

# Numerical Investigation of Heat Transfer of Nanofluid Flow Past Two Cylinder in Tandem Arrangements

Özge Yetik

**Abstract**— In this paper, numerical method is developed and simulations of flow and heat transfer of nanofluids is presented. The effect of nanoparticle volume fraction for the enhancement of heat transfer is examined for several sets of values of Reynolds numbers. The study is conducted for pure water and water-copper nanofluid. The study is carried out for Reynolds numbers of 100 and 150 and for volume fraction of nanoparticles of 0, 0.05, 0.10 and 0.15. The mean Nusselt numbers for front and rear cylinders are obtained. The obtained results indicated that the increase of nanoparticles volume fraction has a positive effect on the mean Nusselt number.

**Index Terms**—Nanofluid, Tandem arrangement, Flow past cylinder, Fluent.

## I. INTRODUCTION

Heat transfer from tandem square cylinders has numerous engineering applications such as heat exchanger, heat losses from high-rise buildings, chimneys, pipelines, turbine, piers of a bridge and many other thermal applications. Such fluctuating forced may cause structural vibrations, acoustic noise and resonance and in some cases can trigger structural failure or enhance undesirable flow mixing in the wake [1]. Therefore, it is very important engineering subject.

In tandem circular cylinder with nanofluids have not been studied so far. For this reason, this numerical simulation of steady state and laminar convective heat transfer from two isothermal cylinders of tandem arrangement is carried out by the commercial CFD software -FLUENT. The working fluid is pure water and water based copper nanofluid with different solid fraction values. The analysis is carried out for the Reynolds (Re) numbers of 100 and 150.

The single square cylinder has examined extensively in the studies [2-12]. They examined the change of the Nu and St numbers and drag (CD) coefficient for Re numbers ranges from 100 to 160. All of these studies were simulated in 2D. Mahir [13] studied to investigate the effect of the flow structure in the wake of a square cylinder placed near a plane wall by

applying a fully implicit finite-difference method to the Navier-Stokes equations in two and three-dimensional flow. However, for tandem cases, there are more complex than the single ones because of the existence of two bodies and gap effect.

Chatterjee and Amiroudine [14] carried out to understand the effects of thermal buoyancy and Prandtl (Pr) number on flow characteristics and mixed convection heat transfer over two equal isothermal square cylinders placed in a tandem arrangement within a channel at low Re numbers. The numerical results were presented for the range of conditions as:  $1 \leq Re \leq 30$ ,  $0.7 \leq Pr \leq 100$  (the maximum value of Peclet number being 3000) and  $0 \leq Ri \leq 1$  for a fixed blockage parameter  $B = 10\%$ . The unsteady numerical simulations were performed with a finite volume code based on the PISO algorithm in a collocated grid system.

Mahir and Altaç [15] investigated numerical simulation of unsteady laminar convective heat transfer from two isothermal cylinders of tandem arrangement. This simulation were carried out by the commercial CFD software -FLUENT. The working fluid was air.

But all of these publications are not use nanofluids. Heat transfer of low Re number or of natural convection flow of nanofluid from a single cylinder has received less attention in the literature[16-20].

Vegad et al [16] was investigated two-dimensional steady state flow for water based copper nanofluid with different solid fraction values around a circular cylinder. They showed that local Nusselt (Nu) number over the cylinder surface had found to increase with increase nanoparticle fraction. Selimefendigil and Öztop [17] numerically studied with mix convection at a backward facing with a rotating cylinder. They reported that Nu number is a linear increase in the heat transfer rising with increasing Re number and nanoparticle volume fraction. Akçaoğlu et al [18] were numerical studied of natural convection heat transfer of nanofluid in a square enclosure. Results show that Rayleigh (Ra) numbers, solid volume fraction, nanoparticles and partition heights have a significant effect on the flow and heat transfer. Roslan et al [19] were studied with water-Ag, water-Cu, water- $Al_2O_3$  and water- $TiO_2$  nanofluids. The governing equations were formulated for velocity, pressure and temperature formulation and are modeled in COMSOL, a partial differential equation (PDE) solver based on the Galerkin finite element method (GFEM).

