

# Simultaneous Signal Separation and Denoising in DCT Domain for Wireless Communication Applications

R. Ali, O. Zahran, F. E. Abd El-Samie, and M. Elkordy

**Abstract**—This paper addresses the problem of blind signal separation (BSS) for the system of multiple input and multiple output signals (MIMO). Different modulation techniques such as quadrature phase shift keying (QPSK), minimum shift keying (MSK), and Gaussian minimum shift keying (GMSK) have been considered. Several methods have been investigated to solve this problem such as principle component analysis (PCA), independent component analysis (ICA), and multi user kurtosis (MUK) algorithms. Different modulation techniques and different algorithms in the separation have been investigated to for comparison between results. In this paper, we propose wavelet denoising with PCA, ICA and MUK methods. We use the separation algorithm for the discrete cosine transforms (DCT) for blind of mixed signals, instead of separating the mixtures themselves, as a technique that achieves a great result in eliminating the noise. The proposed algorithm is used with multi user kurtosis (MUK) that used for blind of mixed signals with considering the instantaneous mixture of two sources. The simulation results show a considerable improvement in extracted signals when compared to original signals.

**Keywords**—BSS, MIMO, PCA, ICA, MUK, MSK, GMSK, QPSK, DCT MSE, SNR.

## I. Introduction

Blind source separation (BSS) has an important role in several fields such as biomedical, telecommunication, radar and mobile systems. In BSS, we are interested in recovering a number of source signals that are received in the presence of noise without the use of training sequences. Its goal is to estimate original source signals using only the information of the mixed signals observed in each sensor. In the instantaneous mixture case, the source separation problem can be solved by many approaches [1-6]. In this paper, we considered the case where a number of received MIMO signals are observed in the presence of noise to solve the BSS problem using different types of modulation techniques and different separation algorithms [7-10].

This paper is arranged as follows; transmitted signal model is presented in Section II, modulation techniques that used in transmitted signals are explained in Section III, wavelet denoising that used to reduce noise on signals is presented in Section IV, signal separation algorithms are presented in Section V, separation in DCT is presented in Section VI, computer simulation results are discussed in section VII, and followed by conclusions in Section VIII.

Rania Ali, Osama Zahran, Mohamed El-Kordy, Fathi Abd El-Samie  
 Faculty of Electronic Engineering, Menofia University, Egypt

## II. Transmitted Signal Model

Blind signal separation deals with mixtures of signals in the presence of noise. We assume a number of  $p$  transmitted signals which are mutually independent with zero mean,  $a_j(k)$ ;  $j=1 \dots p$ . These signals share the same statistical properties and are transmitted through  $(p \times m)$  MIMO linear channel matrix  $H$  which introduces inter-user interference (IUI). The number of received noisy signals  $m$  must be  $(m \geq p)$ . These signals are subsequently filtered by an  $(m \times p)$  equalizer  $w$  whose outputs  $Z_j(k)$ ;  $j=1 \dots p$ . Ideally match the transmitted signals  $a_j(k)$ ; the received noisy signals are then:

$$Y(k) = HA(k) + n(k) \quad (1)$$



Figure 1. Block diagram of the transmitted and extracted signals.

## III. Digital Modulation Techniques for Transmitted Signals

We implement three types of digital modulation:

- Quadrature Phase Shift Keying (QPSK)
- Minimum Shift Keying (MSK)
- Gaussian Minimum Shift Keying (GMSK)

The mean square error is used as a performance evaluation metric. The channel matrix  $H$  is of dimensions  $(m \times p)$  and channel output vector is of dimensions  $(m \times I)$ . The received signal  $Y(k)$  is then:-

$$\begin{bmatrix} Y_1(k) \\ \vdots \\ Y_m(k) \end{bmatrix} = \begin{bmatrix} H_{11}(k) & \dots & H_{1p}(k) \\ \vdots & & \vdots \\ H_{m1}(k) & \dots & H_{mp}(k) \end{bmatrix} \begin{bmatrix} A_1(k) \\ \vdots \\ A_p(k) \end{bmatrix} + \begin{bmatrix} n_1(k) \\ \vdots \\ n_m(k) \end{bmatrix} \quad (2)$$

where  $A(k)=[a_1(k) \dots a_p(k)]^T$  is the vector of transmitted source signals;  $H$  is the channel matrix  $(m \times p)$ .  $Y(k)$  is the vector of received signal  $(m \times I)$ ;  $n(k)$  is the vector of additive white Gaussian noise  $(m \times I)$ .

