Potential Game Framework for Interference Avoidance in CRAHNs using Waveform Adaptation and Sub-Carrier Allocation

[Sundus Naseer, Qurratulain Minhas, Zia Muhammad, Hasan Mahmood]

Abstract—**The simultaneous transmissions of multiple users on a channel interfere with each other and it becomes pivotal to devise a mechanism to obtain some form of separation. In a cognitive radio ad hoc network (CRAHN), the users compete to acquire best suitable bandwidth from already scarce spectrum, resulting in a fierce competition. The nodes may not observe desirable etiquettes in selecting channels and waveforms that result in chaos, which degrades the Quality of Service for the entire network. In order to inhibit the nodes while maintaining their autonomous behavior, there is a need of intelligent mechanism to inculcate the nodes in the network. A game theoretic framework is proposed to achieve convergence in selecting sub-channels and waveforms for transmissions. The proposed model is an eigen-iterative framework, in which signature sequences and sub-channels keep on updating after every cycle. Eigenvectors corresponding to the minimum eigenvalue of correlation matrix is used to maximize the utility and ultimately reduce the interference. The simulation results show fast convergence and stable network with Nash equilibrium at a desirable point.**

Keywords—**CRAHN, sub-carrier allocation, waveform adaptation, potential game**

I. Introduction

The primary objective in a design of a wireless network is to deal with interference that is created by other users and the surroundings. Thus, a commotion in the wireless networks due to the increasing number of nodes raises the involvement of distributed decision making, which requires waveform adaptation and optimal channel selection.

Interference that needs to be avoided depends upon the transmission power of the nodes, channel characteristics, and codes (or waveform) selection. In this paper, we investigate the distributed waveform adaptation algorithm that reduces interference and facilitates multiple users by allocating sub-channels in CRAHNs. In ad hoc networks, greedy interference avoidance techniques do not always lead to stable point (or Nash equilibrium). The nodes behave selfishly and try to maximize their own payoffs, making the channel convergence difficult. A potential game framework is designed that leads to the convergence of waveform adaptation games and enhances the overall network performance.

The greedy interference avoidance waveform adaptation algorithms update the codes sequentially [1]. The iterative update produces a set of orthogonal sequences which decreases the total squared correlation. Each sequence is

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replaced by its corresponding normalized minimum mean square error receiver. The distributed algorithms are used for the purpose and Welch Bound Equality is achieved. However, real time scenario MMSE algorithm depends upon stochastic receiver measurement, thus the stochastic convergence needs to be examined. It does not assure the optimal sequence set convergence [2]. Bounded random noise can be added in sequences along with different technique of class warfare, in which users deliberately interfere in comparatively less crowded network area to achieve minimum TSC or maximum sum capacity [3].

Signal space for Interference Aware (IA) techniques is based on eigen-iterations. The stability due to the addition or deletion of nodes and comparison to code division multiple accesses (CDMA) is briefly discussed in [4]. The idea was implemented on a single receiver and unfortunately, no fixed points could be taken for the multiple receivers and power control. Then in [5 - 7] the techniques of IA are extended for multiple receivers and adaptations in asynchronous CDMA systems [8]. Feedback to the transmitting end, however, still remains an issue as the signature sequences can only be estimated at the receiver. Feedback of real-valued sequences is important at each iteration, which causes large network overhead. Feedback with less network overhead is studied in [9] by restricting the signature sequences to the specific signal space, leading to sub-optimal solutions.

Since waveform choice of a user in distributed IA algorithms depends upon the waveform selection of other users, the application of game theoretic techniques becomes natural as the selection of channel and waveform influences other agents. Game theory is the applied mathematics branch that models such dependencies and analyzes them. The analysis of game theory models the synchronous IA scheme with Eigen iterations and fixed power using potential game theory [10]. Restriction of arbitrarily small noise ensures the convergence to orthogonal signature set. The potential game model approach is discussed in [11] and [12]. Analysis of game theory regarding wireless ad hoc networks is given in [13].

