

Altitude Robust Control of a Quad-rotor aircraft using Integral Sliding Mode controller

Iván González-Hernández

Abstract—As we know, sliding mode control methodology is one of the robust control technique to handle systems with model uncertainties, parameter variations and external disturbances. In this paper, a robust altitude control scheme is proposed for a nonlinear quad-rotor aircraft system based on sliding mode controller with an integral action to eliminate the steady-state error effect. The proposed sliding mode controller is chosen to improve the stability and robustness of overall z-dynamics during the altitude control at a desired height. The stability of the system is guaranteed via Lyapunov stability theory. A suitable sliding manifold is designed to achieve the control objective. At last, the theoretical results are supported by different simulation tests to verify the satisfactory performance of proposed robust control scheme under external disturbances applied to autonomous quad-rotor aircraft.

Keywords—Sliding mode, Robust control, Integral action in Sliding mode, Quadrotor aircraft stabilization, steady-state error.

I. Introduction

The complexity of nonlinear feedback control challenges us to come up with systematic design procedures to meet control objectives and design specifications. Nonlinear control is one of the biggest challenges in modern control theory [1], [2]. It is well known that physical systems are nonlinear in nature. Uncertainties, time varying parameters, and input and output disturbances are important to challenge and these necessitate to use nonlinear control methods to design nonlinear controllers. Faced with such challenge, it is clear that we cannot expect one particular procedure to apply to all nonlinear systems. It is also unlikely that the hole design of a nonlinear feedback controller can be based on one particular tool. What a control engineer needs is a set of analysis and design tools that cover a wide range of situations. When working a particular application, the engineer will need to employ the tools that are most appropriate for the problem in hand. Different control methods mostly designed for a quad-rotor aircraft are: nested saturations [13], feedback linearization [15,16], backstepping [17] and sliding mode control [5-12]. Some other methods also used for linearized model of quad-rotor aircraft in literature such as PID and LQR control [19].

A simple approach to robust control, and the main topic of this article, is the so-called sliding mode control methodology. For the class of systems to which it applies (for instance, air, land and sea vehicles) sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecisions. Furthermore, by allowing the trade-offs between modeling and performance to be quantified in a simple fashion, it can illuminate the whole design process.

Sliding control has been successfully applied to robot manipulators, high-performance electric motors, underwater vehicles and Unmanned Aerial Vehicle (UAV). Unfortunately, an ideal sliding mode controller has a discontinuous switching function and it is assumed that the control signal can be switched from one value to another infinitely fast. In practical systems, it is impossible to achieve infinitely fast switching control because of finite time delays for the control computation and limitations of physical. Due to imperfect switching in practice it raises the issue of chattering which is highly undesirable. It appears as a high frequency oscillation near the desired equilibrium point and may excite the unmodelled high-frequency dynamics of the system.

For instance, in [5] the author has proposed a method to improve the altitude control of the quad-rotor aircraft, using Sliding Mode Control for both translational and rotational dynamics of the quad-rotor aircraft. This approach is based on implementing a smoothed *sign* function utilizing the approximation, $\text{sign}(s) \approx \frac{s}{|s| + \epsilon}$ where $\epsilon > 0$. Since the sliding mode entails $s \approx 0$, the noise in the observed quantities becomes highly effective and the controller can generate unnecessarily large control signals. This is known as the chattering in the related literature, [2]. Utilizing the above approximation introduces a boundary layer and eliminates the undesired chattering phenomenon significantly.

In this paper, the objective is to obtain a robust altitude controller using sliding mode where an integral action has been added in order to eliminate the steady-state error in the z-dynamics of the quad-rotor aircraft to reach a desired height. It also has been chosen for its insensitivity to the model errors, parametric uncertainties and external disturbances. Moreover, the simulations results are compared with the traditional and conventional sliding mode control in dynamic responses of the closed-loop control system.

This paper is organized as follows: Section II presents the dynamical model of the quad-rotor aircraft via a Lagrange approach. The robust control algorithm based on

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sliding mode with integral action and the corresponding stability analysis is presented in Section III. The effectiveness of this controller proposed to the altitude control is evaluated and compared through of different simulations in Section IV. Finally, Section V gives a brief conclusion of the proposed robust altitude control design algorithm.