The receiver outputs are:-

$$z(k) = W^T(k)Y(k) = W^T HA(k) + n'(k) = G^T(k)A(k) + n'(k) \quad (3)$$

Where  $W(k)$  and  $G^T(k) = W^T HA(k)$  are the  $(m \times p)$  receiver matrix and  $(p \times p)$  global response matrix respectively.  $Z(k) = [z_1(k) \dots z_p(k)]^T$ ; T denotes transpose,  $n'(k) = W^T(k) n(k)$ , is the colored noise at the receiver output. Equation (3) can be equivalently written as:-

$$Z_j(k) = \sum_{j=1}^p g_{j1} a_1(k) = G_j^T A(k); j = 1 \dots p \quad (4)$$

Where  $G_j$  is the channel/equalizer cascade that contains the contribution of all the  $p$  channel inputs to the  $j$ -the equalizer output. In the absence of noise (in order to simplify the analysis presented later), the signal model reduces to:-

$$Z(k) = G^T(K) A(k) \quad (5)$$

$$G = [G_1 \dots G_p] = \begin{bmatrix} g_{11} & \dots & g_{p1} \\ \vdots & \ddots & \vdots \\ g_{1p} & \dots & g_{pp} \end{bmatrix} \quad (6)$$

## IV. Wavelet Denoising On Received Noisy Signals

Wavelet denoising is a simple operation which aims at reducing noise in noisy signals. After separating mixing signals, noise is removed from received noisy signals by using wavelet denoising. It is performed by choosing a threshold that is sufficiently a large multiple of the standard deviation of the noise in the signal. Most of the noise power is removed by thresholding of wavelet transform values. There are two types of thresholding; hard and soft thresholding [11]. The equation of the hard thresholding is given by:-

$$f_{hard}(x) = \begin{cases} x; |x| \geq Th \\ 0; |x| < Th \end{cases} \quad (7)$$

On the other hand, that of soft thresholding is given by:

$$f_{soft}(x) = \begin{cases} x; |x| \geq TH \\ 2x - TH; TH/2 \leq x < TH \\ TH + 2x; -TH < x \leq -TH/2 \\ 0; |x| < TH/2 \end{cases} \quad (8)$$

The values of  $x$  represent the coefficients of the high frequency components.

## V. Separation Algorithms

Several different approaches have been proposed by many researchers for blind signal separation. Some of these approaches depend on independent component analysis. Others depend on higher order statistics

### A Multi User Kurtosis (MUK) Separation Algorithm

Kurtosis measure the value of banding across the center of distribution of signals. It is known  $K(x) = E(|x^4|) - 2E(x^2) - |E(X^2)|$  is the kurtosis (forth order cumulant) of  $x$ ,  $\sigma_x^2$ ,  $K_a$  are the variance and kurtosis of each  $a_j(k)$ , respectively, and \* denotes complex conjugate. In the rest of the paper we will assume the kurtosis value at each node and select the node that has the highest Kurtosis. We first perform an update of  $W(k)$  in

the direction of the instantaneous gradient (controlled by a step-size  $\mu$ ):-

$$W'(k+1) = W(k) + \mu \text{sign}(K_a) Y^*(k) Z(k) \quad (9)$$

We have:

$$W_j(k+1) = \frac{W'(k+1) - \sum_{i=1}^{j-1} W_i^H(k+1) W_j'(k+1) W_i(k+1)}{\|W'(k+1) - \sum_{i=1}^{j-1} W_i^H(k+1) W_j'(k+1) W_i(k+1)\|} \quad (10)$$

The two conditions for perfect separation are necessary and sufficient for the recovery of all the transmitted signals at the equalizer outputs Multi -User Kurtosis (MUK) maximization criterion for blind source separation.

