

# Advances in Multi objective Decision Making in Information Technology

[M. Ramesh<sup>1</sup>, Anjali<sup>2</sup>, and Kumar, V.S.S<sup>3</sup>]

**Abstract**—Soft In manufacturing industry, complexities will often arise due to the presence of a large number of interacting variables and many of which may defy quantification. Nevertheless, engineers use different skills and strategies for accomplishing and resolving the constraints and achieving the tasks. Operations research (OR) is a widely used technique for management problems through different and appropriate mathematical models. One of the widely used tools of operations research in engineering is the linear programming technique. In this technique all the information pertaining to the problem is expressed in terms of linear constraints on the decision variables. Where the data is precise and the constraints are internally controlled, the technique is good for arriving at the optimized decision. Manufacturing industry is often plagued by uncertainties because of unforeseen factors such as changing weather, breakdown of equipment, labor in efficiency, and lack of coordination. Uncertainty in the supply of resources, i.e., delayed delivery of materials or intermediate products or the availability of shared resources, can lead to inefficiency resulting in lower productivity, delays, and extra cost. In addition, rates of resources are not steady and are difficult to match during the execution of the project. Since, there are no effective methods to minimize the uncertainty, some flexible strategies need to be adopted to reduce the interaction or dependence between activities

**Keywords**—Fuzzy sets, linear programming, constraints, decision.

## I. Introduction

Engineers use several techniques, with varying degrees of complexity, to handle projects. Bar charts are one of the early tools for project scheduling. While bar charts are improved into sophisticated networks, operations research techniques such as linear programming, simulation, and value engineering are increasingly used in the manufacturing industry for project scheduling. Essentially, the initial function of operations research was the analysis of existing operations to find more efficient performance methods.

---

Research Scholar, Department of Mechanical Engineering  
University College of Engineering, Osmania University,  
Hyderabad, India

P G Scholar, Department of Civil Engineering  
University College of Engineering, Osmania University,  
Hyderabad, India

Principal, University College of Engineering, and  
Prof. of Civil Engineering, Osmania University,  
Hyderabad, India

In 1956 the critical path method (CPM) was first formulated and implemented on a computer to schedule activities of projects. In 1957, a technique called the Program Evaluation and Review Technique (PERT) was developed to integrate and co-ordinate contractors working on a single project. This method uses probability theory and enables management to plan projects by knowing the probabilities of occurrence of events.

The CPM provides a practical tool for planning and controlling projects. Many new algorithms and techniques have been developed to enhance the usefulness of CPM. Among these algorithms and techniques, time-cost trade-off analyses have been one of the most important enhancements for using CPM to plan and control projects. In general, there is a certain relationship between time and cost to complete the activities within a project. In real project, activities must be scheduled under limited resources, such as limited crew sizes, limited equipment amounts, and limited materials (Leu 1999). However, many of these constraints are possibly externally controlled and the variations cannot be predicted to a reliable extent (Bellman and Zadeh 1970). If there is a variation in the constraints, the variabilities cannot be easily taken care of by classical linear programming for arriving at the values of decision variables.

This has proved to be one of the most difficult aspects of linear programming, since this variation cannot be converted into mathematical equivalents (Zadeh 1965). To adequately represent them by just keeping in the conventionally quantifiable variables is obviously a stumbling block. Consequently, the results could be erroneous as decision indicators. Thus there is a need to accommodate these variations in the pre implementation stages of projects for multi objective decision making.

## II. Uncertainties in Resource Requirements

Some activities cannot be planned for rigid demands on the quantities of materials for each cycle of operation. Examples include the "spread" activity in an earthmoving operation and the "mix" activity in concrete placement. Spread can start when the supply of soil, an intermediate product from the preceding activity such as "dump," has reached certain levels without achieving the optimal one. When the quantity of material, e.g., cement and sand, which may not be steadily supplied, has reached certain ranges, the mix can start. Meanwhile, there is subjectivity in assessing and monitoring the quantities of resources to activate an activity. Therefore,

checking the quantities of resources is carried out in a vague or imprecise environment, especially when these resources are demanded on a flexible basis.

The quantities of resources involved may be different for each cycle due to the above uncertainties. In addition to unforeseen reasons, e.g., weather, traffic conditions, and the efficiency of workers, the duration of the activity may vary with the quantities of resources involved in each cycle of operation, resulting in a nonlinear relationship between the duration and the quantity of resources involved (Dhingra et al. 1990). Linear programming algorithms employ searching methods to identify local optimum solutions for the given problem (Rao 1987).

