

Optimum Design of Corrugated Web Beams Using Stochastic Search Techniques

[Ferhat Erdal, Erkan Doğan, Osman Tunca, Serkan Taş]

Abstract— In the present research, the designation of corrugated steel web beams is formulated as optimum design problem. The minimum weights of these new generation steel beams are taken as the objective functions while the design constraints are respectively implemented from BS EN1993-1:2005 (Annex-D, Eurocode 3) ve DIN 18-800 Teil-1. Firefly and hunting search stochastic optimization techniques are used for obtaining the solution of the design problem. The thickness and height of web, distance between the peaks of the following two web curves, the width and thickness of flange will be considered as the problem design variables. The formulation of the design problem considering the limitations of the above mentioned turns out to be a discrete programming problem. Number of design examples considered to demonstrate the efficiency of the algorithms.

Keywords—structural optimization, corrugated web beams, stochastic search methods, firefly algorithm, hunting search.

I. Introduction

The use of long span steel beams results in a range of benefits, including flexible, free internal spaces and reduced foundation costs. Many large clear-span design solutions are also well adapted to simplify the integration of mechanical or utility services. Corrugated steel web beams provide economical solution and pleasing appearance for space structures.

In steel construction applications, the web part of beam usually carries the compressive stress and transmits shear in the beam while the flanges support the applied external loads. By using greater part of the material for the flanges and thinner web, materials saving could be achieved without weakening the load-carrying capability of the beam. In this case, the compressive stress in the web has exceeded the critical point prior to the occurrence of yielding, the flat web loses its stability and deforms transversely.

Corrugated structure of the web cross-section not only increases the resistance of the beam against to shear force and other possible local effects, but also prevents the buckling due to loss of moment of inertia before the plastic limit. This specific structure of the web leads to a decrease in the beam unit weight and increase in the load carrying capacity. These efficient construction materials, commonly used in developed countries over years, can be utilized at the roofs as an alternative to space truss and roof truss, at the slabs as floor beams or columns under axial force. Although the designers are aware of the advantages of the composite systems to be produced with that beams, there is still not a detailed technical specification about their design and behavior. The first studies on the corrugated web beams were focused on the vertically trapezoidal corrugation. Elgaaly investigated the failure mechanisms of trapezoidal corrugation beams under different loading conditions, namely shear mode [1], bending mode [2]. They found that the web could be neglected in the beam bending design calculation due to its insignificant contribution to the beam's load-carrying capability. Besides that, the two distinct modes of failure under the effect of patch loading were dependent on the loading position and the corrugation parameters. These are found agreeable to the investigation by Johnson and Cafolla and were further discussed in their writings [3]. In addition, the experimental tests conducted by Li et al. [4] demonstrated that the corrugated web beam has 2 times higher buckling resistance than the plane web type. According to Pasternak et al., [5], the buckling resistance of presently used sinusoidal corrugated webs is comparable with plane webs.

II. Design of Corrugated Steel Web Beams

Corrugated web beams shown in Fig. 1 are built-up girders with a thin-walled, corrugated web and plate flanges.

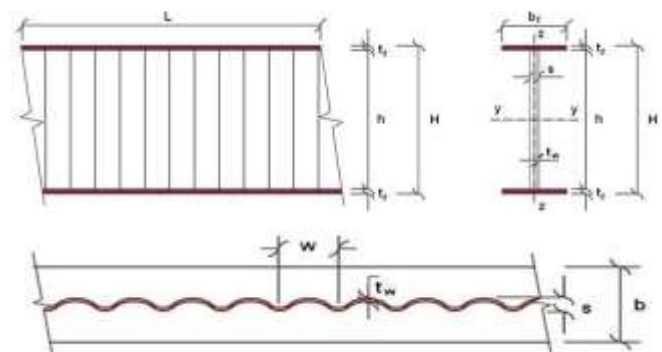


Figure 1. Geometric properties of Corrugated Web Beam

Design constraints include the displacement limitations, buckling capacities of web and flanges, bearing and torsional-flexural buckling capacities of flanges.