- 1)  $|K(Z_j(k))| = K_a; j = 1 \dots p$
- 2)  $E(|Z_j(k)|^2) = \sigma_a^2$

The steps of separation are shown in Table I.

TABLE I. MUK ALGORITHM

1. for $k=0$ initialize $W(0) = W_0$
2. for $K > 0$
3. obtain $W'(k+1)$ from (9)
4. obtain $W_1(k+1) = \frac{W_1'(k+1)}{\ W_1'(k+1)\ }$
5. for $j=2:p$
6. compute $W_j(k+1)$ (10)
7. GO to step 5
8. $W(k+1) = [W_1(k+1) \dots W_p(k+1)]$
9. Go to step2

### B. Principle Component Analysis

Principal Components Analysis (PCA) is a way of signal separation [4]. It involves four steps.

- 1- Covariance (autocorrelation) of received noisy signals.
- 2- Eigen values estimation.
- 3- Direction estimation of principle component with Eigen vectors.
- 4- Finding coordinates of each data point in the direction of principle components.

We assume the scalar weight  $W$  between the original input signal and extracted output signal

$$W_1 = \arg \max_{\|W\|=1} E\{(w^T Z)^2\} \quad (11)$$

$$W_k = \arg \max_{\|W\|=1} E\left\{\left[w^T \left(Z - \sum_{i=1}^k w_i w_i^T Z\right)\right]^2\right\} \quad (12)$$

### C. Independent Component Analysis

Independent component analysis (ICA) is a method for finding underlying factors or components of signals. The standard formulation of ICA requires at least as many sensors as sources. Thus, in this paper, we assume that the number of sources is less by one the number of sensors. In the instantaneous mixture case, the sensors are observed signals  $Y(k)$  and is an unknown full rank mixing matrix  $H$ . In practice, the goal of ICA is to find the inverse of  $H$ , which is the non-mixing matrix  $W = H^{-1}$ . To estimate  $W$ , we have to make certain assumptions and impose some restrictions is included in [3].

Given a set of observations of random variables assuming that they are generated as a linear mixture of independent components  $Y(k) = WZ(k)$  where  $W$  is some unknown matrix. Independent component analysis now consists of estimating both the matrix  $W$  and  $Z(k)$  when observing  $Y(k)$ . For the whitened data find a vector  $w$  such that the linear combination  $Y = W^T Z$  has maximum under the condition.

$I = E\{y^2\}$ ,  $I = E\{w^T Z \cdot Z^T w\}$ ,  $I = w^T E\{Z \cdot Z^T\} w$ ,  $I = w^T \cdot w$   
 Maximize (Kurt)  $W^T Z$  under the simpler constraint that:  $\|w\|=1$ . The steps of separation are shown in Table II.

TABLE II. ICA ALGORITHM

1.	Centering
2.	Whitening
3.	Choose $m$ , No. of ICs to estimate. Set counter: $p \leftarrow 1$
4.	Choose an initial guess of unit norm for $W_p$ .
5.	Let $W_p \leftarrow E\{Z[Z^T Z]^{-1}\} - 3W_p / \ W_p\ ^2$
6.	Do deflation decorrelation
7.	$W_p \leftarrow W_p - \sum_{j=1}^{p-1} (W_p^T W_j) W_j$
8.	Let $W_p \leftarrow W_p / \ W_p\ $
9.	If $W_p$ has not converged $( 1 - W_p^{k+1} \cdot W_p^k  > \epsilon)$ , go to step 5.
10.	Set $p \leftarrow p + 1$ . If $p \leq m$ , go back to step 4.

## New Proposed Scheme

We have used the following methodology:

1. Creation of Random symbols (QPSK, MSK and GMSK symbols) according to number of samples and transmitters.
2. Mapping of random symbols on constellation.
3. Creation of Random Channel Matrix depending on number of transmitter and receivers.
4. Computation of received samples by putting mapped symbols on channel matrix.
5. Creation and addition of white Gaussian noise.
6. Addition of Additive White Gaussian Noise to the signal to be received.
7. Perform different separation (PCA, ICA and MUK) on noisy received signal in each case of modulation techniques.
8. Estimation of source and channel.
9. Calculate MSE between original and estimated signals.
10. Display of constellation mapped data for all the received data and extracted data.