II. SYSTEM MODEL

The CRAHN is considered in the form of a cluster of transmitting and receiving node pairs. The interference caused in the network is affected by the correlation between the waveforms of users, their transmit power and the shared channel characteristics. Since the main focus of this paper is the sub-channel selection on the basis of correlation between waveforms, the transmit power levels are assumed to be fixed initially, and then distributed variably for all user

nodes. A signal space characterization is used for the representation of the waveforms of nodes [4]. The orthogonal signal dimensions (either in time, frequency, or spreading code) represent the waveforms of nodes and are named as the signature sequences.

A. *PN Sequences in CRAHNs*

The properties of pseudo random sequences resemble the properties of white noise that with minimum auto correlation and very low cross-correlation. Therefore, pseudo random sequences are chosen for attaining orthogonal sequence set. Direct sequence spread spectrum multiplies the base band data pulses directly with pseudo noise sequences. A single pulse of waveform represents a chip. Resultant data signal is represented as non-overlapping rectangular pulses in time sequence with amplitude +1 or -1 [15]. In classical communication model, transmitted signal is:

$$s(t) = b\sqrt{p}s(t) \qquad (1)$$

In interval [0, T] [4], p is received power and $s(t)$ is signal waveform of PN sequences with an amplitude equal to +1 or -1 [15]. These sequences are of finite energy. Putting a constraint on $s(t)$ of unit energy $\sum_n s_n^2 = 1$. SIR can be maximized within the network by choosing $s_n(t) = 1$ [14]. Signal space dimensions (time or frequency) are used to represent the signature sequences that are kept near orthogonality.

B. *Network Design*

Let K be the number of signal space dimensions and N be the number of nodes in the network. K-dimensional signature sequence of the transmitting node n is represented as s_n. Now let the signature sequence, corresponding to the ith node, be $s_i \in R^{K*1}$. It is assumed that the received signal experiences single path and that the signature sequences are synchronized at the receiver. The extension of the system model is possible for multipath channels and asynchronous systems (similar to the analysis in [11] and [12], respectively). The received signal at the jth receiver node can be expressed as

$$r_j = \sum_{j=1}^{N} b_i \sqrt{p_i} h_{ij} s_n + Z \qquad (2)$$

where $r \in R^{K*1}$ and $Z \in R^{K*1}$ is the additive white Gaussian noise with zero mean and unit variance. p_i represents the transmitting power of ith node and b_i is the bit symbol transmitted by the user node i. h_{ij} is the fading/channel coefficient between transmitting node i and receiving node j. This fading coefficient h_{ij} is an $N \times N$ fixed matrix as the assumption is made that it depends on the distance between nodes and network topology remains the same throughout the adaptation process. Nodes, however, change their channels after selecting signature sequences iteratively. Ultimately they converge towards maximum utility by achieving equilibrium where no further change can cause flipping over channel.

III. **GAME THEORY IN CRAHNS**

Since there is no centralized structure in CRAHNs, the CR users need to coordinate to maintain QoS in transmission. This coordination capability is provided by the spectrum sharing techniques that are (1) intra-network spectrum sharing and (2) inter-network spectrum sharing. However, CRAHNs are focused only on intra-network spectrum sharing, and all sharing and adaptive decisions must be made in a decentralized manner. This gives rise to the game theoretic approach in CRAHNs.

Game theory deals with the interactive rational decision situations and is used for modeling the interaction and coordination among independent nodes in CRAHNs. In CRAHNs game theory is applied for *power control* and *waveform adaptation* at physical layer. Medium access control (MAC) protocol handles the sensing control and spectrum access at link layer, and packet forwarding is done at network layer. The aim to apply game models on these stages is to discourage selfish behavior of nodes in ad hoc networks [13].