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A. Transverse load carrying capacity of corrugated webs

Based upon the experimental tests and finite element analysis results, the following design procedure has been suggested: The corrugated web is regarded as an orthotropic plate with rigidities D_x and D_y . According to [5], the following formula therefore applies to the corrugated web:

$$D_x = \frac{E \times w \times t^3}{12 \times s}, D_y = \frac{E \times I_y}{w} \text{ for } D_x \leq D_y \quad (1)$$

For transverse buckling stress of corrugated web;

$$\tau_{EG} = \frac{162}{5 \times t_w \times h^2} \sqrt{(D_x \times D_y^3)} \quad (2)$$

For slenderness parameter of corrugated web;

$$\lambda_{GN} = \sqrt{\frac{f_y}{\sqrt{3} \times \tau_{EG}}} \quad (3)$$

With the buckling coefficient of corrugated web;

$$K_B = \frac{1}{(\lambda_{GN})^{3/2}} \quad (4)$$

the transverse force load carrying capacity for the corrugated web finally results in:

$$V_{TK-MAX} = \frac{K_B \times f_y \times h \times t_w}{\sqrt{3}} \quad (5)$$

B. Normal load carrying capacity of flanges

In determining the normal bearing force of the flanges, a distinction must be made between tensile and compressive stresses. In the case of tensile stress, the load carrying capacity of the flange is derived as follows:

$$\sigma_{ALLOW} = \frac{N_{T-MAX}}{b_f \times t_f} \quad (6)$$

Reformulation of the expression for $\psi = 1$ leads to the following elastic limit stress:

$$\sigma_{EL} = \frac{4000}{(b_f \times t_f)^2} \quad (7)$$

Therefore the reduced normal force on the flange:

$$N_{NORMAL} = \sigma_{EL} \times b_f \times t_f \quad (8)$$

Global failure of stability - lateral buckling of the flange - is equivalent to the verification against torsional-flexural buckling. If the restraining effect of the web is ignored, the torsional-flexural verification is carried out as the buckling verification for the "isolated" flange in accordance with [5]. The following condition for the distance between lateral supports is obtained:

$$\tau_{EG} = \frac{\pi}{4\sqrt{3}} \sqrt{E \times f_y} \times \frac{b_f^2 \times t_f}{k_c \times c} \quad (9)$$

III. Stochastic Optimization Techniques

A combinatorial optimization problem requires exhaustive search and effort to determine an optimum solution which is computationally expensive and in some cases may even not be practically possible. Metaheuristic search techniques are established to make this search within computationally acceptable time period. Amongst these techniques are simulated annealing [6], evolution strategies [7], particle swarm optimizer [8], tabu search method [9], ant colony optimization [10], harmony search method [11], genetic algorithms [12] and others. All of these techniques implement particular metaheuristic search algorithms that are developed based on simulation of a natural phenomenon into numerical optimization procedure. They have gained a worldwide popularity recently and have proved to be quite robust and effective methods for finding solutions to discrete programming problems in many disciplines of science and engineering, including structural optimization.

A. Hunting Search Algorithm

Hunting search method based optimum design algorithm has six basic steps, which is outlined in the following [13].

Step 1 Initializing design algorithm and parameters: *HGS* defines the group size which is the number of solution vectors in hunting group, *MML* represents the maximum movement toward the leader and *HGCR* is hunting group consideration rate which varies between 0 and 1.

Step 2 Generation of hunting group: On the basis of the number of hunters (*HGS*), hunting group is initialized by selecting randomly sequence number of steel sections (I_i) for each group.

$$I_i = \text{INT}[I_{\min} + r(I_{\max} - I_{\min})] \quad i = 1, \dots, n \quad (10)$$

where; the term r represents a random number between 0 and 1, I_{\min} is equal to 1 and I_{\max} is the total number of values in the discrete set respectively. n is the total number of design variables.

Step 3 Moving toward the leader: New hunters' positions are generated by moving toward the leader hunter.

$$I_i^j = I_i + r \text{MML}(I_i^L - I_i) \quad i = 1, \dots, n \quad (11)$$

where; I_i^L is the position value of the leader for the i -th variable.

