

Finite Element Modelling of Different 2D Re-entrant Structures of Auxetic Materials

[Mozafar Shokri Rad, Zaini Ahmad*]

Abstract— Auxetic materials exhibit a unique characteristics when subjected to uniaxial loading. Various structures have been used to model these materials. Among most important auxetic structures, re-entrant structures are of interest in this present study. These structures have different shapes in which are known as lozenge grids, sinusoidal ligaments, square grids, double arrowhead, and structurally hexagonal re-entrant honeycomb could be named. In this paper, finite element approach for the abovementioned structures was employed to obtain basic mechanical properties including Poisson's ratio and elastic modulus. The study aims at investigating the effect of cross sectional geometry on mechanical properties. For each structure, three different cross sectional geometries were numerically examined. It is evident that mechanical properties of the material could be controlled by changing the geometry of the cross section. The primary outcome of the study is the design guideline on the effect of cross sectional geometry on mechanical properties of auxetic structures.

Keywords— auxetic; finite element; cross section; re-entrant, Poisson's ratio, elastic modulus

I. Introduction

For the past decades, different geometrical structures exhibiting auxetic behaviour have been fabricated and tested to obtain their mechanical properties. Undoubtedly, these structures are useful since they assist many researchers to comprehend on how auxetic behaviour is obtained and how auxetic materials can be fabricated. Computational procedure is still sparse in predicting their mechanical properties.

Among the most important classes of such auxetic structures are re-entrant structures in which have been investigated by some researchers [1-6]. The other classes are namely chiral structures [7-9], rotating rigid/semi-rigid units[10-14], angle-ply laminates [15, 16], hard molecules [17-20], micro porous polymers [21-23], and liquid crystalline polymer [24 -26]. In particular, the important 2D re-entrant structures are structurally hexagonal honeycomb, double arrow head structures, structures formed from lozenge grids, structure formed from sinusoidal ligaments, and structure formed from square grids [27]. Man-made auxetic materials were first designed by Theocaris et al. [28]. In this study, the design of material was based on structure of arrays with polygonal-shaped inclusions and re-entrant corners.

Zaini Ahmad
Universiti Teknologi Malaysia
Malaysia

Mozafar Shokri Rad
Universiti Teknologi Malaysia
Malaysia

In the present study, finite element technique has been employed in modelling these structures to represent the characteristic of the auxetic re-entrant structures. Five different shapes of 2D re-entrant structures have been considered to examine the influence of cross sectional geometry on the mechanical properties. It is evident that mechanical properties of auxetic material are controllable, thus facilitating the fabrication technique used in preparing samples in the laboratory.

II. Finite element modeling of different 2D re-entrant structures of auxetic materials

There are different re-entrant structures introduced by Liu *et al* [27]. Based on their shape, they were named lozenge grids, sinusoidal ligaments, square grids, double arrowhead, and structurally hexagonal re-entrant honeycomb. These structures are shown in Figure 1.

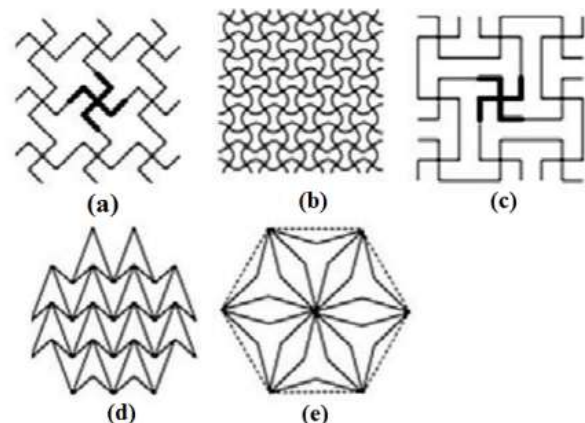


Figure 1. Different shapes of 2D re-entrant structures. (a): Lozenge grids. (b): Sinusoidal ligaments. (c): Square grids. (d): Double arrowhead. (e): Structurally hexagonal re-entrant honeycomb [27]

In the present paper, finite element modelling of the abovementioned structures has been carried out to obtain their basic mechanical properties in terms of Poisson's ratio and elasticity modulus. Element used in the modelling is linear tetrahedral, type C3D4. The optimized mesh size was determined from the convergent study. Each model was loaded along x direction with the imposed normal deflections. The Poisson's ratio and Young's modulus were then calculated.