A. *Waveform Adaptation using Game Theoretic Frame work*

The performance of network nodes is treated as function of signal to interference plus noise ratio (SINR), where interference plus noise is taken as an undesirable signal received along with the desirable signal. Therefore, any change in SINR results in the adaptation of suitable signal and interactive decision making. Signal adaptation occurs through *power control* and *waveform adaptation* techniques. Waveform adaptation process deals with the orthogonal set of waveforms of CR users that are adapted for reducing interference. Interference at the receiver is considered as a cross-correlation function of other CR users within the network. Therefore, game theory is applied to bring all CR users at a cooperation point to minimize interference. Waveform adaptation techniques that reduce interference with minimum adaptation overhead and feedback between receivers and transmitters are required for the convergence within the network. A framework based on potential game theory is best proposed solution for convergent waveform adaptation techniques.

B. *Channel Allocation using Game Theoretic Framework*

Channel allocation is one of the major issues in resource allocation for CRAHNs. If a CRAHN consists of frequency division multiple access (OFDM) having multiple channels in a single spectrum, selection of channels or sub-carriers is needed to satisfy QoS requirements for transmission. Game theory exploits the behavior of CR users for distributed adaptive channel allocation.

However, in case of CDMA, users determine the most suitable channel and corresponding coding rate assuming the constant transmission power. Thus the exact potential game model is suitable in case of cooperative spectrum sharing, and network converges to a pure strategy Nash equilibrium.

IV. **POTENTIAL GAME FRAMEWORK FOR IA**

Potential games are easy to analyze and the global function maximizes by maximizing the utilities of users,

thus are suitable for CRAHNs. Hence the overall network performance measure is improved by only improving the utility of users. Considering a general waveform adaptation game in which players are the node-pairs and signature sequences are the strategies available to each user. Let the utility of each node be

$$u\ (s_i, s_{-i}) = f_1(s_i) - \sum_{j \neq i, j=1}^{N} f_2(I(s_j, s_i), p_j, p_i, h_{ji}, h_{ii})$$

$$- \sum_{j \neq i, j=1}^{N} \gamma_{ij} f_3(I(s_i, s_j), p_i, p_j, h_{ij}, h_{jj}) \qquad (3)$$

The utility function of a potential game quantifies both the benefit and loss associated with a particular choice of the user i, thus f_1 shows the benefit corresponding to a particular choice of waveforms and power. Function f_2 is the interference measure caused by the other users in the network experienced at the receive node i. Function I is an indicator function of two signature sequences s_i and s_j (e.g. the correlation between the sequences). f_3 is the interference created by user i to the other users within the network and γ_{ij} is a weighting factor coefficient [17]. The potential function corresponding to the utility function above can be formulated as

$$P(s) = \sum_{i=1}^{K} \left[\begin{array}{l} f_1(s_i) - \alpha \sum_{j \neq i, j=1}^{N} f_2(I(s_j, s_i), p_i, p_j, h_{ji}, h_{ii}) \\ - \beta \sum_{j \neq i, j=1}^{N} f_3(I(s_i, s_j), p_i, p_j, h_{ij}, h_{jj}) \end{array} \right] \quad (4)$$

where, α and β are the weight coefficients. Convergent waveform adaptation games are easy to formulate under the potential game structures (exact or ordinal potential games) as every user tries to maximize its own utility function unless and until all utility functions are ordinal transformation of each other.

C.   *Strategy Space*

In CDMA technology, the users share same frequency band and orthogonal sequences are assigned to each user for transmission. The limiting factor in a system having same band is self-interference and multi-user interference. Both these factors are dependent on channel characteristics and properties of signature sequences used. Channels are opted after finding minimum correlation between sequences, i.e., mapping different chip of sequence to individual sub-carrier. The basic aim of using both signature sequences and sub-channels together for optimizing the system is to support high data rate services and better QoS in hostile wireless environment [18], [19].

