

Machinery Drives and Dynamic Stresses

A. Abdul-Ameer

Abstract—Distributed-lumped parameter modelling methods are employed in the analysis of manufacturing machine drives comprising rotors, bearings and shaft arrangements for variable speed process systems. Analysis procedures enabling the computation of the torsional shear stress and velocity response of these systems are outlined and block diagram representations are formulated. The dynamic stresses for machine drive arrangements following deterministic input changes are computed. Typical rotor-shaft assemblies are investigated illustrating the peak stress levels encountered.

Keywords— distributed, lumped, modelling, dynamic, stress

I. Introduction

The concept of hybrid system modelling [1], where combinations of lumped point-wise and distributed parameter realisations are inter-connected is introduced [2]. This procedure provides a simple, realistic, compact model for dispersed mechanical systems, in which confidence can be placed.

To demonstrate the techniques involved, the determination of the dynamic stress variations in mechanical drives using methods which can be applied to any distributed, physical system, will be investigated herein. The methods presented enables the stress transients to be accurately predicted in spatially dispersed, power transmission elements.

This includes the identification of peak stress variations and the prediction of stress reversals, amplitudes and frequencies. These results are not easily obtained from alternative methods for distributed parameter system models. However as shown herein, hybrid modelling provides the high integrity, unambiguous answers required.

II. Modelling Torsional Rotor Systems

Although affording the simplest modelling solution, lumped parameter models [3] become increasingly inaccurate with expanding system dispersion. In efforts aimed at improving accuracy, finite element modelling methods are often resorted to [4]. Once again lumped parameter theory is employed.

However, the system is now assumed to comprise a finite number of mass-inertia, damping and stiffness elements arranged in lattice structures to replicate the system topology. This procedure suffers from dimensionality problems, as the number of point-wise equations required increases, to accommodate system complexity. Ultimately, the practice of including further finite elements begins to attract numerical, computational errors, as discussed in [5], [6] and [7] with significant uncertainty regarding the validity of the results obtained.

Of course, in acknowledging system dispersion the prospect of dynamic process descriptions governed by partial differential equations looms. In these representations, system analysis is based on continuous formulations where an infinite number of infinitesimal elements are employed, as part of the system description, as in [8].

Since power transmission systems, as in [9] for example, comprise for analysis purposes rotors, supported on fluid film bearings then arrangements of this type may be derived for a wide variety of applications. Typically, machine and vehicle drives, marine propulsion, process and military systems, as in [10] and [11] may be accurately modelled using distributed-lumped, hybrid realisations, [12], [13] and [14].

Essentially, in these assemblies the relatively concentrated, rotating inertias are lumped and referred to an input or output shaft, as are the frictional coefficients arising from viscous bearing, energy dissipation, see for example [13]. All of the shafts are assumed to be relatively “long” slender components with distributed inertia and stiffness characteristics. Both multiple, parallel and/or profiled shaft sections may be incorporated in the distributed-lumped descriptions employed herein and material non-linear elasticity may be included. Additionally, both the torsional and longitudinal dynamics, of spatially dispersed elements, may be modelled using hybrid, distributed-lumped techniques. Typical application studies are presented in [14].

III. Dual Distributed-Lumped Rotor System Model

A distributed-lumped model, for the rotor system shown in Fig. 1, will be formulated. The rotors and bearings will be modelled as rigid, lumped parameter, point-wise units. Owing to the dimensions of the drive shaft, this component will be described as a distributed parameter element, as in [9], where the inertia and stiffness are continuous functions of shaft length. The notation employed in [2] will be adopted.

Dr A. Abdul-Ameer, Associate Professor in Systems Engineering
The British University in Dubai,
P.O. Box 345015, Dubai-UAE,

In accordance with figure 1, the Laplace transformed model for the distributed-lumped parameter description is:

$$\begin{bmatrix} T_1(s) - J_1 s \omega_1(s) - c_1 \omega_1(s) \\ J_2 s \omega_2(s) + c_2 \omega_2(s) + T_2(s) \end{bmatrix} = \begin{bmatrix} \zeta_1 w_1(s) & -\zeta_1 (w_1^2(s) - 1)^{1/2} \\ \zeta_1 (w_1^2(s) - 1)^{1/2} & -\zeta_1 w_1(s) \end{bmatrix} \begin{bmatrix} \omega_1(s) \\ \omega_2(s) \end{bmatrix}$$