II. Model of the Quad-rotor aircraft

The quad-rotor aircraft used in this study is shown in Figure 1. The dynamic model of this aircraft is basically obtained representing the quad-rotor as a solid body evolving in 3D and subject to one force and three moments that will be explained below. A quad-rotor aircraft is powered by four BLDC motors (M_1, M_2, M_3, M_4) which are attached to a rigid cross frame. Control of quad-rotor aircraft can be achieved by varying the relative speed of each rotor to change the thrust and torque produced by each. The use of four rotors allows each individual rotor to have a smaller diameter than the equivalent single-rotor helicopter, allowing them to store less kinetic energy during flight and thus reduces the damage caused by the rotors hitting any objects. By enclosing the rotors within a frame, the rotors can be protected during collisions. In this type of vehicles, vertical motion is created by collectively increasing and decreasing the speed of all four rotors; pitch or roll motion is achieved by the differential speed of the front-rear set or the left-right set of rotors, coupled with lateral motion; yaw motion is realized by the different reaction torques between the M_1, M_3 and M_2, M_4 rotors.

Let $\mathcal{I} = \{i_I, j_I, k_I\}$ be the inertial frame, $\mathcal{B} = \{i_B, j_B, k_B\}$ denote a set of coordinates fixed to the rigid aircraft as is shown in Figure [1]. Let $q = (x, y, z, \phi, \theta, \psi)^T \in \mathbb{R}^6$, $(\xi, \eta)^T$ be the generalized coordinates vector which describe the position and orientation of the flying machine, so the model could be separated in two coordinate subsystems: translational and rotational. They are defined respectively by

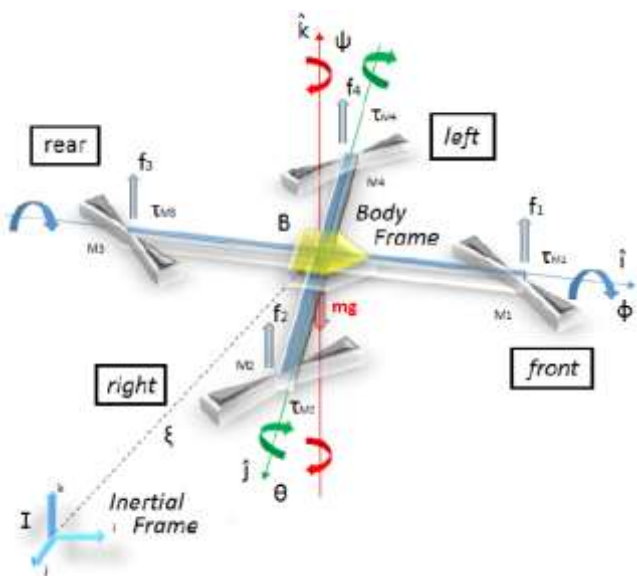


Figure 1. Quad-rotor aircraft scheme

- $\xi = (x, y, z)^T \in \mathbb{R}^3$; denotes the position of the vehicle's mass center relative to the inertial frame \mathcal{I} .
- $\eta = (\phi, \theta, \psi)^T \in \mathbb{R}^3$; describes the orientation of the aerial vehicle, i.e. roll, pitch and yaw angles respectively.

The full quad-rotor aircraft model [3] is obtained from the Euler-Lagrange equations with external generalized force F_B and generalized moment τ_η where F_I is the translational force applied to the quad-rotor aircraft due to the control input.

Then

$$F_B = (0, 0, u)^T \quad (1)$$

where u is the sum of mechanical thrust forces: $u = f_1 + f_2 + f_3 + f_4$ with $f_i = k_i \omega_i^2$ for $i=1,2,3,4$, $k_i > 0$ is a constant and ω_i is the angular speed of motor i , as shown in Figure 1. This force vector can be expressed in the inertial frame as

$$F_I = R^{B \rightarrow I} F_B \quad (2)$$

where $R^{B \rightarrow I}$ is the transformation of vectors from the body-fixed frame to the earth-fixed frame based on Euler angles and the rotation matrix defined by

$$R^{B \rightarrow I} = \begin{pmatrix} c_\theta c_\psi & s_\psi c_\theta & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta c_\phi + s_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi c_\phi & c_\theta c_\phi \end{pmatrix} \quad (3)$$