D.   *Game Model*

Convergent interference avoidance game described here aims to minimize ISIR (Inverse signal to interference ratio) [17]. Function $f_2(.)$ is the interference function of a user and is considered as the ISIR at a specific receiver. It is expressed as:

$$f_2(I(s_j, s_i), p_j, p_i, h_{ji}, h_{ij}) = - \frac{s_i^H s_j s_j^H s_i p_j h_{ji}^2}{p_i h_{ii}^2} \quad (5)$$

Here I is the correlation of two signature sequences. p_i and p_j are the fixed transmitting powers of all users. An exact potential game is formulated by keeping $f_2 (.) = f_3(.)$. The utility of a particular user i is expressed in the form of its signature sequences

$$u_i(s_i, s_{-i}) = - \frac{s_i^H \left(\sum_{j \neq i, j=1}^{N} \frac{s_j s_j^H p_j h_{ji}^2}{h_{ii}^2} \right) s_i}{p_i} \quad (6)$$

$$- p_i s_i^H \left(\sum_{j \neq i, j=1}^{N} \frac{s_j s_j^H h_{ij}^2}{p_j h_{jj}^2} \right) s_i$$

Utility function of each user has an influence on the actions of other users as opposed to the greedy IA games. This influence is incorporated to minimize interference at the particular receiver. The measure of influence due to the utility function is expressed in the form of an exact potential function. In this game framework, potential function is the negative sum of ISIR of the users that can be expressed as

$$P(s) = - \sum_{i=1}^{K} s_i^H \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H p_j h_{ji}^2}{p_i h_{ii}^2} \right) s_i \quad (7)$$

The game is played iteratively; each user chooses its waveform after observing the cross-correlation with the waveforms of other users within the network. This iterative waveform selection results in the maximization of utility function. The correlation function between two users is expressed in the following function

$$X = \frac{\left(\sum_{j \neq i, j=1}^{N} \frac{s_j s_j^H p_j h_{ji}^2}{h_{ii}^2} \right)}{p_i} + p_i \left(\sum_{j \neq i, j=1}^{N} \frac{s_j s_j^H h_{ij}^2}{p_j h_{jj}^2} \right) \quad (8)$$

Since $h_{ij} = h_{ji}$ due to the same topology and fixed transmitting power, X function can be reduced to the general correlation function, which is positive semi-definite matrix [17]. Substituting the X function into the utility, it can be reduced to

$$u_i(s_i, s_{-i}) = - s_i^H (X) s_i \quad (9)$$

Keeping in mind the energy constraint of $\left\| s^2 \right\| = 1$.

Initially signature sequences $s_i \in R^{K \times 1}$ of $N \times K$ are selected randomly. Players replace their sequences with the Eigenvector corresponding to the inferior eigenvalue of correlation matrix X, iteratively which is used to evaluate maximum utility. Similarly, channels are also allocated to each user randomly. However, signature sequence set is same for all users at each channel. Sub-channel allocation is done on the basis of maximum utility. Each user calculates its utility at every channel and selects the maximum utility and least interference channel for it. This process continues until all users make their choices and finally converges to an equilibrium point.

V.   **RESULTS**

The payoff of choosing a particular strategy for each user might not be the optimum as sacrifice is needed for network convergence. However, the general trend of the utility function is increasing, showing the overall benefit each user

attains. This benefit is the incentive to the users for compromising their maximum payoffs and opting negotiable strategies. The utility increases as correlation between the two sequences sharing same channel decreases. The resultant utility of 15 users and 3 signal space dimensions is shown in Fig. 1. Different curves show the utilities of different users that are increasing exponentially. Increasing trend is shown in first few iterations then convergence or Nash equilibrium point is achieved after that.