## VI. Separation using Discrete Cosine Transform

Transform can be used in signal separation are discussed; the DCT, and the DST. All of these domains have some sort of energy compaction, in the transform domain most of the signals energy is concentrate in a few coefficients, most of the transform domain coefficients are close to zero. The main advantage of signal separation in transform domains is decreased the effect of noise on the signals in the transform domains which is smaller than that in the time domain due to the averaging effect of the transform.

## A. Discrete Cosine Transform

The discrete cosine transform (DCT) show in Fig. 3 is a 1-D transform with high energy compaction for signals  $x(n)$ , the DCT studied in [9, 12], the DCT equation is:

$$X(k) = \alpha(k) \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi(2n+1)k}{2N}\right), \quad k = 0, 1, 2, \dots, N-1 \quad (13)$$

where  $\alpha(0) = \sqrt{\frac{1}{N}}$ ,  $\alpha(k) = \sqrt{\frac{2}{N}}$

The Inverse DCT (IDCT) is represented by [12]:

$$x(n) = \sum_{k=0}^{N-1} \alpha(k) X(k) \cos\left(\frac{\pi(2n+1)k}{2N}\right), \quad n = 0, 1, 2, \dots, N-1 \quad (14)$$

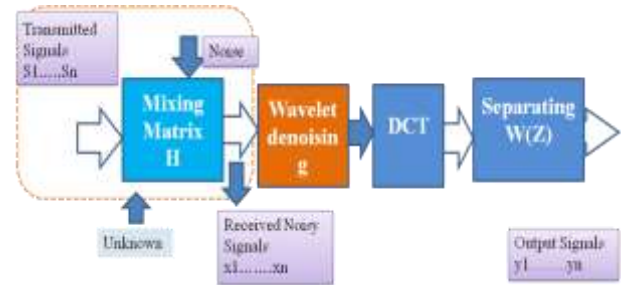


Figure 2. Block diagram of proposed the extracted signals.

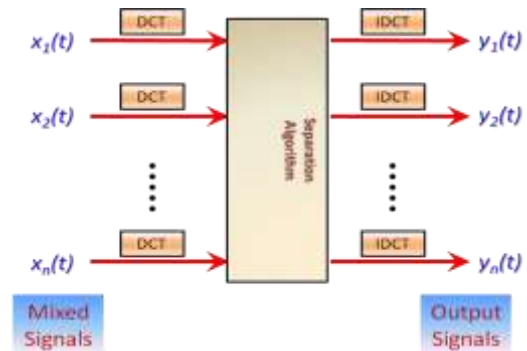


Figure 3. The proposed technique for signal separation algorithm in the discrete cosine transforms (DCT).

## B. Discrete Sin Transform

The discrete sine transform (DST) is another triangular transform with common properties with the DCT. The equation of the DST is represented by [6]:

$$X(k) = \sum_{n=0}^{N-1} x(n) \sin\left(\frac{\pi}{N+1} (n-1)(k+1)\right), \quad k = 0, 1, 2, \dots, N-1 \quad (15)$$

## VII. Results and Discussions

Computer simulations are used to illustrate the usefulness of the algorithm in order to provide numerical evidence for the different separation algorithms using modulation techniques in transmitted signal. We show the result of the separation algorithms. First, the two sources  $P=2$  and  $m=3$  are mixed on the computer with mixing matrix  $H$  with dimensions  $3 \times 2$  with additive white Gaussian. The wavelet denoising is to remove noise. Since the true sources are available, we can evaluate the performance of the algorithms to give better separation of extracted sources. We use mean square error (MSE) between original and extracted signals to compare between results.