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As a result, the best heat transfer was obtained at a high nanoparticle concentration with a good conductivity value.

## II. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

A schematic description of the physical problem and coordinate system considered in this study is shown in Fig. 1. and a typical generated grid is shown in Fig. 2. The circular diameter, height and width of channel are  $D$ ,  $20D$  and  $32.5D$ , respectively.

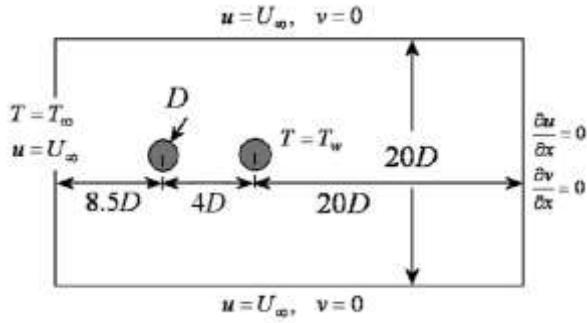


Fig. 1. Schematic diagram of the computational domain.

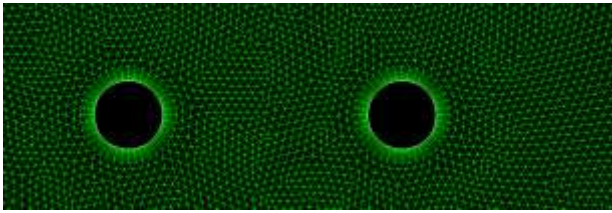


Fig. 2. Grid distribution of the computational domain.

The flow was investigated two dimensional, steady state, laminar and incompressible. The dimensionless governing equations for continuity, two-dimensional Navier-Stokes and the energy equations are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \beta_{nf} (T - T_c) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

Where  $u$  and  $v$  are the velocity components in  $x$  and  $y$  coordinates,  $T$  is the temperature of the fluid,  $p$  is the pressure.  $\alpha$ ,  $\rho$  and  $\nu$  denote the thermal diffusivity, density and kinematic viscosity, respectively. The thermo-physical properties of nanofluids are defined using equations. Density of the nanofluid is calculated using particle volume fraction  $\phi$  and densities of pure fluid  $\rho_f$  and of solid nanoparticles  $\rho_p$  as [20]:

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p \quad (5)$$

$\phi$  is the solid volume fraction of nanoparticles. Thermal diffusivity of the nanofluids is:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (6)$$

The thermal expansion coefficient of the nanofluids can be determined by:

$$(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_{bf} + \phi (\rho \beta)_p \quad (7)$$

The heat capacitance of the nanofluid can be expressed as [21]:

$$(\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_p \quad (8)$$

The ratio of dynamic viscosity of the nanofluids given as [22]:

$$\mu_{nf} = \mu_f (1 - \phi)^{-2.5} \quad (9)$$

The ratio of thermal conductivity of the nanofluids approximated by the Maxwell-Garnett formula [23]:

$$k_{nf} = k_f \left[ \frac{(k_p + 2k_f) - 2\phi(k_f - k_p)}{(k_p + 2k_f) + \phi(k_f - k_p)} \right] \quad (10)$$

Local Nusselt number is defined as [16]:

$$Nu = \left( \frac{\frac{k_{nf}}{k_f} \left( \frac{\partial T}{\partial Y} \right)_{cylinder\ surface}}{(T_{wall} - T_{basefluid})} \right) \quad (11)$$

### III. NUMERICAL METHODOLOGY

In this study, the finite volume method based on the ANSYS FLUENT package program is used. Problem is two-dimensional and steady state. Flow is laminar and slip boundary condition adopted in all surfaces. Second order upwind discrimination method is used momentum and energy equations to reduce errors.

The boundary conditions can be stated as:

- At the inlet, velocity is unidirectional and constant, temperature is uniform.  $(u = u_{\infty}, v = 0, T = 300K)$
- At the outlet, gradients of all variables in the x direction are set to zero.  $\left( \frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial T}{\partial x} = 0 \right)$
- At the cylinder walls, velocity is zero and temperature is uniform.  $(u = 0, v = 0, T = 400K)$
- At the bottom and top walls, velocity is unidirectional and constant, temperature is uniform.  $(u = u_{\infty}, v = 0, T = 300K)$

In this study, the basic fluid is water and nanoparticles is Cu. Fluid and solid particles thermo physical properties are showed Table 1.

