

# Reduced-Complexity Channel Estimation for ISDB-T One-Seg using Modified Orthogonal Matching Pursuit

Ryan Paderna, Takeshi Higashino, Minoru Okada

**Abstract**—Integrated Services Digital Broadcasting for Terrestrial (ISDB-T) One-Seg is a Japanese standard for digital television specifically for mobile reception. It uses Orthogonal Frequency Division Multiplexing (OFDM) that provides robustness against multipath fading. A novel approach called Compressed Sensing (CS) has been implemented for estimating the Channel State Information (CSI). The CS improves the spectral efficiency by estimating the CSI with less measurement. However CS requires high computational cost that makes the method difficult for actual implementation. This paper proposes a new approach for channel estimation using reduced size sparse measurement matrix combined with Modified Orthogonal Matching Pursuit (MOMP) algorithm. The simulation shows that the proposed method provides less execution time for CSI calculation.

**Keywords**—OFDM, compressed sensing, matching pursuit, channel estimation

## I. Introduction

Since 2012, Japan successfully shifted from analog to digital television system because of the several advantages that ISDB-T can offer. ISDB-T uses wireless broadband technology such as OFDM and capable of receiving good reception with less transmitting power [1]. OFDM is a multicarrier modulation system that provides robustness against multipath interference and co-channel interference. Examples of multicarrier system that support OFDM are the 3rd and 4th generation cellular systems [2].

Another advantage of OFDM is its higher spectral efficiency that can carry more information in a given bandwidth.

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That is why ISDB-T exploits the advantage of OFDM by providing other services aside from High Definition TV (HDTV). These services include Emergency Notification Services and One-Seg.[1]

In ISDB-T, the OFDM sub channels are divided into thirteen segments. One-Seg uses one out of the thirteen segments of the OFDM for mobile reception purposes. Fig 1 shows the bandwidth allocation for ISDB-T One-Seg.

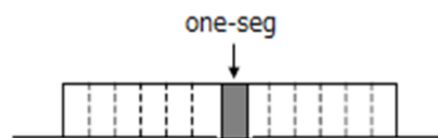


Figure 1. One-Seg bandwidth allocation

Since One-Seg only uses a small portion of the total bandwidth for ISDB-T, the complexity for signal recovery such as processing time and hardware requirements will be reduced. These advantages are ideal for mobile reception such as mobile phone or portable TV. However, mobile reception for One-Seg is prone to double selective fading channel wherein the channel state information (CSI) is rapidly changing over time. Double selective fading channel is due to the Doppler frequency shift caused by high-speed moving reception.

In order to determine the variation of CSI, a periodic measurement of the channel will be done using channel estimation. Channel estimation is a process of determining the channel characteristic using pilot-assisted scheme [3]. The channel was analyzed using the response of the known pilot that was inserted on the selected subcarriers of OFDM. The performance of the system is based on how accurate the channel estimation can offer.

Although there are many types of channel estimation available in the literature, [4-8] all methods has its own pros and cons. The most common tradeoff for different channel estimation methods is between spectral efficiency vs. performance. If the system requires better performance, it needs higher channel measurements which means higher number of pilots to be inserted in the subcarriers. If higher number of pilots are inserted, the spectral efficiency is reduced since pilots do not contain any information at all.

Research shows that the channel for broadband communication systems including ISDB-T is a sparse channel. Sparse channel is composed of a very large multipath channel where few paths have high transmission energy while most

paths are weak or approximately zero transmission energy [9]. This assumption can be exploited using compressed sensing. Compressed Sensing is a novel technique for sparse signal recovery using few measurements [10]. By taking advantage of the compressed sensing method for channel estimation, we can obtain better performance and at the same time increase the spectral efficiency since few pilots are needed to measure the CSI.

However, CS requires higher computational cost due to huge size matrix calculation which is very difficult for actual implementation. Two of the authors [11] used a linear program for CS called Modified Orthogonal Matching Pursuit (MOMP) which reduces the computational cost for channel estimation in ISDB-T. The assumption is that the channel impulse response has a small change in every succeeding symbol. This assumption will drastically reduce the computational cost since every channel impulse response for every succeeding symbol was correlated. Every channel estimation for every present symbol was based on the previous symbol calculation. MOMP is used to determine the best projection of the observed frequency response to the measurement matrix for signal recovery. This method was done in frequency domain.