### A. Linear Programming Model

In general, a linear programming problem (LPP) is as follows

Optimize  $Z = C_1x_1 + C_2x_2 + \dots + C_nx_n$

Subject to constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1 \quad (1)$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2 \quad (2)$$

$$\dots \dots \dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m \quad (3)$$

And  $x_1, x_2, \dots, x_n \geq 0$  and is a non-negativity restriction.

Here  $x_j$ 's are decision variables;  $C_j$ 's represent the cost coefficients;  $a_{ij}$ 's are the technological coefficients;  $b_i$ 's are the resource values. Since, the real world problems does not go with the rigidity there is a need to introduce flexibility in various elements of the LP model (Zadeh 1973). By introducing flexibilities, the above LP model is converted into the following fuzzy linear programming model.

### B. Fuzzy Linear Programming

Fuzzy linear programming is extensively used for decision making in an uncertain environment. The conventional LP model is

$$\text{Maximize } Z = CTX$$

$$\text{Subject to } AX \leq b, \text{ and } x \geq 0$$

Where A, b and C describe the relevant state variables; x, the decision variables; Z, the event resulting from combination of the state and the decision variables. The objective function is expressed by the requirement to maximize Z. Here the elements A, b, C can be fuzzy numbers rather than crisp numbers, the constraints can be represented by fuzzy sets rather than by crisp inequalities and objective function can be represented by either a fuzzy set or a fuzzy function (Zimmermann 1978). The solution can be either a fuzzy or a crisp solution. The goal of decision-maker is expressed as a fuzzy set and solution space defined by constraints are modeled by fuzzy sets. In such situation, the optimization model is expressed as

Find x, such that

$$CTX \geq Z$$

$$Ax \leq b$$

$$X \geq 0$$

Here  $\leq$ , denotes fuzzified version of  $\leq$  and the objective function is a minimizing goal in order to consider Z as an upper bound. Now, the objective function and the constraint equations are fully symmetric and considering Z as an upper bound. It can be shown as

Find x such that

$$Bx \leq d$$

$$x \geq 0$$

where

$$\begin{pmatrix} -c \\ A \end{pmatrix} = B \text{ and } \begin{pmatrix} -z \\ b \end{pmatrix} = d \quad (4)$$

Each of m+1 rows are represented by fuzzy sets each with membership values of  $\mu_i(x)$ . Therefore, the membership function of fuzzy set 'decision' is

$$\mu_D(x) = \min \{ \mu_i(x) \} \quad i=1,2,\dots,m+1$$

By introducing the flexibility  $\pi_i$ , membership function  $\mu_i(x)$  will increase monotonically from 0 to 1, ie,

$$\mu_i(x) = \begin{cases} 1 & \text{if } B_i(x) \leq d_i \\ [0,1] & \text{if } d_i < B_i(x) \leq d_i + \pi_i \\ 0 & \text{if } B_i(x) > d_i + \pi_i \end{cases} \quad (5)$$

Here,  $i = 1, 2, \dots, m+1$

By introducing a new variable  $\lambda$  and flexibility  $\pi_i$ , the model becomes

Maximize  $\lambda$

Such that,

$$\lambda \pi_i + B_i x \leq d_i + \pi_i$$

$$0 \leq \lambda \leq 1$$

and

$$x \geq 0 \quad \forall i = 1, 2, \dots, m+1$$

$$\mu_i(x) = \begin{cases} = 1 & \text{if } A_i x \leq b_i \rightarrow \partial \pm \\ = \frac{b_i + \pi_i - A_i x}{\pi_i} & \text{if } (b_i < A_i x \leq b_i + \pi_i) \\ = 0 & \text{if } A_i x > b_i + \pi_i \end{cases} \quad (6)$$

The symmetry is achieved between the objective function and constraints.

Therefore, an equivalent model is

Maximize  $\lambda$

Such that



(7)

$$\mu_G(x) = \begin{cases} = 1 & \text{if } f_0 \leq C^T x \\ \frac{C^T x - f_1}{f_0 - f_1} & \text{if } (f_1 < C^T x \leq f_0) \\ = 0 & \text{if } C^T x > f_1 \end{cases}$$

$$\lambda(z_0 - z_1) - CTx \leq -z_1$$

$$\lambda p + Ax \leq b + p$$

$$Dx \leq b'$$

$$\lambda \leq 1$$

$$\lambda, x \geq 0$$

The flexibility introduced in the various elements of the fuzzy linear programming model is elucidated by using the trapezoidal representation.

