

# Mathematical Modeling of the Drying Process of Coconut Meat

Lola Domnina B. Pestaño

**Abstract**— Drying process is a method of food preservation that thrives by removing water from the food, which inhibits the growth of bacteria and has been practiced worldwide since ancient times to preserve food. In the coconut industry, drying the coconut meat reduces its moisture content that inhibits the growth of bacteria and the action of enzymes that causes spoilage. The coconut meat is named copra once drying starts. Well-dried copra contains 7% moisture and is processed into coconut oil (CNO) and copra cake after storage when the moisture content is reduced to about 4-5%, the equilibrium moisture content of copra at most existing storage conditions (Guarte et al., 1996). This study aims to determine the best mathematical model that describes the drying characteristics of thinly-sliced coconut meat. Two mathematical models were examined to describe the drying behavior of thinly sliced coconut meat at 60, 70 and 80°C using a hot-air batch dryer. The Page model gave the best fit to the experimental data with a high coefficient of determination values that could adequately describe the drying characteristic of sliced coconut meat that predicted a drying temperature of 80°C in the shortest average drying time of 181.05 min. to reach the desired moisture content of 7 per cent wet basis.

**Keywords**—copra, coconut oil (CNO), Mathematical Model, correlation factor, time constant, total error

## Nomenclature

Symbols:

$M$	= mass of coconut meat and water, g
$MC$	= moisture content at any time, g water/g dry matter
$MR$	= moisture ratio
$M_e$	= equilibrium mass of coconut meat and water, g
$M_i$	= initial mass of coconut meat and water, g
$R$	= drying rate, g of liquid evaporated per s-m <sup>2</sup> of solid surface
$W_s$	= weight of dry solid
$\bar{X}'$	= bulk moisture content of the solid, g of liquid/g of dry solid at time $t$ .
$\bar{X}'_1$	= bulk moisture content at time 0
$\bar{X}'_2$	= bulk moisture content at time, $t$
$\tau$	= the time constant
$k$	= drying constant
$a$	= drying constant
$n$	= order of the reaction, positive integer

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## I. Introduction

In the coconut industry, drying the coconut meat reduces its moisture content that inhibits the growth of bacteria and the action of enzymes that causes spoilage. The coconut meat is named copra once drying starts. Well-dried copra contains 7% moisture and is processed into coconut oil (CNO) and copra cake after storage when the moisture content is reduced to about 4-5%, the equilibrium moisture content of copra at most existing storage conditions (Guarte et al., 1996).

At present, the copra industry is beset with quality and technical problems that resulted to very low prices of copra-related products in the world market. Quality problems include the contamination by aflatoxin B<sub>1</sub> in copra and copra cake, a cancer-causing toxic metabolite produced by the *Aspergillus* sp. group of moulds, specifically *Aspergillus flavus*, the presence of high contents of free fatty acid (FFA) that increases the refining cost and reduces the oil recovery by 1.4 times (Flynn, 1973), and a carcinogenic and mutagenic polycyclic aromatic hydrocarbon (PAH) in CNO. Among them benzo[*a*]pyrene and dibenzo[*a,h*]anthracene that are present in smoke-cured foods (Larsson, 1986; Head et al, 1999). Aflatoxin and FFA problems are attributed to the high moisture content of copra after drying and during storage (Head, 1991), while PAH-related problems are due to direct contamination with smoke during copra drying. Many of these difficulties stemmed from technical problems of inappropriate drying conditions and poor dryer design. This study is carried out to determine and model the drying kinetics of coconut meat dried in a hot-air batch dryer to eliminate aflatoxin B<sub>1</sub> and benzo[*a*]pyrene in Philippine CNO.

## II. Mathematical Model

A mathematical model is a representation in mathematical terms of the behaviour of real devices and objects. Mathematical modeling is a principled activity that has both principles behind it and methods that can be successfully applied (Dabbaghian and Mago, 2014). Mathematical models are used in the natural sciences and engineering disciplines, as well as in the social sciences. Physicists, engineers, statisticians, operations research analysts, and economists use mathematical models most extensively.

