Chaotic Phenomena in Nonlinear Magneto-Piezoelectric Energy Harvester

Marek Pietrzakowski and Danuta Sado

Abstract—**The presented analysis concerns the two-degree-of-freedom lumped parameter model, which is combined of the magneto-piezoelectric energy harvester and the mass-spring subsystem used as a dynamic magnifier. The simplified single-degree-of-freedom energy harvester describes the tip beam dynamics and the voltage generated by the piezoelements. The equivalent mass and stiffness of the cantilevered beam, and also the piezoelectric properties characterize this subsystem. The repulsive force created by the tip magnet and another one mounted to the cantilever base is involved. It is assumed that the repulsive loading component is a nonlinear odd function of the cantilever tip displacement and the energy harvesting subsystem can be modelled as Duffing oscillator. Changing the distance between the magnets, monostable or bistable behaviour of the beam harvester can be created. The presence of bistability improves the harvester effectiveness and also may create chaotic behaviour. For characterizing an irregular chaotic response in this work the time histories, bifurcation diagrams, Poincarè maps and maximal exponents of Lyapunov have been constructed.**

Keywords—**nonlinear energy harvester, bimorph piezoelectric beam, chaos**

I. Introduction

Techniques based on application of distributed piezoelectric transducers have found a relevant role in vibration control of thin walled flexible structures to improve their operational behaviour and ability to reduce unwanted vibration (see e.g. [1], [2], [3], [4], [5]). In engineering applications such as structural health monitoring and environmental monitoring, practical solutions for a wireless power supply of electronic devices are of great importance. Energy harvesting is a promising technique for development of self-powered wireless electronics. Piezoelectric transducers occur quite effective to convert mechanical energy from usually ambient vibrations into electrical energy. The most popular harvesters are design as a cantilever with piezoceramic layers or patches bonded to the beam sides. The cantilever, which is mounted to a vibrating host structure, is subjected to excitation related to the base movement.

Marek Pietrzakowskir
Warsaw University of Technology
Poland

Danuta Sado
Warsaw University of Technology
Poland

This type of energy harvesters is effective when they operate at resonance, and a shift of the excitation frequency causes a drastic decrease of the generated electrical power [6], [7]. In order to increase the harvester bandwidth and also improve its effectiveness a nonlinear magnetic force is applied to create bistable structure behaviour, where two equilibrium positions are present. The magnetic repulsive force can be produced by two permanent magnets oppositely polled. One of them is attached to the beam tip and the other is mounted to the base of the system near the beam free end. The contribution to energy harvesting from the higher vibration modes, which occur far away from the fundamental frequency, is usually neglected. Hence, a conventional beam harvester can be reduced to a single-mode device with effective properties parameters calculated and introduced [8].

The presented analysis concerns the two-degree-of-freedom lumped parameter model, which is combined of the magneto-piezoelectric energy harvester and the subsystem used as a dynamic magnifier to amplify the vibration of the harvester [9]. The dynamic magnifier consists of the mass and spring, and is placed between the energy converter and the base whose movement is assumed as a harmonic excitation. The simplified single-degree-of-freedom energy harvester describes the tip beam dynamics and the voltage generated by the piezoelements. The equivalent mass, stiffness and damping of the cantilevered beam, and also the piezoelectric properties characterize this subsystem. The repulsive force created by the tip magnet and that mounted to the cantilever base is involved. For relatively small vibrations it is reasonable to assume that the repulsive force is constant in magnitude and changes in direction when the tip magnet moves. In the considered case the repulsive loading component is a nonlinear odd function of the cantilever tip displacement and the energy harvesting subsystem can be modelled as Duffing oscillator [10]. Changing the distance between the magnets, monostable or bistable behaviour of the beam harvester can be created. The presence of bistability makes the system capable to rapidly switch between the two stable positions improving the harvester effectiveness and also may cause chaotic behaviour. For characterizing an irregular chaotic response in this work the time histories, bifurcation diagrams, Poincarè maps and maximal exponents of Lyapunov have been constructed. These descriptors are available to observe chaos, and to better understand it (see e.g. [11], [12], [13]). When a bifurcation diagram is plotted, several phenomenon can be observed: existence of a simple attractor with low period, or existence of a chaotic attractor and various bifurcation. All these phenomena have to be verified in the phase space. Nonperiodic attractors have been traced by solving an initial value problem. The maximum Lyapunov exponents have been calculated in order to characterize the chaotic orbits.