Since $T_2(s)$ in this application is zero the impedance description becomes:

$$\begin{bmatrix} T_1(s) \\ 0 \end{bmatrix} = \begin{bmatrix} \zeta_1 w_1(s) + \gamma_1(s) & -\zeta_1 (w_1^2(s) - 1)^{1/2} \\ \zeta_1 (w_1^2(s) - 1)^{1/2} & -\zeta_1 w_1(s) - \gamma_2(s) \end{bmatrix} \begin{bmatrix} \omega_1(s) \\ \omega_2(s) \end{bmatrix} \quad (1)$$

In equation (1):

$$\gamma_1(s) = J_1 s + c_1, \quad \gamma_2(s) = J_2 s + c_2$$

$$L_1 = \rho_1 J_s \quad \text{and} \quad C_1 = \frac{1}{(G_1 J_s)}$$

Hence:

$$\zeta_1 = \sqrt{\frac{L_1}{C_1}} = J_s \sqrt{(\rho_1 G_1)}$$

and since:

$$w_1(s) = \frac{e^{2l_1 \Gamma_1(s)} + 1}{e^{2l_1 \Gamma_1(s)} - 1} \quad (2)$$

where in (2):

$$\Gamma_1(s) = s \sqrt{L_1 C_1} = s \sqrt{\rho_1 / G_1}$$

Then following the inversion of (1):

$$\begin{bmatrix} \omega_1(s) \\ \omega_2(s) \end{bmatrix} = \begin{bmatrix} \zeta_1 w_1(s) + \gamma_2(s) \\ \zeta_1 (w_1^2(s) - 1)^{1/2} \end{bmatrix} T_1(s) / \Delta_1(s) \quad (3)$$

where in equation (3):

$$\Delta_1(s) = \zeta_1 (\gamma_1(s) + \gamma_2(s)) w_1(s) + \gamma_1(s) \gamma_2(s) + \zeta_1^2$$

It is very easy to compute the outputs $\omega_1(s)$ and $\omega_2(s)$ of (3) following any arbitrary input $T_1(s)$. To accomplish this, in delay form:

$$w_1(s) = \frac{(1 + e^{-l_1 \Gamma_1(s)})}{(1 - e^{-2l_1 \Gamma_1(s)})}$$

and upon evaluation:

$$(w_1^2(s) - 1)^{1/2} = \frac{2e^{-l_1 \Gamma_1(s)}}{(1 - e^{-2l_1 \Gamma_1(s)})}$$

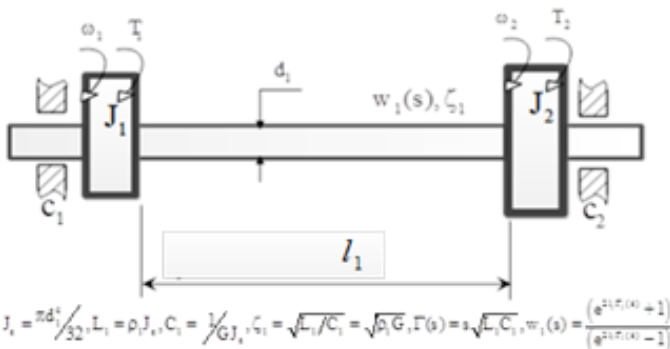


Figure 1, Two Rotor Lumped-Distributed Parameter Model

iv. Three Rotor System

Following the same route as in sections 3 the model for the three rotor system of figure 2 becomes:

$$\begin{bmatrix} T_1(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \zeta_1 w_1(s) + \gamma_1(s) & -a_1(s) & 0 \\ a_1(s) & -\zeta_1 w_1(s) - \gamma_2(s) - \zeta_2 w_2 & a_2(s) \\ 0 & -a_2(s) & -\zeta_2 w_2(s) + \gamma_3(s) \end{bmatrix} \begin{bmatrix} \omega_1(s) \\ \omega_2(s) \\ \omega_3(s) \end{bmatrix} \quad (4)$$