A model may help to describe the behavior of a system, to explain why that behavior and results occurred as they did and to study the effects of different components, and to make predictions about future behaviors that are as yet unseen or unmeasured.

### III. The Drying Procedure

The hot-air batch dryer was installed in an environment with the relative air humidity of about 30–40% and the ambient air temperature about 34°C. The air velocity through the drier was controlled by the speed of the blower. Single layers of thinly sliced coconut meat were arranged on three (3) trays and were heated at temperatures of 60-80°C at 10°C intervals every 30 minutes until constant weight is obtained. Before beginning the experiments, the dryer system was started in order to achieve a desirable steady state condition of temperature and at constant air velocity of 0.80 m s<sup>-1</sup>.

Weighing of air-dried coconut meat was done using a precision balance with an accuracy of 0.01 g. Drying experiments for each temperature were conducted in duplicate and values were recorded.

### IV. Examination of Mathematical Models for Simulation of Drying Data

To select a suitable model for describing the drying process of coconut meat, drying curves were fitted with two thin-layer drying equations. There are a number of mathematical models available to represent the drying mechanism of an object. Page model and Laplace Transform Model yield the highest correlation factor, which is the basis of effective drying parameters.

#### A. The Page Model

The Page model in this study is derived by linearizing the moisture rate:

$$MR = a \exp(-kt^n) \quad (1)$$

Equation (2) is the linearized equation of (1):

$$\ln MR = \ln a - kt^n \quad (2)$$

#### B. The Laplace Transform Model

The Laplace Transform Model used in this study is derived from the material balance obtained from the system in the experiment. Equation (3) was carried out as the input minus the output out of the system equals the rate of accumulation within the system.

$$\tau \frac{dM}{dt} = M_i - M \quad (3)$$

Equation (3) is then formulated to its Laplace transform, integrated and simplified forming (4) used in modelling the drying rate characteristics of the coconut meat.

$$M = M_e - M_e \exp\left(-\frac{t}{\tau}\right) + M_i \exp\left(-\frac{t}{\tau}\right) \quad (4)$$

### V. Calculations

#### A. Moisture Ratio

A number of theoretical, semi-theoretical and empirical drying models have been reported in the literature. The most frequently used model for thin layer drying is the lumped parameter type such as the Newton equation (Liu and Bakker, 1997; Kingsly et al., 2007). The moisture ratio during drying is determined by (5).

$$MR = \frac{M - M_e}{M_i - M_e} = \exp(-kt) \quad (5)$$

For the mathematical analysis, it is assumed that the moisture gradient driving force during drying is a liquid concentration gradient. The effect of heat transfer is neglected as a simplifying assumption. For all experimental conditions, the value of  $(M - M_e)/(M_i - M_e)$ , expressing dimensionless moisture content are obtained.

#### B. Drying Rate and Drying Time

From the experimental data, coconut meat drying rate and drying time were calculated using (6):

$$R = -\frac{W_S}{A} \frac{d\bar{X}'}{dt} \quad (6)$$

Equation (6) can be rearranged and integrated. The integral form (7) was used to obtain the drying time.

$$\int_0^t dt = -\frac{W_S}{A} \int_{\bar{X}'_1}^{\bar{X}'_2} \frac{d\bar{X}'}{R} \quad (7)$$

#### C. Determination of heating time to attain 7% moisture

Lagrange interpolation is a method for producing n<sup>th</sup>-order polynomials that pass through  $n + 1$  points. These polynomials attempt to produce interpolation values of increased accuracy by assuming a curvature in the relationship of the data. Its general equation is represented by (8). The three versions are (9), (10) and (11).

$$f_n(x) = \sum_{i=0}^n L_i(x) f(x_i) \quad (8)$$

First-Order Version:

$$f_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1) \quad (9)$$

Second-Order Version:

$$f_1(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2) \quad (10)$$

Third-Order Version:

$$f_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}f(x_3) \quad (11)$$

Lagrange (third-order version) interpolation was used to determine the heating time of coconut meat when 7% moisture content is desired.