The generalized moments on the η variables are denoted by $\tau_\eta = (\tau_\phi, \tau_\theta, \tau_\psi)^T$ where

$$\begin{aligned} \tau_\phi &= (f_3 - f_1)l \\ \tau_\theta &= (f_2 - f_4)l \\ \tau_\psi &= ((f_1 + f_2) - (f_3 + f_4))d \end{aligned} \quad (4)$$

where l is the distance to the center of gravity and d is the drag coefficient produced by coordinated reactive torque involving the four rotors because of the geometry of the quad-rotor aircraft. Since the lagrangian contains no cross terms in the kinetic energy, combining ξ and η vectors in the Euler-Lagrange equation can be partitioned into the dynamics for the ξ coordinates and the η dynamics. So, we obtain

$$\begin{aligned} F_I &= m\ddot{\xi} + mg \\ \tau_\eta &= J\ddot{\eta} + \dot{J}\dot{\eta} - \frac{1}{2} \frac{\partial}{\partial \eta} (\dot{\eta}^T J \dot{\eta}) \end{aligned} \quad (5)$$

Defining the Coriolis terms and gyroscopic and centrifugal terms as

$$C(\eta, \dot{\eta}) \dot{\eta} = J\ddot{\eta} - \frac{1}{2} \frac{\partial}{\partial \eta} (\dot{\eta}^T J \dot{\eta}) \quad (6)$$

Finally, the dynamic model of the quad-rotor aircraft is the following

$$\begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{pmatrix} = \begin{pmatrix} -u \sin \theta \\ u \sin \phi \cos \theta \\ u \cos \phi \cos \theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \quad (7)$$

$$J\ddot{\eta} = \tau_\eta - C(\eta, \dot{\eta}) \dot{\eta} \quad (8)$$

III. Nonlinear control law strategy

In this section we present a control law for the altitude control of the quad-rotor aircraft based on sliding-mode technique with integral action in order to eliminate the steady-state error.

The goal of control is the stabilization in altitude at a given reference along some flight mission. Such that the coordinated control of all four rotors will provide the desired altitude control.

A. Altitude robust control

Since altitude control concerns only the displacement in the z-axis one can consider the following reduced model described in (7), where

$$\ddot{z} = \frac{1}{m} (u \cos \phi \cos \theta - mg) \quad (9)$$

or we can have the following state space variable set,

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{1}{m} (u \cos \phi \cos \theta - mg) \end{aligned} \quad (10)$$

Now, the design problem is to enforce the behavior of the system states towards the desired trajectories which are known. Denote the reference trajectories by \dot{z}_d and z_d which is velocity and altitude desired respectively. Afterwards, we define the tracking errors by $e_z = z - z_d$, $\dot{e}_z = \dot{z} - \dot{z}_d$ and $\ddot{e}_z = \ddot{z} - \ddot{z}_d$ where z_d is the altitude desired.

B. Sliding mode with integral action

First of all, the sliding mode control with integral action scheme introduces a "sliding surface" along which the sliding motion is to take place. This surface is denoted by "s" and is defined as follow

$$\begin{aligned} s &= \left(\frac{d}{dt} + \lambda \right) e_z + k_I \int_0^{\infty} e_z dt \\ \dot{s} &= \dot{e}_z + \lambda e_z + k_I \int_0^{\infty} e_z dt \end{aligned} \quad (11)$$

where $\lambda > 0$ is the slope of the sliding line and k_I is the integral gain. And the derivative of the sliding surface in (11) can be given as

$$\dot{s} = \ddot{e}_z + \lambda \dot{e}_z + k_I e_z \quad (12)$$

C. Stability analysis

In order to provide global asymptotic stability about the equilibrium point of (11), we propose the following Lyapunov function candidate given as

$$V = \frac{1}{2} s^2 \quad (13)$$

The time derivative of the Lyapunov function candidate in (13) can be computed as follows

$$\dot{V} = s \dot{s} \quad (14)$$

then, doing the mathematical operations corresponding, we have that

$$\begin{aligned} \dot{V} &= s (\ddot{e}_z + \lambda \dot{e}_z + k_I e_z) \\ \dot{V} &= s (\ddot{z} - \ddot{z}_d + \lambda (\dot{z} - \dot{z}_d) + k_I (z - z_d)) \end{aligned} \quad (15)$$

after, we need that $\ddot{z}_d = 0$ and $\dot{z}_d = 0$ therefore

$$\dot{V} = s (\ddot{z} + \lambda \dot{z} + k_I (z - z_d)) \quad (16)$$

then, substituting (9) in (16) leads to

$$\dot{V} = s \left(\frac{1}{m} (u \cos \phi \cos \theta - mg) + \lambda \dot{z} + k_I (z - z_d) \right) \quad (17)$$

