

# Performance of Convolutionally Coded Underwater Acoustic Ad-Hoc Networks

Andrej Stefanov

**Abstract**—The paper considers the performance of underwater acoustic ad-hoc networks. A uniform distribution of nodes over a finite area is assumed. The node-to-node channel is modeled through the Ricean fading model. The paper adopts a communication theoretic approach and studies the impact of point-to-point convolutional channel coding on the sustainable number of hops through the network. Numerical examples are presented to illustrate the results of the analysis.

**Keywords**—underwater acoustic networks, fading.

## I. Introduction

Motivated by the recent advances in underwater acoustic communications [1], research efforts have extended to also incorporate the analysis and design of underwater networks [1—4]. This includes the progress made towards the design of medium access control [5—7], and routing protocols [8—11] for underwater networks. The design of underwater acoustic networks is presented with a number of challenges that include a significant limitation of the operational bandwidth, path loss that is dependent not only on the transmission distance, but also on the signal frequency, and high latency due to the low speed at which sound propagates under water.

The paper studies an underwater network of nodes that are bottom mounted, therefore a two-dimensional model is considered for the network. The nodes are also assumed to be uniformly distributed over a finite area. Multihop routing among nearest neighbor nodes is considered, due to more beneficial conditions in terms of bandwidth and path loss. The focus is on the impact of point-to-point convolutional channel coding on the performance of the multihop network.

The paper is organized as follows. The underwater acoustic propagation model and the ad-hoc network set-up are discussed in Sections II and III, respectively. A communication theoretic approach [12] is adopted and the impact of point-to-point convolutional coding on the connectivity of the network, that is, the sustainable number of hops in the network, is investigated. Numerical examples are presented in Section IV. Conclusions are provided in Section V.

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## II. Underwater Acoustic Propagation

Underwater acoustic communications experience a path loss that is dependent on the distance between the transmitting and the receiving node, and the transmission signal frequency. The path loss increases both with distance and with frequency setting a limit on the transmission bandwidth.

Hence, underwater acoustic propagation experiences attenuation, (a path loss), which at distance  $d$  between the transmitting and the receiving node, and for a signal transmitted on frequency  $f$ , is

$$A(d, f) = A_0 d^\kappa a(f)^d \quad (1)$$

where  $A_0$  designates a unit-normalizing constant that incorporates fixed losses,  $a(f)$  designates the absorption coefficient and  $\kappa$  designates the spreading factor. The spreading factor is usually  $1 \leq \kappa \leq 2$ . For practical spreading,  $\kappa = 1.5$ . Using Thorp's formula which gives  $a(f)$  in dB/km for  $f$  in kHz, the absorption coefficient is

$$10 \log a(f) = \frac{0.11f^2}{1+f^2} + \frac{44f^2}{4100+f^2} + \frac{2.75f^2}{10^4} + 0.003. \quad (2)$$

The above formula typically holds for frequencies above several hundred Hz, which is the common range for most systems.

The ocean ambient noise is modeled as composed of turbulence, shipping, waves and thermal noise, characterized by Gaussian statistics and a continuous power spectral densities (p.s.d.). The four noise components have empirical formulae that give their p.s.d.'s in dB re  $\mu\text{Pa}$  per Hz as a function of frequency in kHz [13]:

$$\begin{aligned} 10 \log N_t(f) &= 17 - 30 \log f, \\ 10 \log N_s(f) &= 40 + 20(s - 0.5) + 26 \log f \\ &\quad - 60 \log(f + 0.03), \\ 10 \log N_w(f) &= 50 + 7.5\sqrt{w} + 20 \log f \\ &\quad - 40 \log(f + 0.4), \\ 10 \log N_{th}(f) &= -15 + 20 \log f, \end{aligned} \quad (3)$$

where  $s$  designates the shipping activity factor and  $w$  designates the wind speed in m/s. The total p.s.d. of the ambient noise is

$$N(f) = N_t(f) + N_s(f) + N_w(f) + N_{th}(f). \quad (4)$$

### III. Ad-Hoc Network Setup

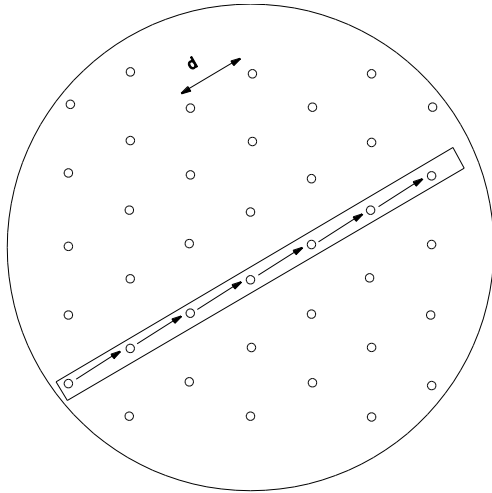


Fig. 1. Uniform network setup.