$$MSE = \sum_{i=1}^N \frac{(Z(i) - A(i))^2}{length(A(i))} \quad (16)$$

Where the original signal  $A(i)$  and extracted signal  $Z(i)$ ,  $i$  is the sample index and  $N$  is the number of sample using different modulations technique QPSK, MSK, GMSK and different separation algorithms PCA, ICA, MSK and wavelet denoising to remove noise. Table III shows MSE between original and extracted signals using QPSK modulation technique and different separation algorithms. From results shown in Table III, MUK algorithm give better result in separation compared to another algorithms ICA, PCA in the same condition. Table IV shows MSE between original and extracted signals using MSK modulation technique and different separation algorithms. When comparing between values in Tables (III and IV), MSK give better result in separations rather than QPSK modulation. Table V show MSE between original and extracted signals using GMSK modulation technique and different separation algorithms. The simulation results that shown in Tables (III,IV, and V) illustrate that increased number of sampling of signals, gives better result in separation of signals and performance of the MUK algorithm. MSK modulation in transmitted signal gives better estimated sources rather than QPSK and GMSK because the power spectral density has low side lobes that help to control adjacent-channel interference, however the main lobe becomes wider than the QPSK after modulation. In final we use MSK modulation and MUK algorithm in BSS. Table VI display MSE between original and extracted signals using methods of separation algorithm, types of modulation and white Gaussian noise, at varying noise variance (0.1, 0.01, and 0.05). Table VII display MSE between original and extracted signals using methods of separation algorithm, types of modulation and using wavelet denoising at varying noise variance. Compare between table (VI,VII) the results show when using wavelet denoising before the algorithm used in separation to reduce the noise Using wavelet denoising which aims at reducing noise in noisy signals that give better result and improvement separation algorithms.

TABLE III

Method of separation	MSE using QPSK			
	Number of Sampling Points (N)			
	N=500	N=1000	N=5000	N=10000
PCA	0.0012	6.9e-4	1.7e-4	8e-5
ICA	0.0011	4.9e-4	2e-4	5e-5
MUK	8e-4	4.8e-4	1e-4	4e-5

TABLE IV

Method of Separation	MSE using MSK			
	Number of Sampling Points (N)			
	500	1000	5000	10000
PCA	5.5e-4	2.8e-4	5.3e-5	2.8e-5

TABLE V

Method of Separation	MSE using GMSK			
	Number of Sampling Points (N)			
	500	1000	5000	10000
PCA	0.0089	0.0079	9e-4	5.5e-5
ICA	0.0086	0.0063	7e-4	5.3e-5
MUK	0.0084	6.9e-4	6e-4	5e-5

TABLE VI

Noise Variance	MSE								
	Separation Algorithm and Type of Modulation								
	QPSK			MSK			GMSK		
	PCA	ICA	MUK	PCA	ICA	MUK	PCA	ICA	MUK
0.1	0.0022	0.002	0.0011	5.7e-4	5.6e-4	5.5e-4	0.0086	0.0084	0.0083
0.01	0.0019	0.0018	0.001	5.6e-4	5.5e-4	5e-4	0.0084	0.0065	0.002
0.05	0.0012	0.0011	8e-4	5.5e-4	5.4e-4	4.5e-4	0.0069	0.0064	0.001

TABLE VII

Noise Variance	MSE								
	Separation Algorithm & Type of Modulation with Wavelet Denoising								
	QPSK			MSK			GMSK		
	PCA	ICA	MUK	PCA	ICA	MUK	PCA	ICA	MUK
0.1	0.0011	0.001	0.0008	5.6e-4	5.5e-4	5.3e-4	0.0085	0.0083	0.0078
0.01	9.5e-4	9.4e-4	9e-4	5.5e-4	5.4e-4	5e-4	0.0076	0.0073	0.007
0.05	9.3e-4	9e-4	7e-4	5.5e-4	5.3e-4	4.3e-4	0.0022	0.0021	0.0018

The experiments is separating mixtures of signals in DCT and DST and compare results of separating mixtures of signals in time domain to study the effect of wavelet denoising. The experiment is performed on two mixtures signal in the presence of noise so we using wavelet denoising on noisy mixtures after separating signals to remove the noise from extracted signals in all domains. When applied wavelet denoising decreasing the noise results showed that a lower error has been achieved with wavelet denoising. Results showed that a lower error has been achieved in discrete cosine transform (DCT). We use signal to noise ratio (SNR) between original and extracted signals to compare between results when separated in the presence of noise and using wavelet denoising to remove the noise from the signals after using separation algorithms.