II. Model of the System

Figure 1a gives a schematic view of the magneto-piezoelectric beam harvester joined with the magnified subsystem. The dynamic magnifier consists of the mass m_1 and spring k_1, and is used to amplify the vibration of the harvester. The nonlinear energy converter consists of a cantilevered beam covered by piezoelectric layers on both sides. To create nonlinear repulsive force two permanent magnets with opposite polarization are used. One of them is attached to the beam tip and the other is stationary mounted to the base of the harvesting subsystem near the cantilever free end. The excitation is assumed as the base vertical movement described by a harmonic single frequency time function.

The analysis of the system chaotic behaviour is performed for the simplified two-degree-of-freedom model presented in Fig. 1b. Here, the system simplification concerns the beam energy harvester. The reduced energy harvesting subsystem consists of the equivalent mass m_2, linear spring with equivalent stiffness k_2, and the nonlinear spring representing the magnet force. The mass m_2 account for the effective mass of the first flexural mode of the cantilever plus the magnet, and the effective stiffness k_2 relates to the beam bending stiffness calculated for the vertical tip displacement. On the schema the piezoelectric element, which is responsible for producing the voltage in relation to the tip beam deflection, is placed between the mass m_1 and m_2. Displacements of the mass m_1 and m_2 are indicated as x_1 and x_2, respectively. Considering the base movement $u(t)$ the relative displacements of the lumped-elements are defined: $w_1 = x_1 - u$ and $w_2 = x_2 - x_1$.

As mentioned above the interaction between magnets can be reduced to the vertical component F_{RV} and described as the nonlinear repulsive force acting on the mass m_2 (cf [10]). The repulsive force is approximated as the Duffing equation

$$F_{RV} = \alpha w_2 - \beta w_2^3 \quad \text{with} \quad \alpha = \frac{F_R}{s} \text{ and } \boldsymbol{\beta} = \frac{F_R}{2s^3} \quad (1)$$

where F_R and s is the interaction and distance between magnets, respectively.

The piezoelectric element in Fig. 1 inserted between the masses m_1 and m_2 is used for calculation the voltage V_p generated by the converter basing on the relative displacement w_2. For this reason a following relation can be used

$$\frac{V_p}{w_2} = \frac{\theta}{c_p W(l)} \quad (2)$$

where $W(l)$ is the first mode shape value for the bimorph length, θ is the electromechanical coupling, c_p is the piezoelectric layers capacitance.
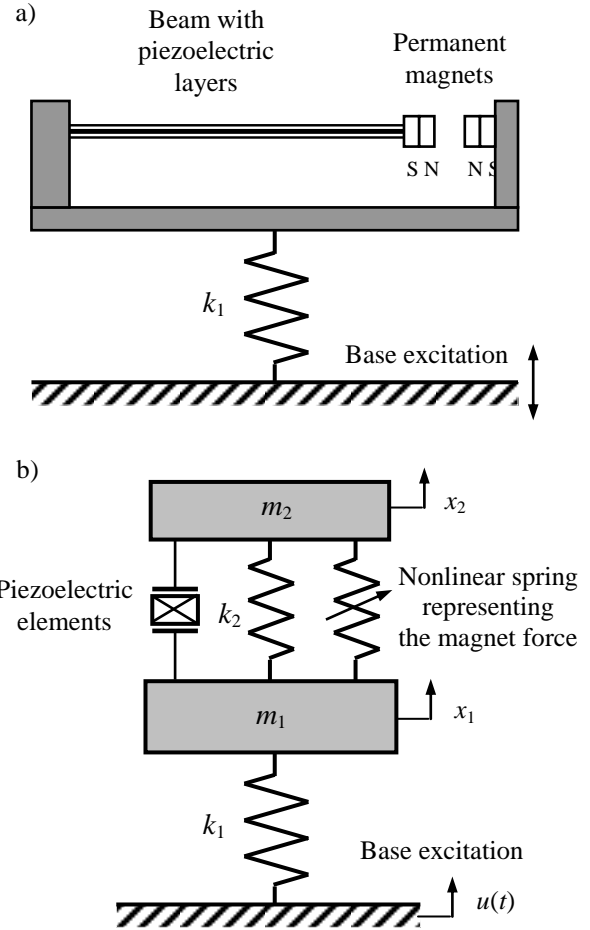