This paper proposed the same CS based method for channel estimation but in time domain calculation. Time domain calculation has advantage in terms of sparse measurement matrix. This proposed method will further reduced the computational cost using MOMP because we only compute the non-zero entries of the measurement matrix. The next sections of this paper will cover the following. Section II introduces the system model, section III discusses the proposed channel estimation, section IV discusses the numeral results, and lastly section 5 discusses the conclusion.

## II. System Model

Figure 2 shows the system model considered in this paper.

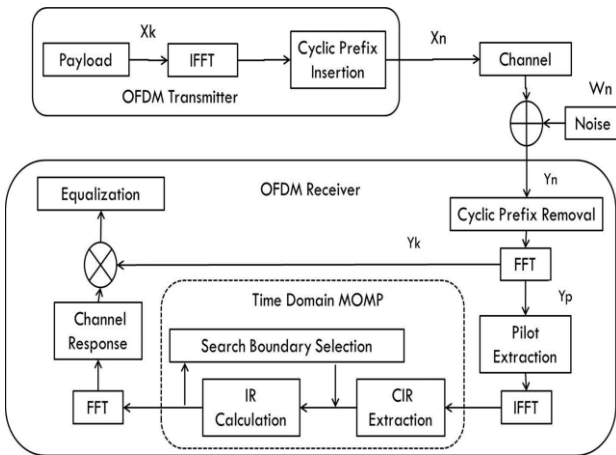


Figure 2. System Block Diagram

The payload  $x_k$  which is composed of data and pilot subcarriers, is transformed by  $N$ -point Inverse Fast Fourier

Transform (IFFT). The transmission signal  $x_n$  after cyclic prefix insertion  $N_{cp}$  can be represented as

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{j \frac{2\pi n k}{N}}, -N_{cp} \leq n < N \quad (1)$$

The transmission signal  $x_n$  is then transmitted and propagated through a multipath fading channel with Additive White Gaussian Noise (AWGN). The fading channel can be represented by time varying impulse response  $h_{n,l}$  where  $l \in [0, L)$  represent  $l^{th}$  tap. The received signal  $y_n$  after cyclic prefix removal can now be represented as

$$y_n = \sum_{l=0}^{L-1} h_{n,l} x_{n-1} = \mathbf{H}_t \mathbf{F}^H \mathbf{X}_k + \mathbf{w}_n \quad (2)$$

where  $\mathbf{H}_t$ ,  $\mathbf{F}^H$  and  $\mathbf{w}_n$  denotes time domain channel matrix, Hermitian transpose  $(\cdot)^H$  of Fast Fourier Transform Matrix  $\mathbf{F}$ , and AWGN, respectively.

The received signal  $y_n$  is then inputted to the  $N$ -point Fast Fourier Transform (FFT) for frequency-domain representation defined as

$$\mathbf{Y}_m = \mathbf{H}_f \mathbf{X}_m + \mathbf{W}_m \quad (3)$$

where  $\mathbf{H}_f = \mathbf{F} \mathbf{H}_t \mathbf{F}^H$  is the frequency domain channel matrix and  $\mathbf{W}_m$  is the FFT of  $\mathbf{w}_n$ . The  $(m, k)^{th}$  element of  $\mathbf{H}_f$  can be expressed as

$$\mathbf{H}_f(m, k) = \frac{1}{N} \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} h(n, l) e^{j \frac{2\pi(k-m)n}{N}} e^{j \frac{2\pi l n}{N}} \quad (4)$$

The next section will discuss the proposed channel estimation for One-Seg.

## III. Proposed Channel Estimation

This section discusses the proposed channel estimation. The proposed channel estimation is divided into three parts namely Linear Impulse Response Estimator, Compressed Sensing based Impulse Response Estimator, and Modified Orthogonal Matching Pursuit:

### A. Linear Impulse Response Estimator

The aim of this research is to estimate the impulse response which can be done in time domain. Since the channel estimation used is a pilot based scheme, the information of the channel is carried out by the response of the known pilot. The response of the pilot can be describe by the observed impulse response  $\hat{\mathbf{g}}$ .