The users select their channels at every iteration with minimum interference and maximum utility. This process continues until all users fix their channels so that there is no further increase in payoff of a user by moving to some other channel. Channel convergence is shown in Fig. 2. Potential function of the proposed game follows the same trend as of utility as an exact potential function is formulated. By satisfying the condition that is [16]

$$u_i(\widehat{a}_i, a_{-i}) - u_i(a_i, a_{-i}) \Leftrightarrow P(\widehat{a}_i, a_{-i}) - P(a_i, a_{-i})$$
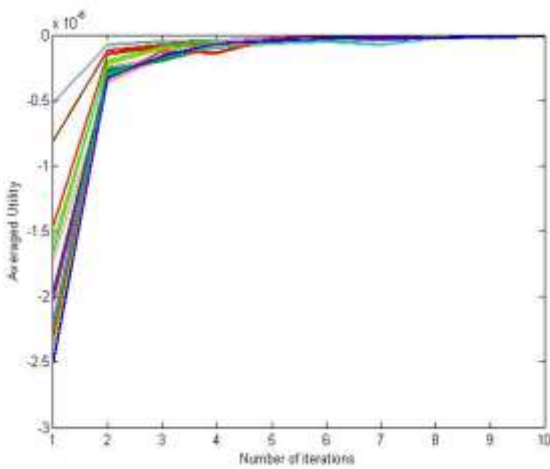

Fig. 1: Utility Function

Plot of the potential function is shown in Fig. 3, which converges where users start coordinating with each other. Any further change due to unilateral deviation can cause non-convergence providing no benefit to any user.

Correlation between two sequences depicts the interfering effect one user creates for the other. Decrease in correlation
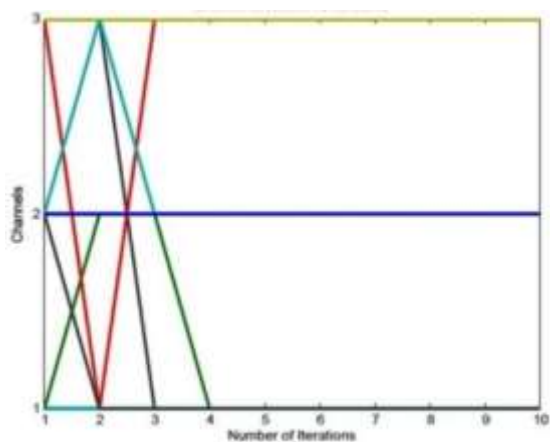

Fig. 2: Channel Convergence

decreases interference and hence reduces the ISIR. As signature sequences are updated by the inferior eigenvectors,

correlation between the signature sequences of users sharing same channel decreases iteratively. This results in the minimization of ISIR, due to change in correlation, leading to convergence. The decrease in the ISIR shows better QoS.

VI. Effect of Power Control

The game model above, though, results in equilibrium but the network performance can further be improved by changing the transmission power instead of keeping it fixed. This will make the mechanism more practical to inculcate the nodes in the network.

Since the objective is to reduce ISIR, negative inverse of power of each user ultimately helps in achieving the goal. Each CR user has its own random transmission power,
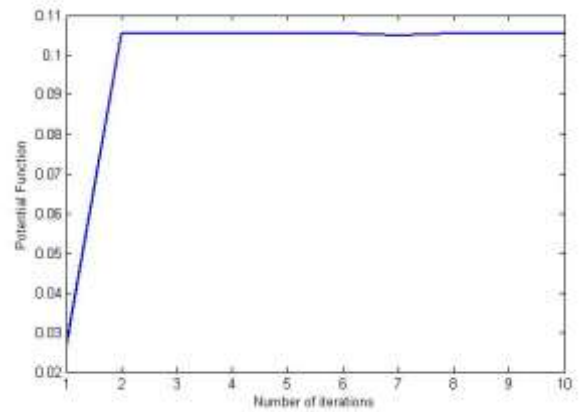

Fig. 3: Potential Function

which is then adapted through water filling mechanism according to the channel it chooses. The function $f_2(.)$ that depicts ISIR is

$$f_2(I(s_j, s_i), p_j, p_i, h_{ji}, h_{ij}) = \frac{s_i^H s_j s_j^H s_i p_j h_{ji}^2}{p_i^2 h_{ii}^2} \quad (10)$$