#### D. Brute-Force Method

Brute-force search or exhaustive search, also known as generate and test, is a very general problem-solving technique that consists of systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the problem's statement. It is simply the evaluation of multiple (or every) possible answer(s) for fitness.

Brute-force method is used to find the tau ( $\tau$ ) in the Laplace Transform Model that yields the least total error and was used as a basis in the selection of a mathematical model.

## VI. Results and Discussion

### A. Drying Behaviour of Coconut Meat

The drying curves in Fig. 1 shows the changes in moisture content vs. drying time of the coconut meat at indicated temperatures (60, 70 and 80°C). The drying of coconut meat exhibited the characteristic moisture desorption behaviour. An initial high rate of moisture removal was followed by slower moisture removal in the latter stages. This characteristic behaviour is due to the various forms in which water is present in food products. As the drying time progressed the moisture ratio was observed to decrease non-linearly with increase in drying time at all temperatures. This is a general trend reported for other food products e.g. mulberry, eggplant, tomatoes, sweet pepper, and peach slices (Doymaz, 2004; Ertekin and Yaldiz, 2004; Doymaz, 2007; Vengaiah and Pandey, 2007; Kingsley et al., 2007).

Fig. 1 shows the drying characteristics of copra, indicated by the progressive change of moisture content, was significantly affected by the drying temperature, being fastest at high temperatures.

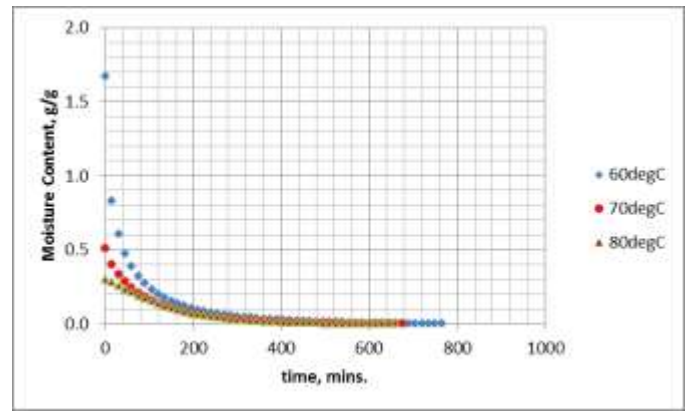


Figure 1. Drying Characteristic of Copra, Corrected Values MC vs. time (60, 70 and 80°C Treatment)

### B. Mathematical Model Interpretation

The drying data obtained (moisture ratio vs drying time) for the hot-air dryer at 60, 70 and 80°C were fitted by Page and Laplace Transform Model. The Page Model was evaluated based on the coefficient of determination ( $R^2$ ) while the Laplace Transform Model was evaluated based on the time constant tau,  $\tau$ . The selection of the model that best describe the drying behavior of coconut meat will be based on the high  $R^2$  and the goodness of fit of a model and the least total error on the time constant, tau,  $\tau$ . Both models gave good results.

### C. Page Model Interpretation

Figs. 2 and 3 show the comparison between the experimental and corrected values of MR vs time using Page Model. The 70°C treatment temperature gave the best coefficient of determination with the value of 0.9961, which was obtained when the rate of the reaction is at 0.88 with the slope and y-intercept values of -0.093 and -1.1313 respectively. Slope is observed to be negative since weight and time are inversely proportional. Other values of the correlation factor obtained ranges from 0.9454 to 0.9834, 0.9816 to 0.9961 to 0.996 and 0.9655 to 0.9934 for 60°C, 70 °C and 80 °C.