#### A. Network Topology

A two dimensional network of bottom mounted nodes that provides coverage over a certain area is considered. The distribution of nodes is assumed to be uniform as depicted in Figure 1. Given the number of nodes in the network,  $N$ , and the area of the network,  $\mathcal{A}$ , the network density is

$$\rho = \frac{N}{\mathcal{A}} \quad (5)$$

Given the uniform node distribution and circular area of the network, the distance between nodes is

$$d = \frac{c}{\sqrt{\rho}} \quad (6)$$

where  $c$  is a constant that depends on the node placement (grid pattern). We let  $c = 1$ .

We assume multihop transmission based on nearest neighbor routing, which is an energy saving strategy that is appealing for networks with nodes that are powered by batteries. As the longest multihop route in the network is along the diameter of the network the maximum number of hops is  $n_h^{\max} = (2/\sqrt{\pi})\sqrt{N}$ . The average number of hops for a multihop route is designated by  $\bar{n}_h$ . Then, given that the probability distribution of the number of hops is symmetric, we have  $\bar{n}_h = \sqrt{N/\pi}$  [12].

#### B. Coded Multihop Transmission

In the case of point-to-point convolutionally coded BPSK transmission, the union upper bound on the bit error probability (BEP) can be obtained as [14]

$$p_b < \sum_{d=d_{\min}} c_d p_d \quad (7)$$

where  $c_d$  is the error coefficient, that is, the information error weight for error events of distance  $d$ ,  $d_{\min}$  is the minimum distance of the code, and  $p_d$  designates the pairwise error probability.

#### 1) Interference Model:

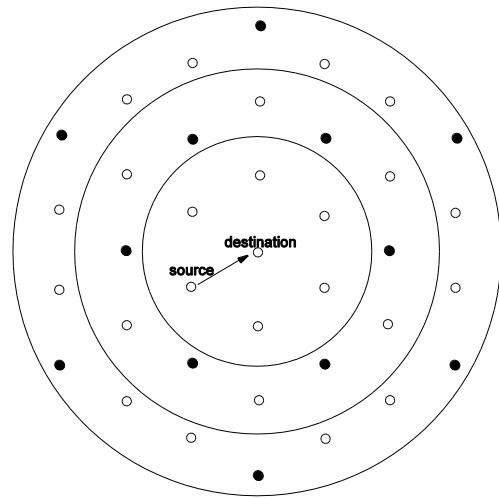


Fig. 2. The model of interference.

Consider a source node to a destination node transmission. The interference model imposes a protocol constraint, as depicted in Figure 2. Nodes that are at the same distance from the destination node as the source node are not allowed to transmit in the same time slot and on the same frequency band as the source during the source node's transmission [4]. Under the assumption that all nodes transmit with a constant p.s.d.  $S$ , the combined interference from the nodes in the first and second tier is

$$I(f) \approx \frac{c_1 S}{A(2d,f)} + \frac{c_2 S}{A(3d,f)} \quad (8)$$

where  $c_1 \leq 12$  and  $c_2 \leq 18$  are constants indicating the number of interfering nodes in tiers 1 and 2, respectively, for a hexagonal grid. In the Figure 2 example,  $c_1 = c_2 = 6$ . As there are multiple interfering nodes in the network, we assume that the interference is Gaussian with p.s.d.  $I(f)$ .