TABLE I. FLUID AND SOLID PARTICLE THERMO PHYSICAL PROPERTIES

Properties	Water	Cu
$\rho$ (kg/m <sup>3</sup> )	998.2	8978
$C_p$ (J/kgK)	4182	381
$k$ (W/mK)	0.6	387.6
$\alpha \times 10^7$ (m <sup>2</sup> /s)	1.47	1163.1
$\beta \times 10^6$ (1/K)	210	16.7

The study is carried out for Reynolds numbers of 100 and 150 and for volume fraction of nanoparticles of 0, 0.05, 0.10 and 0.15. Grid testing study is carried out to obtain an independent mesh. Five different grid sizes are controlled and checked. The Nu numbers is given in Table 2 for different grid size at Re number 100 and  $\phi = 0$ . Grid size of 29565 is decided from this table.

TABLE II. NUSSLETT NUMBERS FOR DIFFERENT GRID SIZE.

Grid Size	Averaged Nu Number (front cylinder)	Averaged Nu Number (rear cylinder)
9341	10.35868	5.57857
7785	10.27586	5.68952
14577	10.13548	5.49895
29565	10.13372	5.47693
39748	10.12458	5.46786

To validate the computational code, the available data in the literature which are mostly studied to air. Figure 3 and figure 4 compares the Nu number and drag coefficient ( $C_d$ ) in this study and literature at Re 100, 120, 140 and 160 for single cylinder.

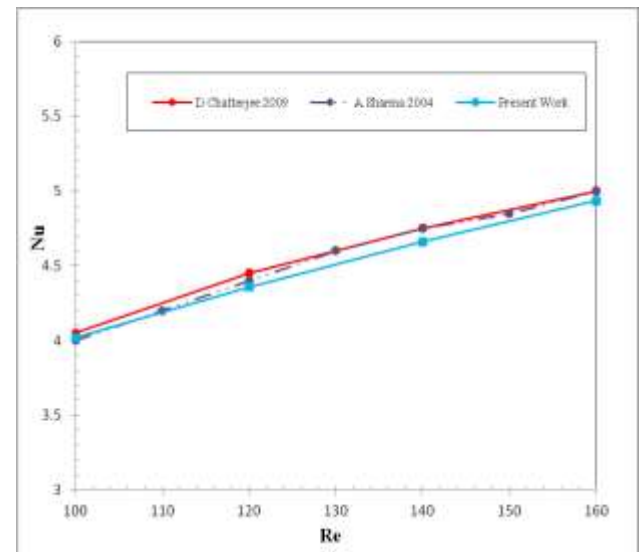


Fig. 3. Comparing the values of Nu number with the data in the literature.

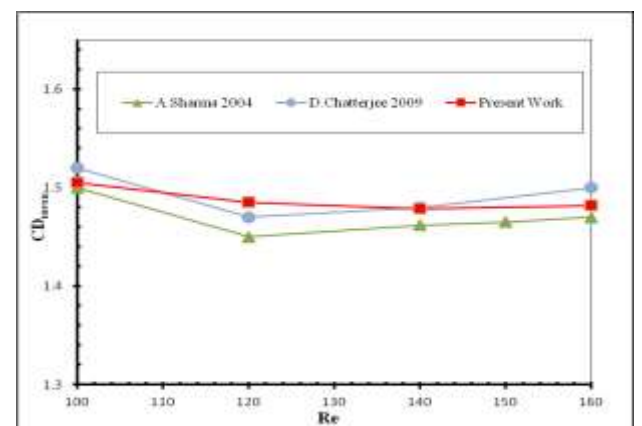


Fig. 4. Comparing the values of  $C_d$  number with the data in the literature.