$$SNR = 10 \log_{10} \frac{\sum_{i=1}^N x^2(i)}{\sum_{i=1}^N (x(i) - y(i))^2} \quad (17)$$

Where the original signal  $x(i)$  and extracted signal  $y(i)$ ,  $i$  is the sample index and  $N$  is the number of sample using different modulations technique. The experiments have been performed on two random signals in the presence of noise with SNR= -10 dB, 0 dB, and +10 dB. Fig. 4a shows signal to noise ratio between original signal 1 and extracted signal 1 in time domain and (DCT, DST) when separation in higher order statistics using MUK separation algorithm. Fig. 4b shows signal to noise ratio between original signal 2 and extracted signal 2 in time domain and (DCT, DST) when separation in higher order statistics using MUK separation algorithm. Fig. 4c shows signal to noise ratio between original signal 1 and extracted signal 1 in time domain with wavelet denoising and without wavelet denoising and separation in other domain in

the discrete cosine transform (DCT) when using wavelet denoising to remove noise from the signal and without denoising to compare between results. Fig. 4d shows signal to noise ratio between original signal 2 and extracted signal 2 in time domain with wavelet denoising and without wavelet denoising and separation in other domain in the discrete cosine transform (DCT) when using wavelet denoising to remove noise from the signal and without denoising to compare between results. The comparison shows that separation in DCT and DST domain give better result than in time domain. When using wavelet denoising after separation in the DCT and DST domain to remove the noise from extracted signals give the best performance. Notice that in Fig. 5, show scatter plot of received noisy signal after separation with noise that observed three signals in the receiver but we can transmitted two signal from transmitter. Fig. 6, shows scatter plot of estimated sources that extracted from received noisy signals before separation that give sufficient for perfect recovery of signals. Fig. 7, shows scatter plot of original signals that can transmit. Comparing between extracted 1, input 1 and extracted 2, input 2 from figures. That see extracted 1, 2 signals extracted from received noisy signals using different separation techniques without know original signals.

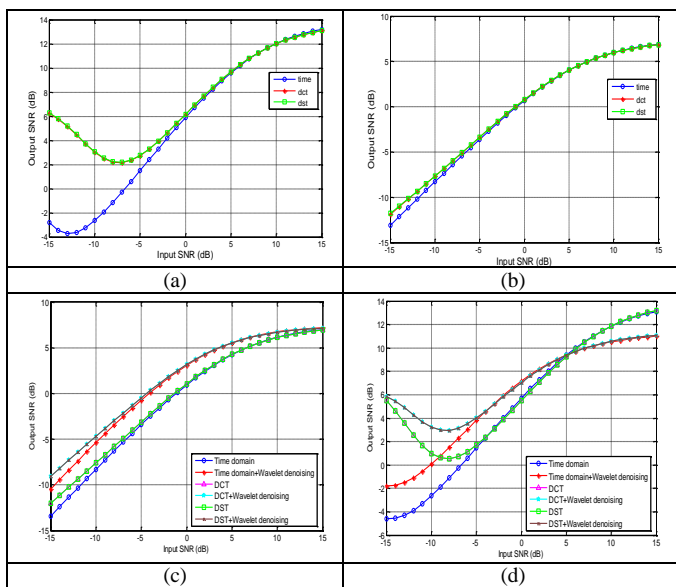


Figure 4. a) SNR between original 1 & extracted 1 signal in TD, DCT/DST  
 b) SNR between original 1 and extracted 1 signal in Time domain and (DCT, DST).  
 c) SNR between original 1 and extracted 1 signal in Time domain and DCT using wavelet denoising  
 d) SNR between original 2 and extracted 2 signals in Time domain and DCT using wavelet denoising.