Using the defined values for the lengths of connecting rods and their rigid property, one can determine strains ϵ_x , ϵ_y and Poisson's ratio ν_{xy} of the 2D re-entrant structures shown in Figure 1 parametrically by using the following equations.

$$\varepsilon_x = \frac{\Delta L}{L}, \varepsilon_y = \frac{\Delta H}{H} \quad (1)$$

$$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{(\Delta H)(L)}{(\Delta L)(H)} \quad (2)$$

Where H and L are height and width of the 2D structure, respectively.

Normal stress along x , σ_1 , is defined as the whole external force applied to the structure divided by the area perpendicular to the loading direction at the origin. This can be expressed as:

$$\sigma_1 = \frac{F}{A_1} = \frac{F}{(Ht)} \quad (3)$$

Where t is the thickness of the structure.

As a result, one can determine elastic modulus of the 2D structure parametrically:

$$E_1 = \frac{\sigma_1}{\varepsilon_1} = \frac{\frac{F}{(Ht)}}{\frac{\Delta L}{L}} \quad (4)$$

III. Results and discussion

As abovementioned, finite element for each structure has been developed. For each model, three different cross sections were taken into consideration. These cross sections are square, circle, and triangle in which cross section areas for each model were held fixed. The displacement contours obtained from finite element modelling is shown in Figures 2 and 3.

TABLE I. FINITE ELEMENT RESULTS OF POISSON'S RATIO AND ELASTIC MODULUS.

Model No	Cross Section	ε_x	ε_y	$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x}$	$E_1 = \frac{\sigma_1}{\varepsilon_1}$
1	Square	0.1127	0.0292	-0.2591	1051.5 MPa
	Circle	0.1127	0.0291	-0.2472	1480 MPa
	Triangle	0.1127	0.0278	-0.2378	2617 MPa
2	Square	0.0556	0.0387	-0.696	2131.1 MPa
	Circle	0.0556	0.0345	-0.621	2580.3 MPa
	Triangle	0.0556	0.0355	-0.639	3124.5 MPa
3	Square	0.0478	0.02	-0.421	2832.5 MPa
	Circle	0.0478	0.0187	-0.392	3721.2 MPa
	Triangle	0.0478	0.0148	-0.311	4239.6 MPa
4	Square	0.0232	0.0283	-1.2241	4020 MPa
	Circle	0.0232	0.0258	-1.1124	6010 MPa
	Triangle	0.0232	0.0214	-0.925	8040 MPa
5	Square	0.025	0.0292	-1.168	3608.4 MPa
	Circle	0.025	0.0268	-1.0721	4728.4 MPa
	Triangle	0.025	0.022	-0.885	6121.2 MPa

Table 1 shows the results of finite element simulation in which the first three models, second three models, third three models, fourth three models and last three models are lozenge grids structures, sinusoidal ligaments, square grids, double arrowhead, hexagonal re-entrant honeycomb structures, respectively.

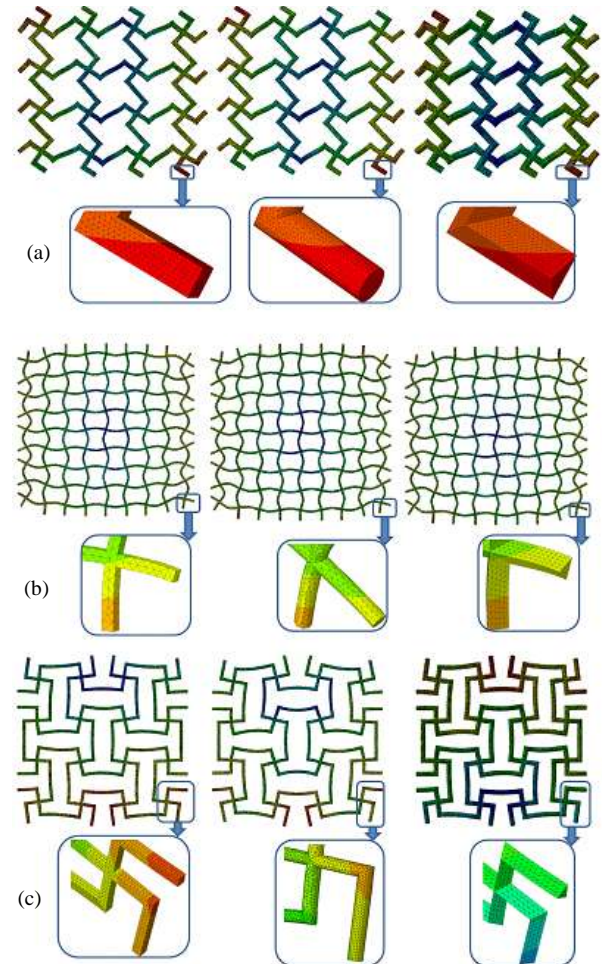


Figure 2. Finite element models of 2D re-entrant structures. (a): lozenge grids. (b): sinusoidal ligaments. (c): square grids.