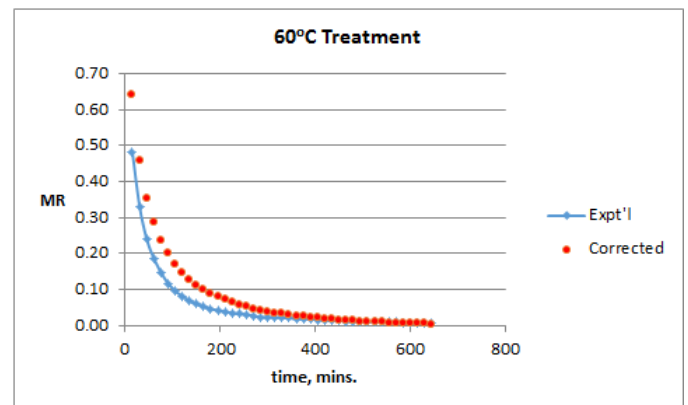


Figure 2. Comparison of the Experimental and Corrected Values of MR vs. time (60°C Treatment, Trial 1 Tray 3)

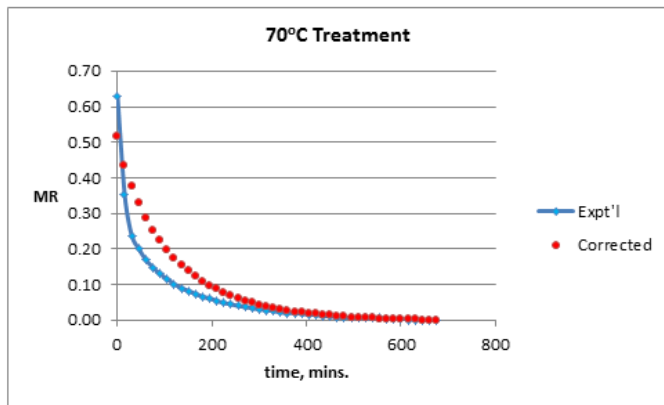


Figure 3. Comparison of the Experimental and Corrected Values of MR vs. time (70°C Treatment, Trial 1, Tray 2)

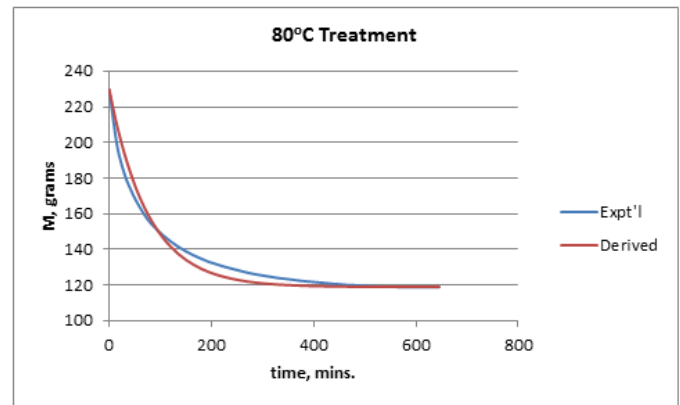


Figure 5. Comparison of the Experimental and Derived Values of M vs. time (80°C Treatment, Trial 1, Tray 2)

#### D. Laplace Transform Model Interpretation

Figs. 4 and 5 show the comparison between the experimental and derived values of MR vs time using the Laplace Transform Model at 60 and 80°C respectively. Brute-force method is used to find the tau ( $\tau$ ) that yields the least total error.

The total error ranges from 167.96 to 291.73, 332.15 to 578.47 and 266.66 to 821.42 for 60°C, 70°C and 80°C respectively. The 60°C treatment temperature, with the total error of 167.96 showed the highest yield among the three temperatures as it gives the lowest error.