Let us suppose that the pilot equalization vector in  $K$  subcarriers is defined as

$$\mathbf{e} = [e_0, e_1, \dots, e_{K-1}]^T \quad (5)$$

where

$$e_{Mm+i} = \begin{cases} \frac{1}{p_m} & (i = 0) \\ 0 & (1 \leq i < M) \end{cases} \quad (6)$$

and  $p_m$  is the  $m$ -th pilot symbol assigned in every  $M$  subcarriers.

The frequency response can now be solved by

$$\hat{\mathbf{h}} = \mathbf{E} \mathbf{Y}_m \quad (7)$$

where  $\mathbf{E} = \text{diag}(\mathbf{e})$  is the diagonal matrix whose diagonal elements correspond to the inverse of the pilot symbols. The frequency response  $\hat{\mathbf{h}}$  is then inputted to IFFT to solve for the Observed Impulse Response  $\hat{\mathbf{g}}$  given by

$$\hat{\mathbf{g}} = \mathbf{F}^H \hat{\mathbf{h}} \quad (8)$$

## B. Compressed Sensing based Impulse Response Estimator

Compressed sensing (CS) is a new method to reconstruct a sparse signal using few measurements [12]. This method is best fit for the proposed channel estimation since the impulse response  $\mathbf{g}$  is a sparse vector. Fig 3 shows the vector and matrix illustration for solving the impulse response  $\mathbf{g}$  using compressed sensing.

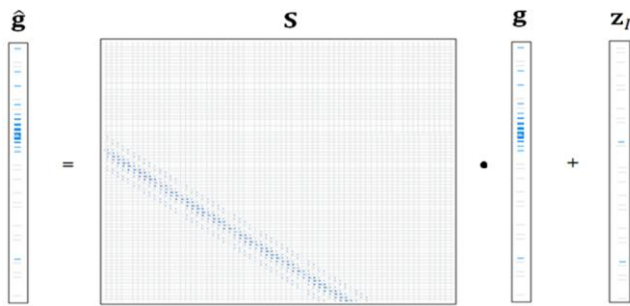


Figure 3. Compressed Sensing

where  $\mathbf{S}$  and  $\mathbf{z}_l$  denotes measurement matrix and noise, respectively. What compressed sensing does is to determine the best basis of the observed impulse response  $\hat{\mathbf{g}}$  to the measurement matrix  $\mathbf{S}$  in order to solve the impulse response  $\mathbf{g}$ . CS methods basically focus on optimization using several linear programming methods. In this case, the Modified Orthogonal Matching Pursuit (MOMP) will be used to solve the problem in CS which will be discussed in the next section.

Measurement Matrix  $\mathbf{S}$  is a dictionary matrix that contains all the possible basis of the observed impulse response  $\hat{\mathbf{g}}$ . Since the Observed Impulse Response  $\hat{\mathbf{g}}$  is a sparse vector, then the measurement matrix  $\mathbf{S}$  can be a sparse matrix since only non-zero entries of the observed impulse response  $\hat{\mathbf{g}}$  will be projected to the measurement matrix  $\mathbf{S}$ . In that way, the computational cost will be drastically reduced.

The measurement Matrix can be constructed by

$$\mathbf{S} = \mathbf{Q}_M \mathbf{F}^H \mathbf{Q} \mathbf{Q}_s \mathbf{Q}^T \mathbf{F} \quad (9)$$

where

$$\mathbf{Q}_M = \begin{bmatrix} \mathbf{O}_{N_T, N_{GI}-N_T} & \mathbf{I}_{N_T} \\ \mathbf{I}_{N_{GI}-N_T} & \mathbf{O}_{N_{GI}-N_T, N_T} \end{bmatrix} \mathbf{O}_{N_{GI}, N_A} \quad (10)$$

denotes mask matrix with  $N$ ,  $N_{GI}$ ,  $N_T$ , and  $N_A = N - N_{GI}$  signifies as FFT size, guard interval size, number of tail samples, and the number of masked samples, respectively.