Moreover, we can consider presence of bounded disturbances $f(z, \dot{z})$, as follows

$$\dot{V} = s \left(\frac{1}{m} (u \cos \phi \cos \theta - mg) + \lambda \dot{z} + k_I (z - z_d) + f(z, \dot{z}) \right) \quad (18)$$

where we have to drive the variable "s" in (11) to zero in finite time by means of the control u . Therefore, assuming that

$$u = \frac{m (-\lambda \dot{z} - k_I (z - z_d) + mg)}{\cos \phi \cos \theta} + \nu \quad (19)$$

and substituting it into (18) we obtain

$$\dot{V} = s (f(z, \dot{z}) + \nu) = s f(z, \dot{z}) + s \nu \leq |s| L + s \nu \quad (20)$$

where L is known constant $L > 0$, that follows

$$\dot{V} \leq |s| L + s \nu \quad (21)$$

and selecting

$$\nu = -\rho \text{sign}(s) \quad (22)$$

where $\rho > 0$ and $\text{sign}(s) = \frac{s}{|s|}$ substituting into (21) we obtain

$$\dot{V} \leq |s| L + s \left(-\rho \frac{s}{|s|} \right) = |s| L - |s| \rho \quad (23)$$

therefore

$$\dot{V} \leq -|s| (\rho - L) \quad (24)$$

Finally, the sliding mode control law u proposed below will drive the quad-rotor aircraft to the desired altitude,

$$u = \frac{m (c_1 + mg)}{\cos \phi \cos \theta} \quad (25)$$

for simplicity the term c_1 includes the sliding mode action, which is described as follows:

$$c_1 = -\lambda \dot{z} - k_I (z - z_d) - \rho \text{sign}(s) \quad (26)$$

From control law (25) described above. it follows that the pitch and roll angles must belong to $\phi, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ in order to avoid singular positions, which represents a real situation in hover flight. Then, using (25) into (9) we get

$$\ddot{z} = -\lambda \dot{z} - k_I (z - z_d) - \rho \text{sign}(s) \quad (27)$$

choosing the correct values of λ , k_I and ρ in (27) ensures a stable performance of the altitude of the quad-rotor aircraft.

D. Attitude control

The yaw angular position ψ can be controlled by applying

$$\tau_{\psi} = -a_{\psi_1} \dot{\psi} - a_{\psi_2} (\psi - \psi_d) \quad (28)$$

where ψ_d is the desired yaw angular position. Now, we propose a control law to stabilize the attitude using robust nested saturation method in [13] for the *roll and pitch control*. This control law is used to avoid abrupt behavior in the performance of the quad-rotor aircraft.

$$\begin{aligned} \tau_{\phi} = & -\sigma_{\phi_1} (k_1 \phi_2 + \sigma_{\phi_2} (k_2 (\phi_2 + k_1 \phi_1) \\ & + \sigma_{\phi_3} (k_3 ((\phi_2 + k_1 \phi_1) + k_1 k_2 y_2 + k_2 \phi_1) \\ & + \sigma_{\phi_4} (k_4 ((\phi_2 + k_1 \phi_1) + k_1 k_2 y_2 + k_2 \phi_1) \\ & + k_3 \phi_1 + k_1 k_2 k_3 y_1 + (k_1 + k_2) k_3 y_2))) \end{aligned} \quad (29)$$

and

$$\begin{aligned} \tau_{\theta} = & -\sigma_{\theta_1} (k_1 \theta_2 + \sigma_{\theta_2} (k_2 (\theta_2 + k_1 \theta_1) \\ & + \sigma_{\theta_3} (k_3 ((\theta_2 + k_1 \theta_1) - k_1 k_2 x_2 + k_2 \theta_1) \\ & + \sigma_{\theta_4} (k_4 ((\theta_2 + k_1 \theta_1) - k_1 k_2 x_2 + k_2 \theta_1) \\ & + k_3 \theta_1 - k_1 k_2 k_3 x_1 - (k_1 + k_2) k_3 x_2))) \end{aligned} \quad (30)$$

where $\sigma(s)$ is a saturation function is defined as

$$\sigma(s) = \begin{cases} M & \text{if } s > M \\ s & \text{if } -M \leq s \leq M \\ -M & \text{if } s < -M \end{cases} \quad (31)$$

and constant M is defined as $M > 0$ and it represents the amplitude (range) of the function of saturation.