Step 4 Position correction-cooperation between hunters:

After moving toward the leader, hunters tend to choose another position to conduct the 'hunt' efficiently, i.e. better solutions. Position correction is performed in two ways, one of which is real value correction and the other is digital value. In this study real value correction is employed for the position correction of hunters.

$$I_i^j \leftarrow \begin{cases} I_i^j \in \{I_i^1, I_i^2, \dots, I_i^{HGS}\} \text{ with probability } HGCR \\ \text{INT}(I_i^j = I_i \pm Ra) \text{ with probability } (1-HGCR) \end{cases} \quad (12)$$

Step 5 Reorganizing the hunting group: Hunters must reorganize themselves to get another chance to find the global optimum. If the difference between the objective

function values obtained by the leader and the worst hunter in the group becomes smaller than a predetermined constant (ϵ_i) and the termination criterion is not satisfied, then the group reorganized. By employing the Eq. 6, leader keeps its position and the others randomly select positions.

$$I_i^L = I_i^L \pm r \left(\max(I_i) - \min(I_i) \right) \alpha (-\beta EN) \quad (13)$$

Where; I_i^L is the position value of the leader for the i -th variable, r represents the random number between 0 and 1, $\min(I_i)$ and $\max(I_i)$ are min. and max. values of variable I_i , respectively, EN refers to the number of times that the hunting group has trapped until this step. α and β are determine the convergence rate of the algorithm.

Step 6 Termination: The steps 3 and 5 are repeated until a pre-assigned maximum number of cycles is reached.

B. Firefly Algorithm

Firefly (FFO) algorithm is originated by Yang [14, 17] and it is based on the idealized behaviour of flashing characteristics of fireflies. These insects communicate, search for pray and find mates using bioluminescence with varying flaying patterns. The firefly algorithm is based on three rules. These are:

- All fireflies are unisex so they attract one another.
- Attractiveness is propositional to firefly brightness. For any couple of flashing fireflies, the less bright one moves towards the brighter one. Attractiveness is proportional to the brightness. Attractiveness and brightness decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
- The brightness of a firefly is affected or determined by the landscape of the objective function.

Mathematical interpretation of the above rules is given in following as explained in [14].

In the firefly algorithm attractiveness of a firefly is assumed to be determined by its brightness which is related with the objective function. The brightness i of a firefly at a particular location x can be chosen as $I(x) \propto f(x)$ where $f(x)$ is the objective function. However, the attractiveness β is relative, it should be judged by the other fireflies. Thus, it will vary with the distance r_{ij} between firefly i and firefly j .

In the firefly algorithm the attractiveness function is taken to be proportional to the light intensity by adjacent fireflies and it is defined as;

$$\beta(r) = \beta_0 e^{-\gamma r^m}, \quad (m \geq 1) \quad (14)$$

where β_0 is the attractiveness at $r = 0$.

The distance between any two fireflies i and j at x_i and x_j is calculated as

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (15)$$

Where $x_{i,k}$ is the k^{th} component of the spatial coordinate x_i of the i^{th} firefly.

The movement of a firefly i which is attracted to another more brighter firefly j is determined by

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha \left(\text{rand} - \frac{1}{2} \right) \quad (16)$$

Where the second term is due to the attraction while the third term is randomization with α being the randomization parameter. "rand" is a random number generator uniformly distributed in [0,1]. The values of parameters in the above equations are generally taken as $\beta_0 = 1$ and $\alpha \in [0,1]$. Randomization term can be extended to a normal distribution $N(0,1)$ or other distributions. γ characterizes the variation of the attractiveness, and its value determines the speed of convergence and performance of the firefly algorithm. In most applications its value is taken between 0 and 100.

The pseudo code of the algorithm is given in [14] which repeated in Fig. 2. Engineering design application of firefly algorithm is given in [15]. In [16] FFO algorithm is used to determine optimum solution of six engineering design problem that are taken from the literature and its performance is compared with other metaheuristic algorithms such as particle swarm optimizer, differential evolution, genetic algorithm, simulated annealing, harmony search method and others. It is stated that the results attained from the optimum solutions of these design examples firefly algorithm is more efficient than particle swarm optimizer, genetic algorithm, simulated annealing and harmony search method. In [17], the permutation flow shop is formulated as a mixed integer programming problem which is classified as hard to solve nonlinear programming problem. FFO algorithm is applied to find the optimum solution of this problem. It is concluded that the preliminary results indicated that FFO performs better than the ant colony algorithm on benchmark problems taken from literature.