Figure 1. Magneto-piezoelectric energy harvesting system with a dynamic magnifier: a) schematic view, b) configuration of the equivalent two-degree-of-freedom system

The electromechanical coupling is given by the formula

$$\theta = d_{31} Y_p \frac{t_p + t_s}{2} b_p \left[\frac{dW(x)}{dx} \bigg|_{x=l} - \frac{dW(x)}{dx} \bigg|_{x=0} \right] \quad (3)$$

where d_{31} is the piezoelectric material constant, Y_p is the Young modulus, t_p, and t_s indicate the piezoelectric and beam substrate layer thickness, respectively, b_p is the bimorph beam width.

A. Equations of Motion

The equations of motion of the system derived as Lagrange's equations take the following form

$$m_1 \ddot{w}_1 + k_1 w_{1+} + F_{RV} - k_2 w_2 = -m_1 \ddot{u} \quad (4)$$

$$m_2 (\ddot{w}_1 + \ddot{w}_2) + k_2 w_2 - F_{RV} = -m_2 \ddot{u} \quad (5)$$

where $u(t)$ is the base displacement assumed as a harmonic function

$$u = A\sin\nu t \qquad (6)$$

where A and ν indicate the excitation amplitude and frequency, respectively.

After transformations the equations of motion can be written in as follows

$$m_1\ddot{w}_1 + k_1 w_1 - (k_2 - \alpha)w_{2+} - \beta w_2^3 = -m_1 A\nu^2 \sin\nu t \qquad (7)$$

$$m_2(\ddot{w}_1 + \ddot{w}_2) + (k_2 - \alpha)w_2 + \beta w_2^3 = -m_2 A\nu^2 \sin\nu t \qquad (8)$$

The following dimensionless time and dimensionless parameters are introduced

$$\tau = \omega_0 t, \quad \omega_0^2 = \frac{\alpha}{m_1}, \quad \omega_1^2 = \frac{k_1}{m_1}, \quad \omega_2^2 = \frac{k_2}{m_2}, \quad d = \frac{m_1}{m_2}$$

$$\overline{w}_1 = \frac{w_1}{A}, \quad \overline{w}_2 = \frac{w_2}{A}, \quad a = \frac{\omega_1^2}{\omega_0^2}, \quad b = \frac{\omega_2^2}{\omega_0^2}, \quad c = \frac{\beta A^2}{\alpha}, \quad \omega = \frac{\nu}{\omega_0} \qquad (9)$$

After using (9) the equations of motion can be written in the dimensionless form

$$\ddot{w}_1 + aw_1 + (1-b)w_2 - cw_2^3 = -\omega^2 \sin\omega\tau \qquad (10)$$

$$\ddot{w}_1 + \ddot{w}_2 + d(b-1)w_2 + bcw_2^3 = -\omega^2 \sin\omega\tau \qquad (11)$$

Bars in (10) and (11) are omitted for convenience.

III. Numerical Results

The equations of motion in dimensionless form (7) have been solved numerically. Calculations have been done for different values of the system parameters. The calculations incorporated the following initial conditions: $w_1(0)=0$, $w_2(0)=0.0001$. Exemplary results the external resonances for displacement w_1 and w_2 are presented in Fig. 2. As we can see near the external resonances the system presents some interesting nonlinear phenomena.

For characterizing of the system response are constructed [13]: bifurcation diagrams (where the parameter ω is the bifurcation parameter), Poincaré maps and Lyapunov exponents. Bifurcation diagrams versus bifurcation parameter ω for displacement w_1 and displacement w_2 are presented in Fig.3. The Lyapunov maximal exponents are shown in Figs 4 and 5 and the Poincaré maps are presented in Fig. 6.