#### IV. RESULT AND DISCUSSION

Altaç and Mahir [15] said that the  $L/D$  values are small, vortex doesn't occur between the cylinders in their study. The vortex street is formed by the boundary layer separated from the upstream cylinder at the downstream region. Because a vortex formation between the two cylinders is not observed for  $L/D = 1.5$ , the oscillations increase to form a vortex street for  $L/D = 2.5$  and a vortex occur for  $L/D = 4$ . For this reason  $L/D$  value is considered 4. Figure 5 and Figure 6 show the contours of temperature and velocity for the case of  $\phi = 0$  at  $Re = 150$ . Figure 7 shows vectors of velocity for the case of  $\phi = 0$  at  $Re = 150$ .

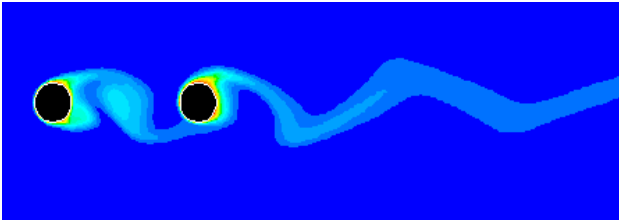


Fig. 5. Contours of temperature for the case of  $\phi = 0$  at  $Re=150$ .

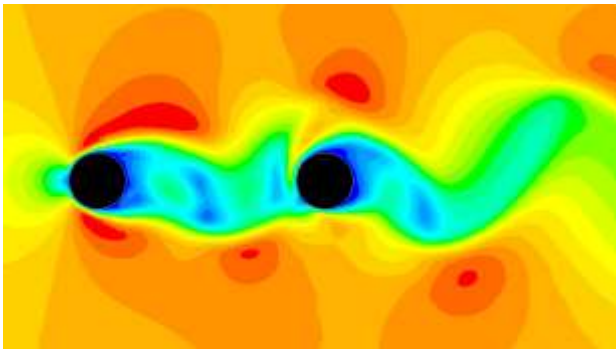


Fig. 6. Contours of velocity for the case of  $\phi = 0$  at  $Re=150$ .

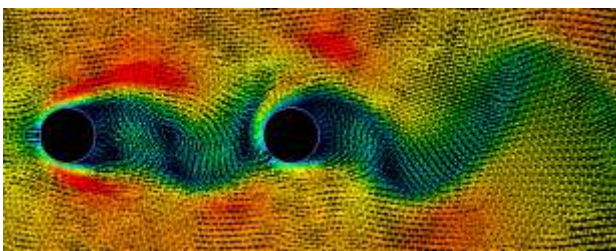


Fig. 7. Vectors of velocity for the case of  $\phi = 0$  at  $Re=150$ .

Figure 8 show that  $Nu$  number variation of nanofluid volume fraction at  $Re=100$  and  $Re=150$ . the  $Nu$  number is increased as the particle concentration and  $Re$  number increases.

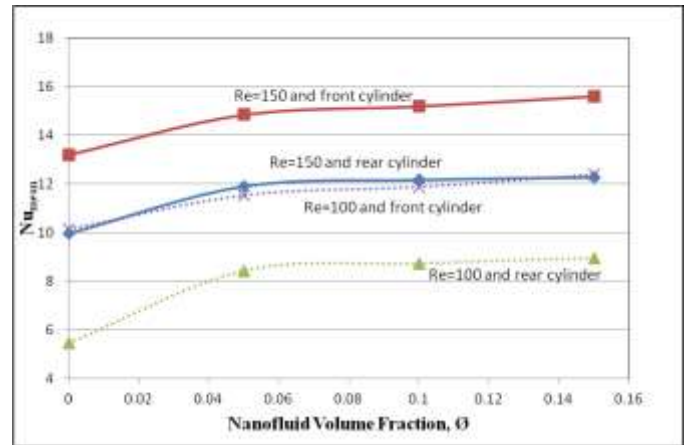


Fig. 8.  $Nu$  number variation of nanofluid volume fraction at  $Re=100$  and  $Re=150$ .

#### V. CONCLUSION

The paper presents the effect on flow and heat transfer in a tandem circular cylinders with nanofluids. A parametric study is performed for different  $Re$  numbers and solid volume fraction. According to the obtained results, this parameters have important effects on the flow and heat transfer. Due to the water-Cu nanofluids has the high thermal conductivity, heat transfer has increased between about 18 and 22 percent.  $Re$  number is increasing with increasing heat transfer due to the vortex.

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