Assuming perfect channel state information at the receiver and flat Ricean fading for the node-to-node channel [15], the BEP is

$$p_b < \sum_{d=d_{\min}} c_d \left( \frac{1+\mathcal{K}}{1+\mathcal{K}+R\gamma(d,f)} \right)^d \times \exp \left( -dR \frac{\mathcal{K}\gamma(d,f)}{1+\mathcal{K}+R\gamma(d,f)} \right) \quad (9)$$

where  $\gamma$  designates the signal to interference plus noise ratio (SINR) and  $\mathcal{K}$  designates the Ricean fading parameter. We assume that the attenuation, noise and interference are constant over the operational bandwidth  $B$ , so that for a transmit power level  $P$ , the SINR can be calculated at the operating frequency  $f_o(d)$  as

$$\gamma(d, f_o) = \frac{P}{A(d, f_o)(N(d, f_o) + I(d, f_o))B} \quad (10)$$

### C. Sustainable Number of Hops

The end-to-end frame error probability (FEP) for a multihop route with  $n_h$  hops is

$$p_{\text{route}} = 1 - (1 - p_b)^{Ln_h} \quad (11)$$

where  $L$  is the frame size in bits.

The quality-of-service for the network is considered in terms of the maximum allowed end-to-end route FEP, that is,  $p_{\text{route}} \leq p_{\text{route}}^{\text{max}}$ . Let the number of hops that can satisfy the maximum end-to-end route FEP, be designated by  $n_{\text{sh}}$ . The sustainable number of hops is

$$n_{\text{sh}} = \frac{1}{L} \frac{\log(1-p_{\text{route}}^{\text{max}})}{\log(1-p_b)} \approx \frac{1}{L} \frac{p_{\text{route}}^{\text{max}}}{p_b} \quad (12)$$

## IV. Numerical Results

We examine the impact of point-to-point convolutional coding on the network connectivity given by the sustainable number of hops. Independent Ricean fading for each node-to-node channel with  $\mathcal{K} = 10$  is assumed. The maximum allowed end-to-end FEP is  $p_{\text{route}}^{\text{max}} = 10^{-3}$ . The circular network has an area of  $\mathcal{A} = 1000 \text{ km}^2$ . The nodes are able to adjust their powers, so that the sustainable number of hops through the network never exceeds the maximum number of hops. Note that an acoustic signal propagates as a pressure wave whose level is commonly measured in dB relative to  $1 \text{ } \mu\text{Pa}$ . We adapt that convention, hence the power levels are expressed in dB re  $\mu\text{Pa}$ . Fixed losses are neglected, e.g., additional frequency independent losses and background noise level suited for a particular environment. The frame size is  $L = 100$  bits, and the bandwidth is  $B = 4 \text{ kHz}$ . The spreading factor is  $\kappa = 1.5$ , the shipping activity factor is  $s = 0.5$ , and unless otherwise indicated, the wind speed is  $w = 0$ .

Figure 3 presents the sustainable number of hops when there is interference from other nodes in the network, as illustrated in Figure 2. The transmit power is  $P = 135 \text{ dB re } \mu\text{Pa}$ . We observe that in the case of uncoded BPSK transmission with a simple demodulate and forward forwarding strategy, the network cannot even maintain multihop routes with an average number of hops, irrespective of the number of nodes in the network. Note, however that point-to-point coded transmission with the rate  $R = 1/2, (133,171)$ , convolutional code provides full connectivity in the network, that is, all routes satisfy the end-to-end FEP requirement.

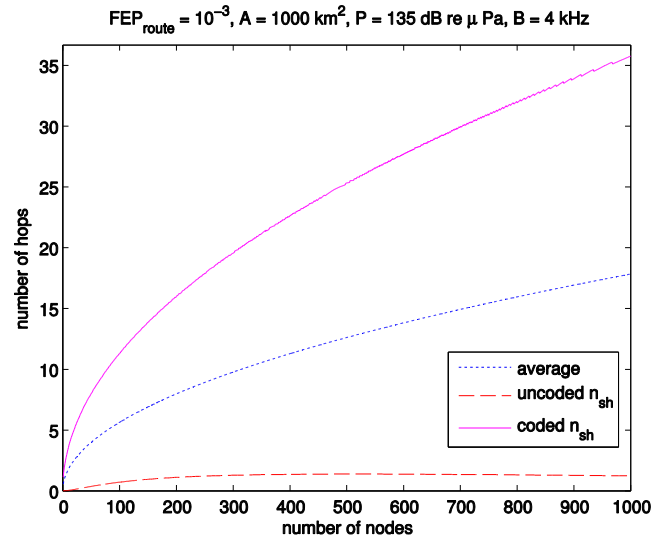


Fig. 3 Sustainable number of node-to-node hops for a uniform network with Ricean fading in the presence of interference. Uncoded BPSK transmission vs. rate  $1/2, (133, 171)$  convolutionally coded transmission.