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begin;
    Objective function  $f(x)$ ,  $(x) = (x_1, \dots, x_d)$ 
    Generate initial population of fireflies  $x_i$ ,  $(i = 1, \dots, n)$ 
    Light intensity  $I_i$  at  $x_i$  is determined by  $f(x_i)$ 
    Define light absorption coefficient  $\gamma$ 
    while (until the termination criteria is satisfied)
        for  $i = 1 : n$  all  $n$  fireflies
            for  $j = 1 : i$  all  $n$  fireflies
                if ( $I_j > I_i$ )
                    Move firefly  $i$  towards  $j$  in  $d$ -dimension
                end if
            Attractiveness varies with distance  $r$  via  $\exp[-\gamma r^2]$ 
            Evaluate new solutions and update light intensity
        end for j
    end for i
    Rank the fireflies and find the current best
    end while
    Post-process results and visualization
end procedure;
    
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Figure 2. Pseudo code of firefly algorithm

IV. Optimum Design Problem

The design of a corrugated web beam requires the selection of width and thickness of a plate from which the corrugated web is to be produced, distance between the peak points of each corrugate, the length of corrugate web, the selection of width and thickness of a plate for upper and lower flanges in the beam. For this purpose a design pool is prepared. The optimum design problem formulated

considering the design constraints explained in the previous sections yields the following mathematical model [18-22]. Find a integer design vector $\{I\} = \{I_1, I_2, I_3, I_4, I_5\}^T$ where I_1 is the sequence number of for the width of upper and lower flanges, I_2 is the sequence number for the thickness values of upper and lower flanges, I_3 is the thickness of corrugated web, I_4 is distance between the peak points of each corrugate web and I_5 the height of corrugate., Hence the design problem turns out to be minimize the weight of the corrugated web beam.

$$W_{SOB} = \rho_s ((2 \times b_f \times t_f \times L) + (h \times t_w \times L_{düz})) \quad (13)$$

where, ρ_s density of steel, b_f the width of flange, t_f thickness of flange, L span of beam, h height of corrugated web, t_w thickness of corrugated web ve $L_{düz}$ span of beam before corrugation process. Design of a corrugated beam requires the satisfaction of some geometrical restrictions that are formulated through Eqns. (14-17).

Web dimensions:

$$333 \text{ mm} \leq h \leq 1500 \text{ mm} \quad (14)$$

$$1.5 \text{ mm} \leq t_w \leq 5.0 \text{ mm} \quad (15)$$

Flange dimensions:

$$120 \text{ mm} \leq b_f \leq 450 \text{ mm} \quad (16)$$

$$6.0 \text{ mm} \leq t_f \leq 30.0 \text{ mm} \quad (17)$$

v. Design Example

Optimum design algorithms presented are used to design a corrugated steel web beam with 5-m span shown in Fig. 3. The beam is subjected to point loading. The upper flange of the beam is laterally supported by the floor system that it supports. The maximum displacement is limited to 17 mm. The modulus of elasticity is 205 kN/mm².

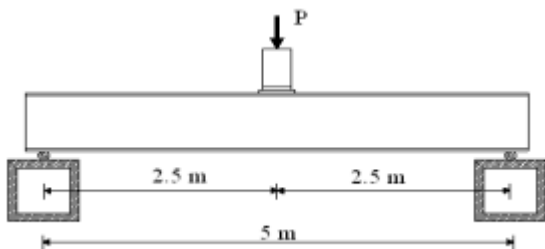


Figure 3. Loading of 5-m span Corrugated Web beam

The design example is separately solved by hunting search (HSM) and firefly algorithm (FFO). The size of the initial population and the maximum number of generations are kept the same in HSM and FFA (Table I).

TABLE I. PARAMETERS OF HSA AND FFO TECHNIQUES

| Technique | The values of parameters |
|-----------|---|
| HSA | $HGS = 90$ $MML = 0.002$ $HGCR = 0.90$ $Ra_{max} = 0.01$, $Ra_{min} = 0$ $par = 0.45$ $\alpha = 0.9$ $\beta = 0.02$ $IE = 25$ $N_{cyc} = 50000$ |
| FFO | $\mu = 40$ $\beta_0 = 0.25$ $\beta = 1$ $\alpha = 0.01$ $\gamma = 1$ $N_{ite} = 50000$ |

The result of the sensitivity analysis carried out for the FFO parameters is given in Table II.

In steel construction applications, the web part of beam usually carries the compressive stress and transmits shear in the beam while the flanges support the applied external loads. By using greater part of the material for the flanges and thinner web, materials saving could be achieved without weakening the load-carrying capability of the beam. In this case, the compressive stress in the web has exceeded the critical point prior to the occurrence of yielding, the flat web loses its stability and deforms transversely.