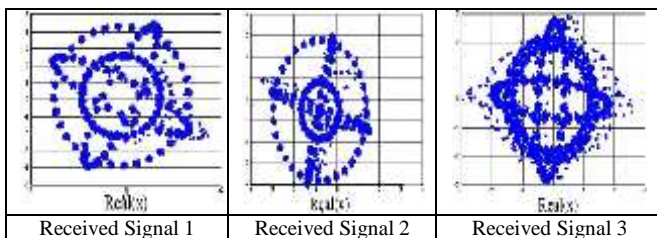


Figure 5. Scatter plot of received noisy signal

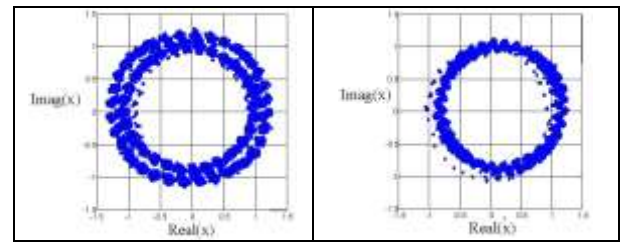


Figure 6. Scatter plot of extracted sources using wavelet denoising, (a) extracted source 1 and (b) extracted source 2.

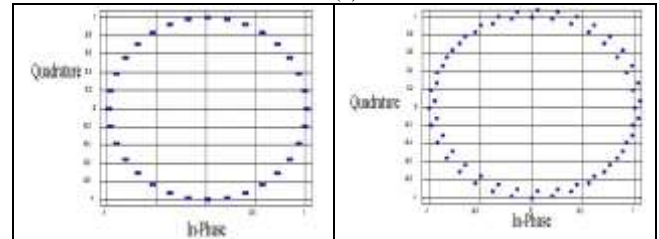


Figure 7. Scatter plot of original signals (a) input 1 and (b) input 2.

## VIII. Conclusion

We have studied the problem of multi-user blind signal separation. We used different modulation techniques for transmitted signals and different algorithms in separation, and then applied wavelet denoising to decrease the noise. Trigonometric transforms are introduced as new techniques to reduce the effect of the noise and to achieve a better performance of blind signal separation algorithms. In order to show the effectiveness of the proposed technique Results showed that a lower error has been achieved with wavelet denoising.

## REFERENCES

- [1] C. Vincent, "Implementation of the MUK algorithm for blind source separation", 22 July 2010.
- [2] A. Hyvarinen, J. Karhunen, and E. Oja, "Independent component analysis," Wiley and Sons, 2001.
- [3] P. Comon, "Blind techniques" laboratory 135, CNRS, University of Nice, 17 January 2010, pp 12.
- [4] V. Zarzoso and A. K. Nandi, "Blind MIMO equalization with optimum delay using independent component analysis", International Journal of Adaptive Control and Signal Processing, vol. 18, pp. 245-263, 2004.
- [5] M. Frigo and S. G. Johnson: *FFTW*, <http://www.fftw.org/>. A free (GPL) C library that can compute fast DCTs (type's I-IV) in one or more dimensions, of arbitrary size. Also M. Frigo and S. G. Johnson, "The Design and Implementation of FFTW3," *Proceedings of the IEEE* 93 (2), 216-231 (2005).
- [6] A. V. Oppenheim, R. W. Schaffer, and J. R. Buck, *Discrete-Time Signal Processing*, second edition (Prentice-Hall, New Jersey, 1999).
- [7] P. Comon, "Independent component analysis, A new concept", *Signal Processing*, vol. 36, pp. 287-314, 1994.
- [8] C. B. Papadias, "A multi-user kurtosis algorithm for blind source separation", presented at the Proc. ICASSP 2000 Conf., vol. V, Istanbul, Turkey, June 5, 2000, and pp. 3144-3147.
- [9] C. B. Papadias, "Blind source separation based on multi-user kurtosis criteria", presented at the Proc. ISIT 2000 Conf., Sorrento, Italy, 30 June, 2000, pp. 245.
- [10] E. Moreau, "A block algorithm for blind signal deconvolution", in Proc. IEEE Signal Process. Workshop Signal Process. Adv. Wireless Communication, Paris, France, Apr. 16-18, 1997, pp. 93-96.
- [11] B. Papadias, "A multiuser kurtosis algorithm for blind source separation", IEEE, 2000.
- [12] Tomas Zeman "BSS - Preprocessing Steps for Separation Improvement" May 2000.