The  $\mathbf{Q}$  implies re-ordering matrix defined as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{O}_{K_P, K_N} & \mathbf{I}_{K_P} \\ \mathbf{O}_{N-K, K_N} & \mathbf{O}_{N-K, K_P} \\ \mathbf{I}_{K_N} & \mathbf{O}_{K_N, K_P} \end{bmatrix} \quad (11)$$

where  $\mathbf{I}_K$  is the identical matrix of size  $K$  and  $\mathbf{O}_{K,N}$  is the zero matrix of size  $K \times N$ .  $K_P$  and  $K_N$  are the numbers of positive and negative frequency subcarriers, respectively.

The  $\mathbf{Q}_s$  implies pilot pattern matrix where  $\mathbf{Q}_s = \text{diag}(\mathbf{q}_s)$ ,  $\mathbf{q}_s = [q_0, q_1, \dots, q_{K-1}]^T$ , and

$$q_{Mm+i} = \begin{cases} 1 & (i = 0) \\ 0 & (1 \leq i < M) \end{cases} \quad (12)$$

The measurement matrix has a unique characteristic of sparseness which is shown in Fig. 4. Fig. 4 shows that as the FFT size increases, the level of sparseness also increases. This characteristic is an advantage for high FFT size such as ISDB-T One-Seg. Using this measurement matrix will further reduce the computational cost since only non-zero entries of the measurement matrix will be used for calculation.

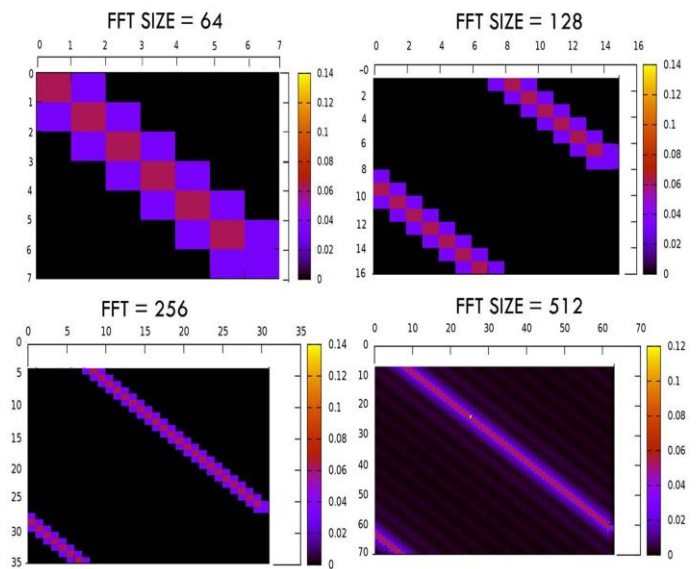


Figure 4. Measurement Matrix for different FFT sizes

## C. Modified Orthogonal Matching Pursuit

The Modified Orthogonal Matching Pursuit (MOMP) is used to solve the CS problem mentioned in the previous section. Basically what MOMP does is to determine the best basis of the Observed Impulses Response  $\hat{\mathbf{g}}$  to the measurement matrix  $\mathbf{S}$  in order to solve the impulse response  $\mathbf{g}$ . The MOMP is a modified version of the Orthogonal Matching

Pursuit (OMP) [11] algorithm that requires low computational cost. The MOMP is valid as long as the observed impulse response does not change much for every succeeding OFDM symbols.

Below are the detailed steps in solving the impulse response  $\mathbf{g}^{omp}$  using OMP [13] with the columns of the measurement matrix denoted as

$$\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \dots, \mathbf{s}_{N_{GI}-1}] = [s_{kn}]_{\substack{0 \leq k < N_{GI} \\ 0 \leq n < N_{GI}}} \quad (13)$$

1. Let the residual vector be  $\mathbf{r}_i = \hat{\mathbf{g}}$ , the index set  $\Lambda_i = \phi$ , and the iteration counter  $i = 1$  as an initialization. Let  $\mathbf{X}_i = []$  be the empty matrix of the chosen atom.
2. Solve for index  $\lambda_i$  by computing the maximum inner product between the residual and columns of measurement matrix.

$$\lambda_i = \arg \max_{k=0,1,\dots,N_{GI}-1} |\mathbf{r}_{i-1}^H \mathbf{s}_k| \quad (14)$$

where  $\mathbf{x}^H$  is the Hermitian transpose of  $\mathbf{x}$ .