IV. Simulation results

In this section, the simulations results of the quad-rotor aircraft using the control technique called integral sliding mode are presented below in order to observe the elimination of steady-state error effect in the input control to the sliding surface on the z-dynamic (*altitude*) for the aerial vehicle.

A short list of the parameters of the sliding mode controller implemented in these simulations are briefly described in Table I.

Table I. Simulations parameters for integral sliding mode.

Parameters	Value
Mass of the quad-rotor aircraft, (m) [kg]	1.00
Gravitational acceleration, (g) [m/s ²]	9.81
Reaching law parameter, (ρ)	0.80
Slope parameter, (λ)	1.20
Integral gain, (k_I)	0.01

In the simulations, a bounded disturbance (d) has been introduced to test stability and convergence rate in the integral sliding mode algorithm for the altitude control of the quad-rotor aircraft as follows,

$$d(t) = 0.15 \sin(15t) + 0.15 \cos(20t)$$

Moreover, in sliding mode algorithm with integral action the resulting system and controller mathematical model were converted to their respective Simulink models for ease of simulations.

A. Altitude simulations results

The results of the simulations of the altitude control based on sliding mode conventional and with integral action are shown in Figures 2 and 3. Observer that altitude of the quad-rotor aircraft achieved in a reasonable time ($t_f = 4.5s$) to the desired altitude with some chattering effect around the altitude reference value which it is an implicit feature of these controllers.

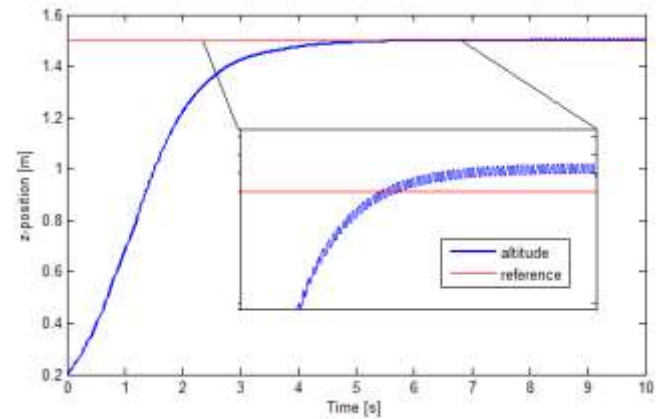


Figure 2. Altitude sliding mode control behavior in the presence of bounded disturbances. There exists steady-state error.

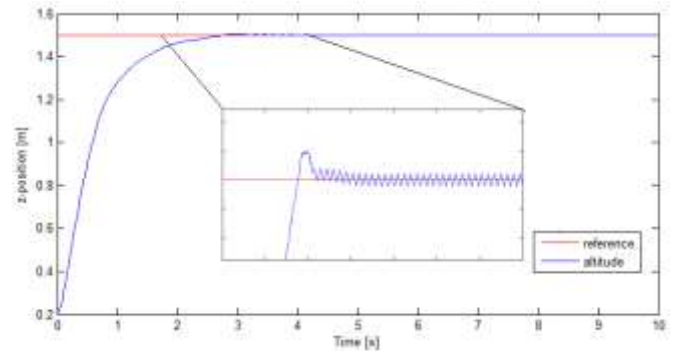


Figure 3. Altitude sliding mode control with integral action behavior in the presence of bounded disturbances. There no exists steady-state error.

In Figure 2 one can clearly observe that there is a steady-state error due to the dynamics of the system while in Figure 3 shows that has been removed completely steady-state error because it has been added the integrator in order to compensate the error towards the altitude reference. This shows that *integral action* in the controller successfully eliminate the steady-state error.

v. Conclusions

In this study, a sliding mode control with an integral action has been adopted to control the altitude for a quad-rotor aircraft in order to eliminate the steady-state error and reduce chattering effect to reach a desired height. This control law consider external perturbations in the input control. Several simulations has been developed to validate the robustness algorithm proposed. In these tests, it has been proposed the bounded disturbance which is used to evaluate the gains of the integral sliding mode controller. We consider that the control altitude is an important issue for a good take-off and landing performance in a future work.

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