When a bifurcation diagram is plotted, phenomena can be observed: existence of a simple or chaotic attractor and various bifurcations. All these effects are verified in the phase space. Poincaré maps and maximal exponents of Lyapunov are available to observe chaos.
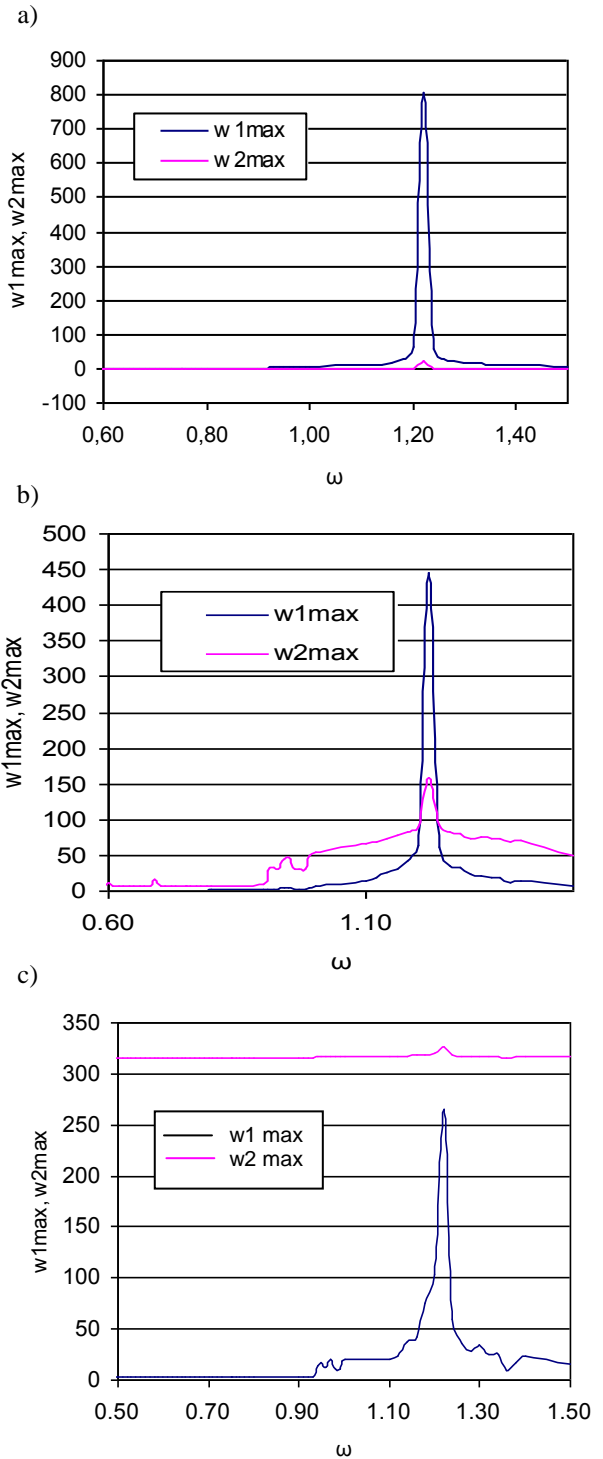
a)

b)

c)

Figure 2. External resonances for $a=1.5$ $b=1.5$ $d=100$ $c=0.00001$ and for different values of parameter b: a) $b=1.5$ b) $b=1.0$ and c) $b=0.5$