Figure 4 compares the sustainable number of hops, when there is interference from other nodes in the network, between coded transmission with the rate  $R = 1/2, (133,171)$ , convolutional code and stop and wait ARQ transmission with various maximum number of retransmissions, designated by  $J$ . The transmit power is  $P = 120 \text{ dB re } \mu\text{Pa}$ . We observe that that in the case of a convolutionally coded transmission the network provides full connectivity whereas stop and wait ARQ, while offering improved performance as the number of retransmissions increases, remains coverage limited.

Figure 5 presents the sustainable number of hops when there is interference from other nodes in the network, in the case of point-to-point coded transmission with the rate  $R = 1/2, (133,171)$ , convolutional code for different wind speeds. The transmit power is  $P = 120 \text{ dB re } \mu\text{Pa}$ . We observe that if the wind speed increases to  $w = 5 \text{ m/s}$ , and  $w = 10 \text{ m/s}$ , the network becomes coverage limited when the number of nodes in the network is  $N \leq 80$ , and  $N \leq 270$ , respectively. Note that the results for an uncoded BPSK transmission are not presented since for the transmit power level and the wind speeds considered in the example the network cannot really provide any connectivity, irrespective of the number of nodes in the network.

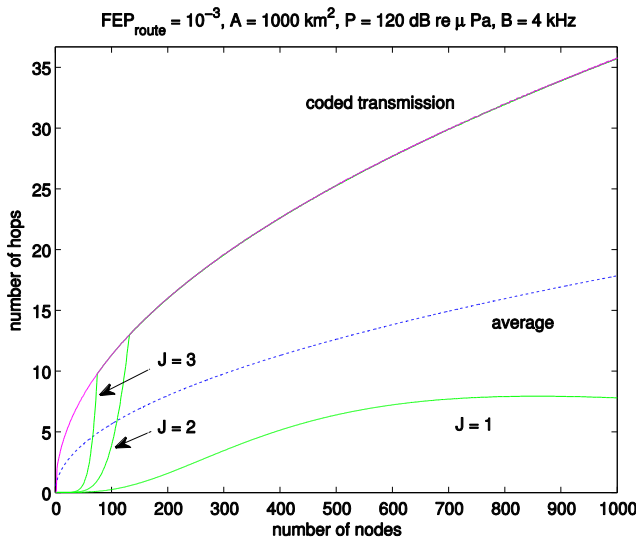


Fig. 4. Sustainable number of node-to-node hops for a uniform network with Ricean fading in the presence of interference for coded transmission with rate  $1/2$ ,  $(133, 171)$ , convolutional code and stop and wait ARQ with various maximum number of retransmissions.

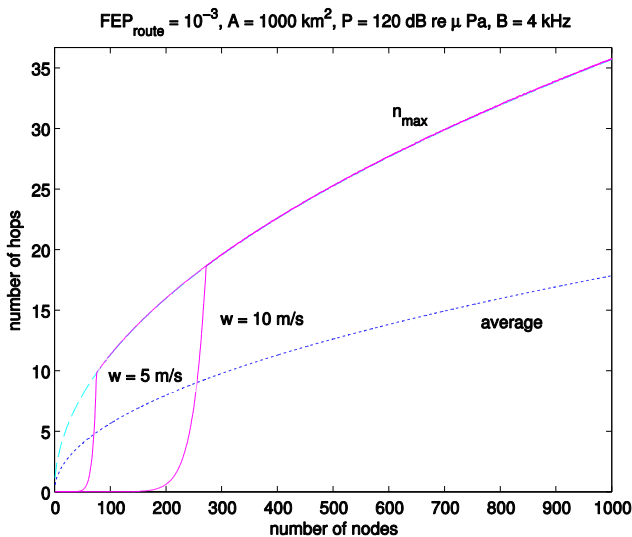


Fig. 5. Sustainable number of node-to-node hops for a uniform network with Ricean fading in the presence of interference for various wind speeds. Rate  $1/2$ ,  $(133, 171)$  convolutional code.

## v. Conclusions

The performance of convolutionally coded underwater acoustic ad-hoc networks was considered in the scenario when there is interference in the network. It was observed that the network connectivity can be significantly improved through point-to-point channel coding. It was found that channel coding also improves the network robustness as considered for the case when the wind speeds are increased. Numerical examples, based on convolutional codes and the union upper bound on the BEP, were presented to illustrate the results of the analysis.

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