3.  $\lambda_i$  is added to the index set as

$$\Lambda_i = \Lambda_{i-1} \cup \{\lambda_i\} \quad (15)$$

While the  $\lambda_i$ -th column vector  $\mathbf{s}_{\lambda_i}$  is augmented as

$$\mathbf{X}_i = [\mathbf{X}_{i-1}, \mathbf{s}_{\lambda_i}] \quad (16)$$

4. Solve the estimated signal using least square.

$$\mathbf{w}_i = \arg \min_{\mathbf{w}} \|\hat{\mathbf{g}} - \mathbf{X}_i \mathbf{w}\|^2 \quad (17)$$

where  $\mathbf{w}_i$  is the  $i$ -dimensional vector.

5. Update the residual as

$$\mathbf{r}_i = \hat{\mathbf{g}} - \mathbf{X}_i \mathbf{w}_i \quad (18)$$

6. Increment the iteration counter  $i$  and repeat 2 through 5 until  $\|\mathbf{r}_i\|^2 \leq \eta_g$  where  $\eta_g$  is the threshold.

7. The OMP estimate is then given by

$$\mathbf{g}^{omp} = [\mathbf{i}_{\lambda_0}, \mathbf{i}_{\lambda_1}, \dots, \mathbf{i}_{\lambda_{m-1}}] \mathbf{w}_m \quad (19)$$

where  $\mathbf{i}_k = \left[ \begin{matrix} 0, \dots, 0 \\ 1, 0, \dots, 0 \end{matrix} \right]^T$  is the  $N_{GI}$ -

dimensional vector, of which  $k$ -th element is one and other elements are set to be zero.

Since the assumption is that the present impulse response  $\mathbf{g}^{(t)}$  does not change much from the previous impulse response  $\mathbf{g}^{(t-1)}$  is being considered, a  $N_{GI}$ -dimension binary vector  $\mathbf{b}^{(t)} = [b_0^{(t)}, b_1^{(t)}, \dots, b_{N_{GI}-1}^{(t)}]^T$  will determined how much change the present and previous impulse responses have acquired.  $\mathbf{b}^{(t)}$  is solved using

$$b_k^{(t)} = \begin{cases} 1; & (|\bar{g}_k^{(t-1)}| \geq \eta_g) \\ 0; & (|\bar{g}_k^{(t-1)}| < \eta_g) \end{cases} \quad (20)$$

where  $\eta_g$  is the threshold for determining the search region that has the highest peak, and  $\bar{g}_k^{(t-1)} = \alpha g_{k-1}^{(t-1)} + g_k^{(t-1)} +$

$\alpha g_{k+1}^{(t-1)}$  is the recalculated impulse response with  $\alpha$  as a weight factor.

And finally, Eq. (14) is modified using

$$\lambda_i = \arg \max_{\{k | b_k^{(t)} = 1\}} |\mathbf{r}_{i-1}^H \mathbf{s}_k| \quad (21)$$

The modification of Eq. (14) to Eq. (21) differs between OMP and MOMP. The inner product operation in Eq. (14) was simply conditioned using binary vector  $\mathbf{b}^{(t)}$  to reduce the computational cost.

## IV. Numerical Result

The proposed channel estimation was evaluated in terms of bit error rate (BER) performance and execution time. A simulation was done using C++ programming with IT++ library for communication systems [14]. It is also assumed that the OFDM transmitter and receiver are in perfect synchronization. The simulation parameter were based on ISDB-T One-Seg standard shown in Table 1 [1].