In Figs 4 and 5 we can observe positive values of maximal exponents of Lyapunov, and in Fig. 6 the Poincaré maps show the strange atractors, so in this case ($\omega=1.22$) motion of the both bodies are chaotic.
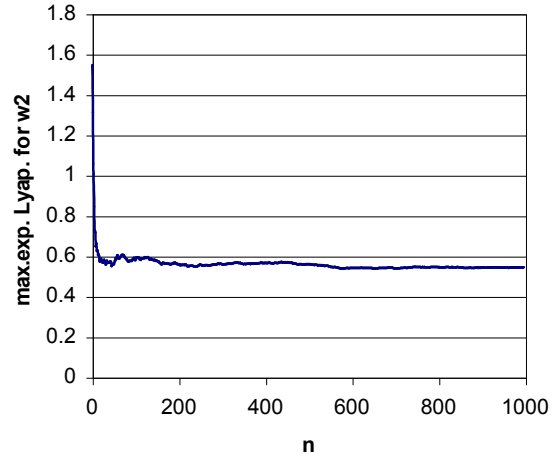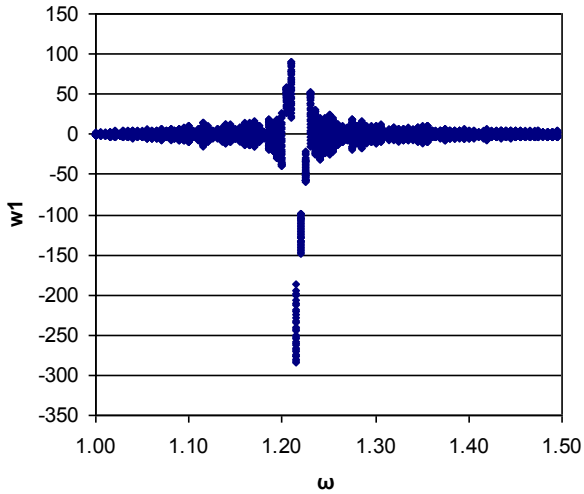
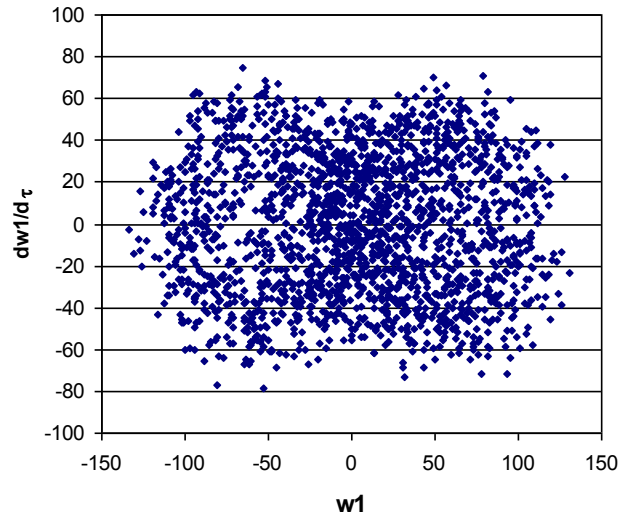Figure 5. Exponents of Lyapunov for w_2 when a=1.5; b=1; c=0.00001; d=100 and ω=1.22
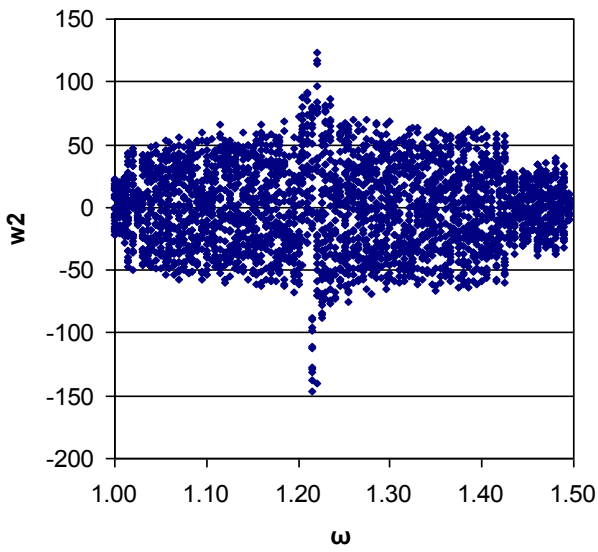