### III. Research Methodology

The research methodology to identify the optimum duration of a project is as follows:

- Identify the activities of the network
- Number the events of the activities in ascending order
- Calculate the slope of the critical activities
- Develop the LP model of the network
- Identify the duration of the project using LINGO 5.0 (fl)
- Introduce flexibilities for the relevant activities
- Calculate the duration of the project (fo)
- Introduce flexibility to objective function (fo-fl)
- Introduce a new variable  $\lambda$  for the acceptability value of the duration
- Convert the objective function into another constraint
- Solve the model
- Review and validate

#### A. Case Study

The construction of building is considered for this work. The duration of the project is nine months. The objective is to identify the realistic evaluation of the project duration using fuzzy linear programming. The project is divided into 19 activities and is listed in Table along with its durations.

TABLE I. FILE NAME

NODE S	TASK	DESCRIPTION OF ACTIVITIES	DURATION	EST	EFT	LST	LFT
0~1	A	Earth work-excavation	30 days	0	30	0	30
1~2	B	Laying PCC	25 days	30	55	30	55
2~3	C	Column footing	30 days	55	85	55	85
2~5	D	Column pedestals	15 days	30	45	30	45
4~10	E	CRS machinery under plinth	5 days	85	92	85	92

		beams					
4~7	F	Refilling the foundation and carting of earth	30 days	85	115	85	115
4~5	G	Plinth beams	7 days	85	92	85	92
5~6	H	Column up to bottom of GF slab	15 days	45	60	92	107
6~8	I	FF column up to bottom of FF slab	14 days	60	74	107	140
8~10	J	FF slab	16 days	74	90	74	90
5~7	K	Brick walls	20 days	45	65	92	115
7~10	L	Wood work	20 days	65	85	65	85
7~8	M	FF plastering	25 days	65	140	115	140
9~10	N	FF brick machinery	15 days	85	100	90	105
10~12	O	FF plastering	15 days	100	115	105	120
8~10	P	Miscellaneous work	45 days	140	185	140	185
10~11	Q	Flouring in GF,FF	30 days	185	215	185	215
6~11	R	Electrification	15 days	60	75	107	122
14~12	S	Painting	30 days	75	105	215	245

#### B. Linear Programming Model Formulation

Let X (A,B,C, ..... ) be the decision variables representing the activities in the network. For example, XA represents the activity A ‘earth work excavation and laying of P.C.C.’, and XB represents the activity of ‘column footings’. The minimum no. of days required to complete the activity ‘A’ is 20 days, hence activity ‘B’ can only start after the completion of activity ‘A’. Therefore the corresponding constraint equation is  $X_b - X_a \geq 14$ . Similarly, the minimum number of days required to complete the activity ‘B’ is 25 days, hence activity ‘C’ can only start after completion of activity ‘B’. Therefore the corresponding constraint equation is  $X_c - X_b \geq 25$ . Accordingly, other constraints equations are formulated using the network data. Here the decision variables are 19 and constraint equations are 30. Using the LINGO 5.0, the minimum duration is identified as 224 days (fu). The appropriate flexibilities have been introduced to relevant activities and the problem has been resolved.

The model is as follows:

$$\text{Min} = xz;$$

$$x_b - x_a \geq 20; x_c - x_b \geq 25; x_d - x_a \geq 30; x_f - x_c \geq 12; x_g - x_c \geq 12; x_e - x_c \geq 12; x_k - x_g \geq 16;$$

$$x_k - x_d \geq 17; x_h - x_g \geq 16; x_h - x_d \geq 17; x_m - x_f \geq 20; x_m - x_k \geq 20; x_l - x_k \geq 20; x_i - x_h \geq 21;$$

$$\begin{aligned}
 &x_j - x_m \geq 20; x_j - x_i \geq 14; x_n - x_j \geq 21; x_n - x_l \geq 20; x_n - x_e \geq 10; x_p - x_i \\
 &\geq 14; x_p - x_m \geq 20; \\
 &x_q - x_p \geq 45; x_q - x_n \geq 12; x_r - x_h \geq 21; x_o - x_n \geq 12; x_o - x_p \geq 45; x_s - x_r \\
 &\geq 15; x_s - x_q \geq 30; \\
 &x_z - x_s \geq 30; x_z - x_o \geq 15; \text{ And} \\
 &x_a \geq 0; x_b \geq 0; x_c \geq 0; x_d \geq 0; x_e \geq 0; x_f \geq 0; x_g \geq 0; x_h \geq 0; x_i \geq 0; x_j \\
 &\geq 0; x_k \geq 0; x_l \geq 0; x_m \geq 0; x_n \geq 0; x_o \geq 0; x_p \geq 0; x_q \geq 0; x_r \geq 0; x_s \\
 &\geq 0;
 \end{aligned}$$