Overall, the results show that the shape of cross section has a great influence on the mechanical properties. In terms of Poisson's ratio, the auxeticity varies with the geometrical shapes of the cross section. It is noteworthy that the square cross section exhibits the highest auxeticity behaviour compared with the other two cross sections. Furthermore, the elastic modulus of triangular cross sections is much more higher than that of other two cross sections as evidenced in Table 1. As a guideline, if the higher stiffness of the auxetic re-entrant structure is needed, the triangular cross-sectional shapes may be employed in simulating auxeticity of any model.

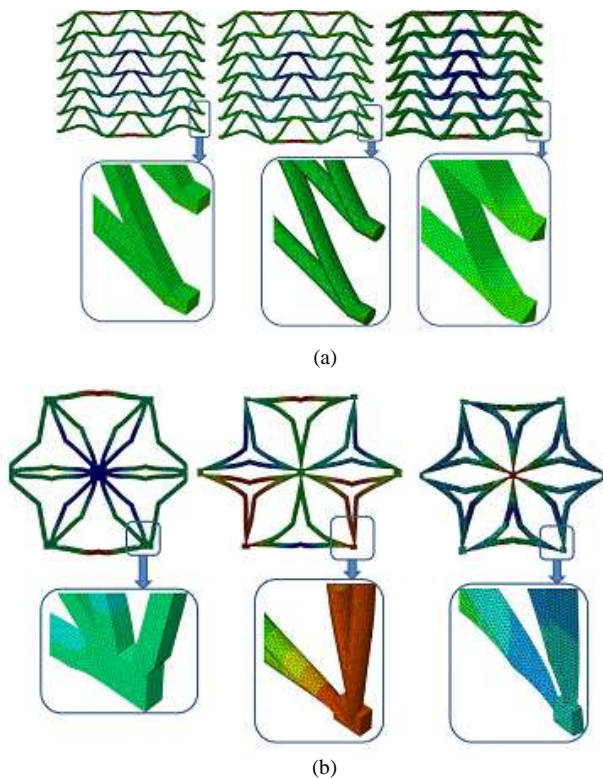


Figure 3. Finite element models of 2D re-entrant structures. (a): double arrowhead. (b): structurally hexagonal re-entrant honeycomb.

IV. Conclusion

Different configurations for 2D re-entrant structures of auxetic materials from previous studies have been developed using finite element technique to examine their mechanical properties. The effect of cross section geometry on Poisson's ratio and elastic modulus has then been investigated numerically. For each structure, three different cross sectional geometries with an identical cross-sectional area were considered. The results show that the shape of cross section has a great influence on the mechanical properties. It is discovered that the elastic modulus for triangular cross sections is greater than that for the other two cross sections. Thereby, manipulation of stiffness properties including Poisson's ratio as well as fracture toughness can be performed by varying the cellular structural parameters. This research provides a basis for experimental additive manufacturing processes such as electron beam melting or selective laser sintering.

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About Author (s):



Zaini Ahmad (CEng MIMechE) is currently a senior lecturer in Universiti Teknologi Malaysia. He received his Master of Science (MSc. in Structures, Crashworthiness& Impact) from Cranfield University, United Kingdom in 2004. He had also obtained his PhD in Structural Impact at Queensland University of Technology, Australia in 2009. Computational solid mechanics is of his primary research interest. Other areas of expertise are finite element modeling, crashworthiness, impact mechanics, crash analysis, thin-walled structures and failure of structure under dynamic loading. He has continuously published a number of refereed scientific publications with Elsevier, Springer, Scientific, etc. He received his professional qualification as Chartered Engineer from the Engineering Council, United Kingdom in 2015 and he is also a member of IAENG, since 2010.



Mozafar Shokri Rad received his BEng and MEng from Iran. He is currently working in the Faculty of Mechanical Engineering, Universiti Teknologi Malaysia, Malaysia as a doctorate student. His research interests are computational solid mechanics, advanced materials, non-linear finite element analysis as well as impact mechanics. He has published several scientific publications with Elsevier, Hindawi, Scientific, etc.