TABLE II. OPTIMUM DESIGN OF CORRUGATED BEAM WITH 5-M SPAN

| Optimum Section | t_w (mm) | h (mm) | t_f (mm) | H_c (mm) | L_c (mm) | Minimum Weight(kg) |
|-----------------|------------|--------|------------|------------|------------|--------------------|
| OGK_330 | 5 | 330 | 8 | 43 | 155 | 176.33 |

It is apparent from the table that FFO produces the least weight for corrugated web beam which is equal to 176.33 kg. The maximum value of the strength ratio is 0.98 which is almost upper bound. This reveals the fact that the strength constraints are dominant in the problem. FFO algorithm presented selects OGK_330 for the root beam. The optimum corrugated web beam should be produced such that it should have 5 mm web thickness 330 mm web height, 8 mm flange thickness and 160 mm flange width. HSA produces 185.76 kg weight for this design example. The design history curve for FFO and HSA techniques is shown in Fig. 4. It is apparent from the figure that FFO method performs better convergence rate and better solution than HSA technique in this design problem.

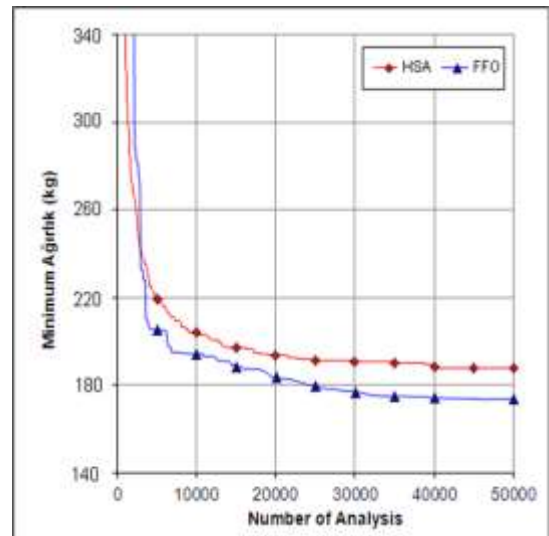
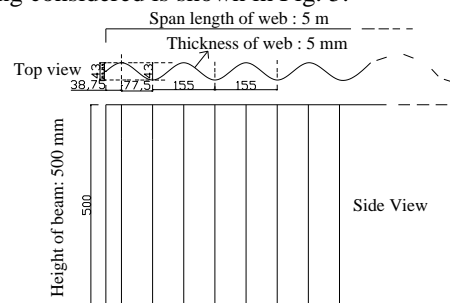


Figure 4. Design History Graphic of 5-m Corrugated Web Beam

The optimum shape of the corrugated steel web beam for the loading considered is shown in Fig. 5.



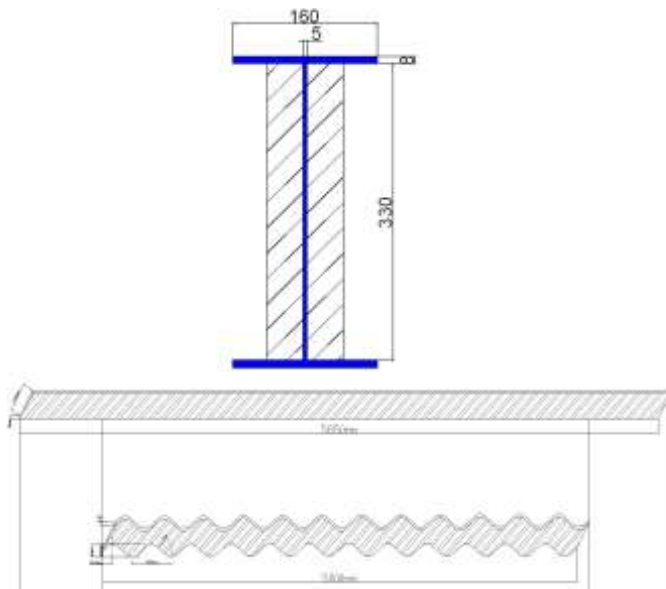


Figure 5. The optimum shape of the corrugated web beam with different views

VI. Conclusions

This study concerns with the application of a hunting search and firefly algorithms to demonstrate the robustness of the proposed algorithms and to find the optimum design of corrugated web beams. The design algorithm is mathematically simple but effective in finding the solutions of optimization problems. Fly-back mechanism is employed for handling the problem constraints and feasible ones being candidate solutions to give the minimum weight are determined. A corrugated web beam example is designed to illustrate the efficiency of the algorithms. Comparison of the optimum designs attained by HSA and FFO clearly shows that the FFO outperforms the latter in the second particular design problem. In view of the results obtained, it can be concluded that the FFO is an efficient and robust technique that can successfully be used in optimum design of corrugated web beams.