The minimum duration for the above model is identified as 218 days (fl). Now, by converting the objective function into another constraint, the model of fuzzy linear programming is as follows:

Max  $\lambda$ ;

$$-6*\lambda - X_z \geq -224; X_B - X_A \geq 20; X_C - X_B \geq 25; X_D - X_A \geq 30; -8*\lambda - X_F - X_C \geq 12;$$

$$-8*\lambda + X_G - X_C \geq 12; -8*\lambda + X_E - X_C \geq 12; 2*\lambda + X_K - X_G \geq 16; 2*\lambda + X_K - X_D \geq 17;$$

$$2*\lambda + X_H - X_G \geq 16; 2*\lambda + X_H - X_D \geq 17; X_M - X_F \geq 20; X_M - X_K \geq 20; X_L - X_K \geq 20;$$

$$X_L - X_H \geq 21; X_J - X_M \geq 20; X_J - X_I \geq 14; X_N - X_J \geq 21; X_N - X_L \geq 20; X_N - X_E \geq 10;$$

$$X_P - X_I \geq 14; X_P - X_M \geq 20; X_Q - X_P \geq 45; -8*\lambda + X_Q - X_N \geq 12; X_R - X_H \geq 21;$$

$$-8*\lambda + X_O - X_N \geq 12; X_O - X_P \geq 45; X_S - X_R \geq 15; X_S - X_Q \geq 30; X_Z - X_S \geq 30; X_Z - X_O \geq 15;$$

$$X_A \geq 0; X_B \geq 0; X_C \geq 0; X_D \geq 0; X_E \geq 0; X_F \geq 0; X_G \geq 0; X_H \geq 0; X_I \geq 0; X_J \geq 0; X_K \geq 0; X_L \geq 0;$$

$$X_M \geq 0; X_N \geq 0; X_O \geq 0; X_P \geq 0; X_Q \geq 0; X_R \geq 0; X_S \geq 0; X_Z \geq 0;$$

By solving the above model the value of  $X_z$  is identified as 222 days with  $\lambda=0.50$ . Here, the optimization is done after the initial overall construction schedule is provided, i.e., the early and late start time ( $ES_i, LS_i, i = 1, 2, \dots, N$ ), are known for the project. The problem mainly concentrates on establishing a mathematical model to optimize overall construction schedule of the project.

## iv. Conclusions

Different probabilistic methods with varying degrees of complexity are being used in the industry. However, when a parameter is expressed in linguistic form rather than mathematical terms, classical probability theory fails to incorporate the information. The linguistic variables can be

translated into mathematical measures by fuzzy sets and theory. In the above case study project duration is 224 days without any flexibility using conventional linear programming. The duration of the project is 218 days with relevant flexibilities. The duration was obtained as 222 days with a satisfaction criterion ( $\lambda$ ) of 0.50. The duration is reduced by 2 days and there is no need to utilize full tolerance. The objective functions as well as the constraints are fuzzy in the project environment. The relationship between constraints and objective function in a fuzzy environment is completely symmetric and the decision is the confluence of goals and constraints. In order to implement the proposed technique, various membership functions need to be estimated, which could be difficult in some cases. However they could be estimated with the assistance of experts, and the information can be refined as this method is used more frequently.

## References

- [1] Blockley, D. I. (1979). "The Role of Fuzzy Sets in Civil Engineering." *Fuzzy Sets and Systems*. 2. 267-278.
- [2] Chapman, C. (1997). "Project Risk Analysis and Management—PRAM the Generic process." *Int. J. Proj. Mgmt.*, 15(5), 273-281.
- [3] Cooper, D. F., and Chapman, C. B. (1987). *Risk Analysis for Large Projects*, Wiley, U.K.
- [4] Institution of Civil Engineers (ICE) and Faculty and Institute of Actuaries (FIA). (1998).
- [5] RAMP: Risk Analysis and Management for Projects, Thomas Telford, London.
- [6] Nabil, D. Parsiani Shull (2006). "Project Evaluation using fuzzy Logic and Risk Analysis Techniques." *Thesis: M.S. in Industrial Engg.* Univ. of Puerto Rico.

About Author (s):



[Like a human expert, it is able to explain the line of reasoning uses for each problem it solves. A user can study the rationale on which the advice is based and is free to accept or reject it.]



[SC approach provides consistent, uniform advice. It is thorough and methodical and does not have lapses that cause it to overlook important factors, slip steps or forget. It is not politically motivated, temperamental or biased.]