^2 - q_3^2)(2\dot{q}_1^2 - \dot{q}_2^2 - \dot{q}_3^2)}{9(R-r)^2} \quad (41)$$

$$+ \frac{2\dot{q}_1^2 + 2\dot{q}_2^2 + 2\dot{q}_3^2}{3} + \frac{2(q_2\dot{q}_2 - q_3\dot{q}_3)^2 + (q_2^2 - q_3^2)(\dot{q}_2^2 - \dot{q}_3^2)}{3(R-r)^2} \quad (41)$$

The abbreviated form of  $\ddot{z}_p$  is found, if equation (22) is rearranged by writing equations (27), (32) and (37).

$$\ddot{z}_p = (HM + JR + KW)\ddot{q}_1 + (HN + JS + KV)\ddot{q}_2 + (HO + JT + KY)\ddot{q}_3 + HP + JU + KZ + L \quad (42)$$

When the expression  $\ddot{z}_p$  given above is located in equation (21), the equation of motion for  $q_1$  is obtained in equation (43).

$$a_1\ddot{q}_1 + b_1\ddot{q}_2 + c_1\ddot{q}_3 = d_1 \quad (43)$$

The equivalence of the statements  $a_1$ ,  $b_1$ ,  $c_1$  and  $d_1$  are given below as equations (44), (45), (46) and (47) orderly.

$$a_1 = m_{motor} + \frac{1}{4}m_p \frac{\partial \ddot{z}_p}{\partial \dot{q}_1} + \frac{1}{4}m_p + B_1 \left( \frac{4q_1 - 4z_p}{6(R-r)} \right) + Q_1HM + Q_1JR + Q_1KW \quad (44)$$

$$b_1 = \frac{1}{4}m_p \frac{\partial \ddot{z}_p}{\partial \dot{q}_1} - B_1 \left( \frac{2q_2 - 2z_p}{6(R-r)} \right) + C_1 \left( \frac{2q_2 - 2z_p}{2\sqrt{3}(R-r)} \right) + Q_1HN + Q_1JS + Q_1KV \quad (45)$$

$$c_1 = \frac{1}{4}m_p \frac{\partial \ddot{z}_p}{\partial \dot{q}_1} - B_1 \left( \frac{2q_3 - 2z_p}{6(R-r)} \right) - C_1 \left( \frac{2q_3 - 2z_p}{2\sqrt{3}(R-r)} \right) + Q_1HO + Q_1JT + Q_1KY \quad (46)$$

$$d_1 = (F_1 - Q_1HP - Q_1JU - Q_1KZ - Q_1L - B_1E - C_1G - A_1) \quad (47)$$

$Q_1$  is expressed in equation (48).

$$Q_1 = \left( -B_1 \frac{(4q_1 - 2q_2 - 2q_3)}{6(R-r)} + C_1 \frac{2(q_3 - q_2)}{2\sqrt{3}(R-r)} + D_1 \right) \quad (48)$$

Also, similar steps can be used for the other arms of Triglide parallel robot. The equations of motion for  $q_2$  and  $q_3$  are found as follows.

$$a_2\ddot{q}_1 + b_2\ddot{q}_2 + c_2\ddot{q}_3 = d_2 \quad (49)$$

$$a_3\ddot{q}_1 + b_3\ddot{q}_2 + c_3\ddot{q}_3 = d_3 \quad (50)$$

After the obtained equations of motion for  $q_1$ ,  $q_2$  and  $q_3$  are written in matrix form,  $\ddot{q}_1$ ,  $\ddot{q}_2$  and  $\ddot{q}_3$  are given as in equations (51), (52) and (53).

$$\ddot{q}_1 = \frac{d_3(b_1c_2 - b_2c_1) - d_2(b_1c_3 - b_3c_1) + d_1(b_2c_3 - b_3c_2)}{(a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1)} \quad (51)$$

$$\ddot{q}_2 = \frac{d_2(a_1c_3 - a_3c_1) - d_3(a_1c_2 - a_2c_1) - d_1(a_2c_3 - a_3c_2)}{(a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1)} \quad (52)$$

$$\ddot{q}_3 = \frac{d_3(a_1b_2 - a_2b_1) - d_2(a_1b_3 - a_3b_1) + d_1(a_2b_3 - a_3b_2)}{(a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1)} \quad (53)$$

## IV. Conclusions

In this work, dynamic equations and accelerations for three degree of freedom triglide parallel robot have been calculated. The obtained equations provide to be known well about dynamic behaviors of the robot.

In the future works, Triglidge parallel robot will be controlled by using suitable nonlinear methods via dynamic equations and accelerations obtained in this paper.

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