Figure 3. Bifurcation diagrams for a=1.5; b=1; c=0.00001; d=100
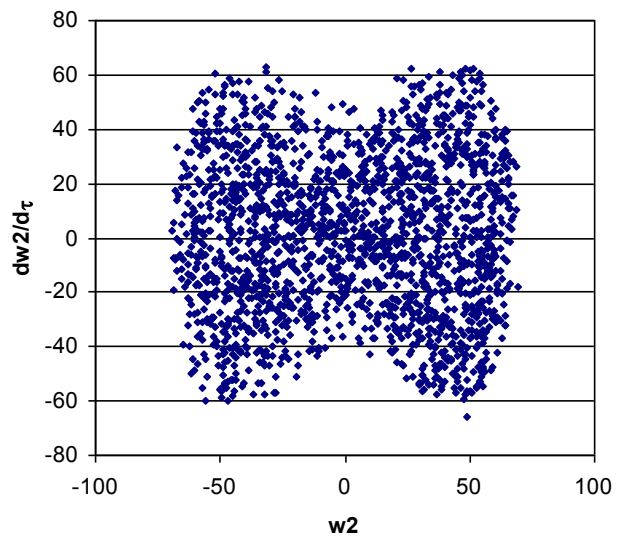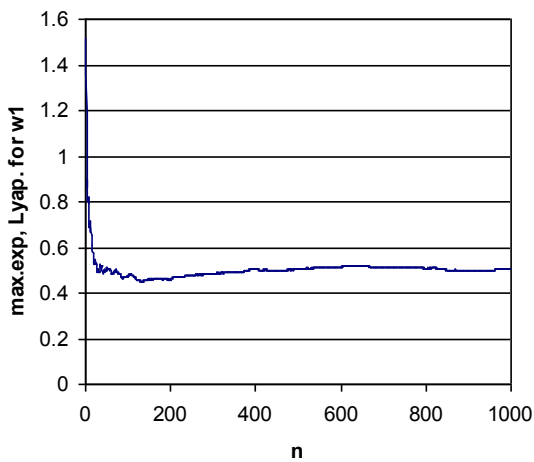

Figure 4. Exponents of Lyapunov for w_1 when a=1.5; b=1; c=0.00001; d=100 and ω=1.22

Figure 6. Poincaré maps for a=1.5; b=1; c=0.00001; d=100 and ω=1.22

IV. **Conclusions**

Several interesting phenomena have been presented. The behaviour of the system near the resonance frequencies is very important. It has been shown that the examined system exhibits very rich nonlinear dynamics. Except periodic vibrations also chaotic vibrations have been found. The chaotic vibrations of the body modelled the beam harvesting subsystem prove the chaotic behaviour of the generated voltage.

References

[1] K. Chandrashekhara, A. N. Agarwal "Active Vibration Control of Laminated Composite Plates Using Piezoelectric Devices: A Finite Element Approach," Journal of Intelligent Material Systems and Structures, 4 (4), 1993, pp. 496-508.

[2] J. N Reddy "On Laminated Composite Plates with Integrated Sensors and Actuators," Engineering Structures, 21, 1999, pp. 568-593.

[3] A. Benjeddou "Advances in Piezoelectric Finite Element Modeling of Adaptive Structural Elements: a Survey," Computers and Structures, 76, 2000, pp. 347-363.

[4] M. Pietrzakowski "Piezoelectric control of composite plate vibration: Effect of electric potential distribution," Computers and Structures, 86, 2008, pp. 948-954.

[5] M. Pietrzakowski "Modelling of piezocomposite functionally graded plates for active vibration control," *Proceedings of the Eleventh International Conference on Computationaql Structures Technology,* B.H.V. Topping (Editor), Civil-Comp Press, 2012, Paper 45, pp. 1-13.

[6] H. A. Sodano, G. Park, D. J. Leo and D. J. Inman "Model of piezoelectric power harvesting beam," Proc. of IMECE'03 2003 ASME International Mechanical Engineering Congress, Washington, D.C., November15–21, 2003.

[7] S.R. Anton, H. A. Sodano "A review of power harvesting using piezoelectric materials (2003-2006)," Smart Materials and Structures, 16, 2007, pp. R1-R21.

[8] A. Erturk, D. J. Inman "On mechanical modelling of cantilevered piezoelectric vibration energy harvesters," Journal of Intelligent Material Systems and Structures 19, 2008, pp.1311-1325.

[9] O. Aldraihem, A. Baz "Energy harvester with a dynamic magnifier," Journal of Intelligent Material Systems and Structures 22, 2011, pp. 521-530

[10] M. Ferrari, V. Ferrari, M. Guizzetti, B. Andò, S. Baglio, C. Trigona "Improved energy harvesting from wideband vibrations by nonlinear piezoelectric converters," Sensors and Actuators A: Physical 162, 2010, pp. 425-431.

[11] F.C. Moon, Chaotic Vibrations, John Wiley & Sons, Inc., 1987.

[12] G.L. Baker and J.P. Gollub, Chaotic Dynamics: An Introduction, 1996.

[13] W. Szemplińska-Stupnicka, "Chaos, Bifurcations and Fractals Around Us," World Scientific, London, 2003.