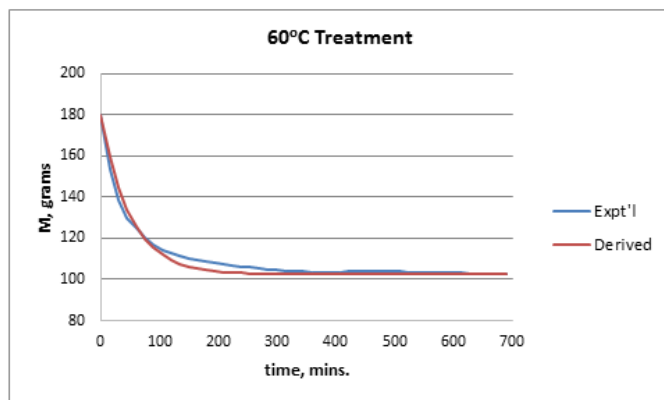


Figure 4. Comparison of the Experimental and Derived Values of M vs. time (60°C Treatment, Trial 2 Tray 3)

#### E. Drying Time Analysis

The drying time for copra to reach the desired MC of 7%, at which attack by *Aspergillus* sp. group of moulds or insects and an increase of free fatty acids (FFA) level in CNO are unlikely to occur (Head, 1991; Thampan, 1975) was significantly affected by the drying temperature. The Lagrange interpolation was used to determine the drying time for copra to reach a MC of 7%. Table I shows a shorter heating time at 80°C was utilized to attain a 7% MC in the coconut meat samples.

TABLE I. CALCULATED DRYING TIME (MINS.) TO ATTAIN 7% MOISTURE AT DIFFERENT TREATMENT TEMPERATURES

Temperature	60°C	70°C	80°C
Trial 1	277.72	253.71	168.19
Trial 2	282.07	243.19	193.9

#### F. Summary of Results

Using for the results for two trials for each of the three trays of the hot-air dryer, values of the coefficient of determination,  $R^2$ , for the Page Model and the time constant, tau,  $\tau$ ; total error from the Laplace Transform Model and the corresponding drying time to attain 7% moisture in the coconut meat samples were recorded and are shown in Table II.

TABLE II. SUMMARY OF RESULTS

Temperature	Trial	Tray	Page Model		Laplace Transform Model		7% Moistur
			n	R <sup>2</sup>	Tau (s)	Total Error	
60 °C	1	1	0.33	0.9454	48.7	176.39	273.30
		2	0.81	0.9902	30.1	291.73	286.51
		3	0.87	0.9834	44.0	205.50	271.25
	2	1	0.38	0.9375	38.2	167.36	296.39
		2	0.89	0.9672	49.2	250.45	278.55
		3	0.83	0.9788	36.1	295.04	271.36
70 °C	1	1	0.98	0.9991	75.0	372.77	244.75
		2	0.88	0.9915	88.1	469.36	258.54
		3	1.00	0.9955	88.3	578.47	257.85
	2	1	1.35	0.9816	74.5	398.65	254.08
		2	0.80	0.9870	95.0	341.53	216.29
		3	1.20	0.9957	58.9	812.15	358.19
80 °C	1	1	1.80	0.9693	74.0	821.42	170.60
		2	1.20	0.9797	75.7	760.40	162.55
		3	1.20	0.9796	83.7	505.14	171.30
	2	1	0.80	0.9958	110.0	332.08	230.71
		2	1.35	0.9655	86.1	526.17	182.76
		3	0.99	0.9915	93.4	286.60	166.34

## VII. Conclusion

The Page Model and the Laplace Transform Model both gave good results in the drying process of coconut meat. Both models gave a good fit to the experimental data. However, results show that the Page Model has a very high coefficient of determination values compared to the Laplace Transform Model that gave a high total error. This concludes that the Page Model could adequately describe the drying characteristic of sliced coconut meat in this study. The Page Model predicted a drying temperature of 80°C in the shortest drying time of 181.05 min. to reach the desired moisture content of 7 per cent wet basis.

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Dr. Pestaño looks forward to advance her study under some form of collaboration with the coconut industry stakeholders.