TABLE I. SIMULATION PARAMETERS

System Model	ISDB-T mode 3
noise type	AWGN
channel delay profile (shift)	{0, 2, 5, 16, 23, 40}
channel power profile (dB)	{-3, 0, -2, -6, -8, -10}
modulation type	16QAM
FFT size	512
data subcarrier	378
pilot subcarrier	54
guard interval ratio	1/8
number of samples	32
search region threshold ( $\eta_g$ )	0.003
weight factor ( $\alpha$ )	1/2

Fig. 5 shows the BER performance against Carrier-to-Noise power Ratio (CNR) in doubly selective channel with normalized Doppler frequency shift  $F_d T_s = 0.0625, 0.125$ . BER performance shows that the proposed method was able to validate a decent channel estimation.

Fig. 6 shows the execution time for different FFT size between the frequency domain MOMP [11] and the proposed time domain MOMP with CNR = 20dB and  $F_d T_s = 0.0625$ . Fig. 6 shows that execution time of the proposed method outperformed the previous approach which is the frequency domain MOMP. It also shows that the execution time of the proposed method has a small change for different FFT sizes because of the unique characteristic of the measurement matrix. Section III discusses that the non-zero entries of the measurement matrix is not scalable to the FFT size. This result to a small change of execution time for different FFT sizes since MOMP will only use the non-zero entries of the measurement matrix.



Table 2 shows the computational complexities of Time-Domain MOMP vs. Frequency-Domain MOMP in terms of flop count. Table 2 shows that the proposed method (Time – Domain MOMP) was able to reduce the computational complexity by 99.2 percent because of the high sparseness of the measurement matrix.

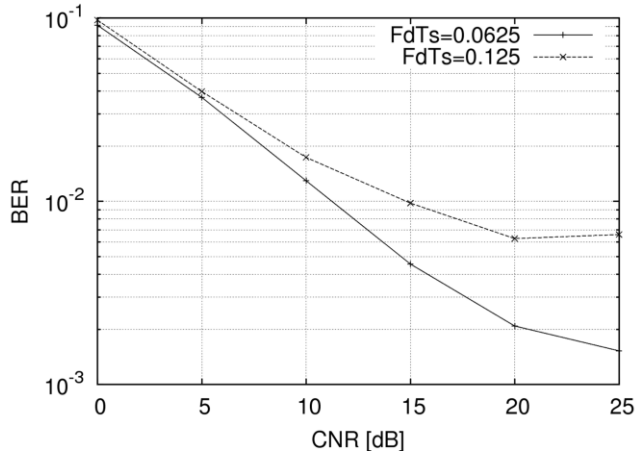


Figure 5. BER performance

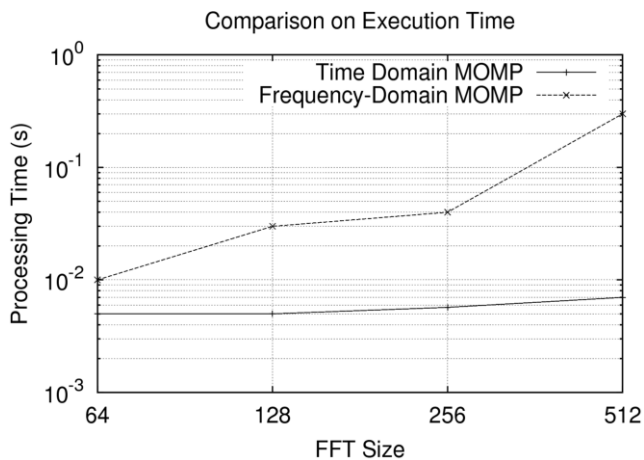


Figure 6. Comparison on Execution Time

TABLE II. COMPUTATIONAL COMPLEXITIES OF FREQUENCY DOMAIN MOMP VS. TIME DOMAIN MOMP

	<i>Frequency Domain MOMP</i>	<i>Time Domain MOMP</i>	<i>Saving</i>
flop count	28,367,118	215,372	99.2%

## v. Conclusion

The paper proposed a new channel estimation method for ISDB-T One-Seg using MOMP with reduced size sparse measurement matrix. The simulation shows that the proposed method was able to validate a decent BER performance in double selective fading channel. Moreover, The computational

complexity was effectively reduced by reducing the execution time and flop count in solving the CS problem. The reason for execution time reduction is the high level sparseness of the measurement matrix that was used.

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