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References

[1] Elgaaly M, Hamilton RW ve Seshadri A, Shear Strength of Beams with Corrugated Webs, *Journal of Structural Engineering (ASCE)*, 122, 4, 390-398, 1996.
 [2] Elgaaly M, Hamilton RW and Seshadri A, Bending Strength of Steel Beams with Corrugated Webs, *Journal of Structural Engineering (ASCE)*, 123, 6, 772-782, 1997.
 [3] Johnson RP ve Cafolla J, Local Flange Buckling in Plate Girders with Corrugated Webs, *Structures and Buildings (ICE)*, 123, 148-156, 1998.
 [4] Li Y, Zhang W, Zhou Q, Qi X ve Widera GEO, Development and Research on H-beams with Wholly Corrugated Webs, *Journal of Materials Processing Technology*, 101, 1-3, 115-118, 2000.

[5] Pasternak H ve Kubieniec G, Plate Girders with Corrugated Webs, *Journal of Civil Engineering and Management*, 16, 2, 166-171, 2010.
 [6] Kirkpatrick, S., Gerlatt, C. D., Vecchi, M. P., Optimization by Simulated Annealing, *Science*, 220, 671-680, (1983).
 [7] Rechenberg I. Cybernetic solution path of an experimental problem. Royal aircraft establishment, library translation no. 1122. UK: Farnborough, Hants;1965.
 [8] Perez, R. E., Behdian, K., Particle Swarm Approach for Structural Design Optimization, *Computers and Structures*, 85 (19-20), 1579-1588, 2007.
 [9] Glover F. Tabu search-part I. *ORSA J Comput*, 1(3):190-206, 1989.
 [10] Dorigo M, Stutzle T. Ant colony optimization a Bradford book. USA: Massachusetts Institute of Technology; 2004.
 [11] Lee K.S., Geem Z.W., A New Structural Optimization Method Based on the Harmony Search Algorithm, *Computers and Structures*, 82, 781-798, 2004.
 [12] Goldberg D.E., Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley Publishing, 1989.
 [13] Oftadeh ., Mahjoob MJ, Shariatpanahi M, A novel meta-heuristic optimization algorithm inspired by group hunting of animals: Hunting search, *Computers and Mathematics with Applications*, 60, 2087-2098, 2010.
 [14] X. S. Yang, (2008) Nature-Inspired Metaheuristic Algorithms, Luniver Press, UK.
 [15] X. S. Yang, (2009) 'Firefly algorithms for multimodal optimization', in: *Stochastic Algorithms: Foundations and Applications* (Eds O. Watanabe and T. Zeugmann), SAGA 2009, Lecture Notes in Computer Science, 5792, Springer-Verlag, Berlin, pp. 169-178.
 [16] X. S. Yang,, Firefly algorithm, stochastic test functions and design Optimisation, *Int. J. Bio-Inspired Computation*, Vol. 2, No: 2, 78-84, 2010.
 [17] X. S. Yang,, and X. He, Firefly algorithm: Recent advances and applications, *International Journal of Swarm Intelligence*, Inderscience Publishers, Artificial Intelligence and Intelligent Systems, Vol. 1, 36-50, Science, Engineering and Technology, 2013.
 [18] British Standards, BS 5950, Structural Use of Steelworks in Building, Part 1. Code of Practice for Design in Simple and Continuous construction, hot rolled sections, British Standard Institution, London, U.K., 2000.
 [19] DIN V ENV 1993-1-1, EUROCODE 3: Design of Steel Structures; Part 1- General Rules and rules for buildings.
 [20] DIN 18 800 Teil1-3, Stahbauten; Bemessung und Konstruktion.
 [21] British Standards, BS 8110, Structural Use of Concrete, Part 1. Code of Practice for Design and Construction, British Standard Institution, London, U.K.
 [22] Steelwork design Guide to BS 5950: Part 1, Section Properties, Member Capacities, Vol.1, 4th edition, The Steel Construction Institute, U.K., 1990.