Using the same correlation matrix the utility function is effected with the transmission power of ith node in such a way

$$u_i(s_i, s_{-i}) = \frac{\left(s_i^H(X)s_i\right)}{p_i} \quad (11)$$

Same network topology accommodates CR users in an efficient manner but at the cost of late convergence. The global function is now

$$P(s) = \frac{\left[\sum_{i=1}^{K} s_i^H \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H p_j h_{ji}^2}{p_i h_{ii}^2}\right) s_i\right]}{p_i} \quad (12)$$

Fig. 4 shows the effects of power control on channel allocation with 15 CR users and similar network topology with 3 signal space dimensions and 3 sub-channels. Reduction in ISIR with variable power factor is shown in Fig. 5.
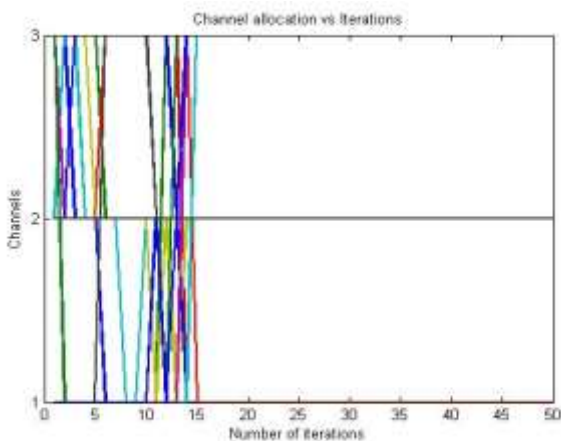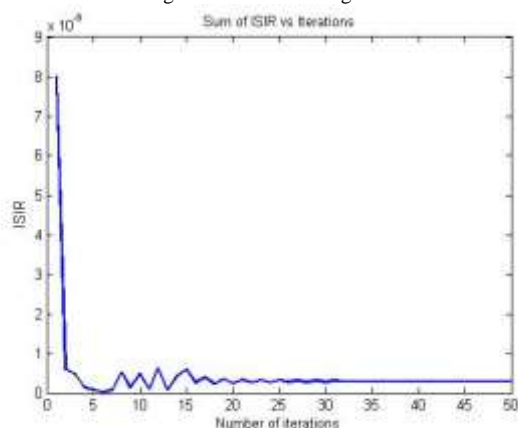
Fig. 4: Channel convergence

Fig. 5: Sum of Inverse Signal to Interference Ratio

VII. EQUILIBRIUM POINT AND CONVERGENCE

Nash equilibrium of the game is the point from where no player can further improve its utility. The IA game has a continuously differentiable and bounded potential function with at least one maxima. Hence, convergence to a NE exists at the maximum of the potential function. The proposed game model exhibits best response convergence. The utility function also has unique maxima as it is a concave function of signature sequences. Users find the set of waveforms and channels at the inferior sum interference. And the best response iteration is known as the convergence point and is considered as the Eigen iteration.

VIII. CONCLUSION

In this paper, a potential game framework is designed to mitigate interference from the CRAHN. This framework is used to formulate the convergent waveform adaptation game for interference avoidance. A feasible target is approached by the users through this distributed adaptation strategy of signature sequences. Further, the channel allocation scheme ensures that the targeted performance is achieved as the minimization of ISIR and maximization of utility. Single channel is shared by multiple CR users having minimum correlation between their signature sequences, which results in interference avoidance. The best response convergence of the potential function assures the better QoS and minimum interference within the network.

However, the convergence under best response dynamics might not be the optimal as users compromise on their maximum benefit to achieve overall suitable environment for transmission and network convergence. Therefore, the convergence of algorithm is attained after running the simulation multiple times, with initial random choices of waveform and channel.

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