Hence, equation (4) upon inversion yields:

$$\begin{bmatrix} \omega_1(s) \\ \omega_2(s) \\ \omega_3(s) \end{bmatrix} = \begin{bmatrix} a_2^2(s) - (-\zeta_1 w_1(s) + \gamma_2(s) + \zeta_2 w_2)(\zeta_2 w_2(s) + \gamma_3(s)) \\ -a_1(s)(\zeta_2 w_2(s) + \gamma_3(s)) \\ -a_1(s)a_2(s) \end{bmatrix} \frac{T_1(s)}{\Delta_2(s)} \quad (5)$$

where in (5):

$$w_j(s) = \frac{(e^{2\Gamma_j(s)l_j} + 1)}{(e^{2\Gamma_j(s)l_j} - 1)}, \quad \zeta_j = \sqrt{\frac{L_j}{C_j}}, \quad \gamma_j(s) = J_j s + c_j,$$

$$\Gamma_j(s) = \sqrt{(L_j C_j)}, \quad L_j = \rho_j d_j^4 \pi / 32, \quad C_j = 1 / G_j J_j,$$

$$a_j(s) = \zeta_j (w_j^2(s) - 1)^{1/2}, \quad 1 \leq j \leq 2$$

and:

$$\Delta_2(s) = \begin{bmatrix} [\zeta_1 \zeta_2 (\gamma_1(s) + \gamma_2(s) + \gamma_3(s)) w_1(s) w_2(s) + \zeta_1 [\gamma_2(s) (\gamma_1(s) + \gamma_3(s)) + \zeta_2^2] w_1(s) \\ + \zeta_2 [\gamma_1(s) (\gamma_2(s) + \gamma_3(s)) + \zeta_1^2] w_2(s) + \gamma_1(s) \gamma_2(s) \gamma_3(s) + \zeta_2^2 \gamma_1(s) + \zeta_1^2 \gamma_3(s) \end{bmatrix}$$

Upon expanding, equation (5) becomes:

$$\begin{bmatrix} \omega_1(s) \\ \omega_2(s) \\ \omega_3(s) \end{bmatrix} = \begin{bmatrix} n_1(s) \\ n_2(s) \\ n_3(s) \end{bmatrix} \frac{T_1(s)}{\Delta_2(s)} \quad (6)$$

where in equation (6):

$$n_1(s) = \zeta_1 \zeta_2 w_1(s) w_2(s) + \zeta_1 \gamma_3(s) w_4(s) + \zeta_2 (\gamma_3(s) + \gamma_2(s)) w_2(s) + \gamma_2(s) \gamma_3(s) + \zeta_2^2$$

$$n_2(s) = \zeta_1 (w_1^2(s) - 1)^{1/2} (\zeta_2 w_2(s) + \gamma_3(s))$$

$$n_3(s) = \zeta_1 \zeta_2 (w_1^2(s) - 1)^{1/2} (w_2^2(s) - 1)^{1/2}$$

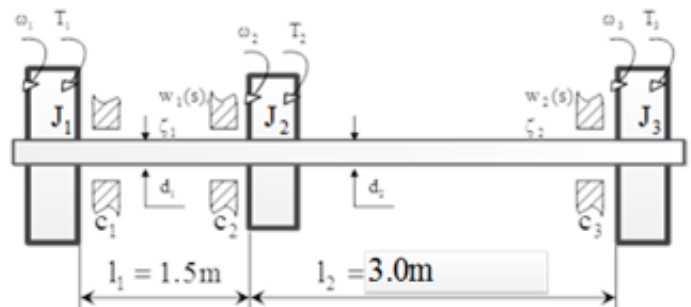


Figure 2, Three Rotor Lumped-Distributed Model

v. Machine Drive Application Study

In this study the dynamics and shaft stress levels in a machine drive system comprising an electric motor, gearbox and load inertia will be investigated. The equivalent system for analysis

purposes is shown in Fig. 2 and the parameters for this system are:

$$J_1 = 3.835 \times 10^{-5}, J_2 = 4.0212 \times 10^{-6}, J_3 = 0.0015 \text{m}^4,$$

$$c_1 = 0.15, c_2 = 0.1, c_3 = 20 \text{ Nmsec/rad}, l_1 = 1.5, l_2 = 3.0 \text{m},$$

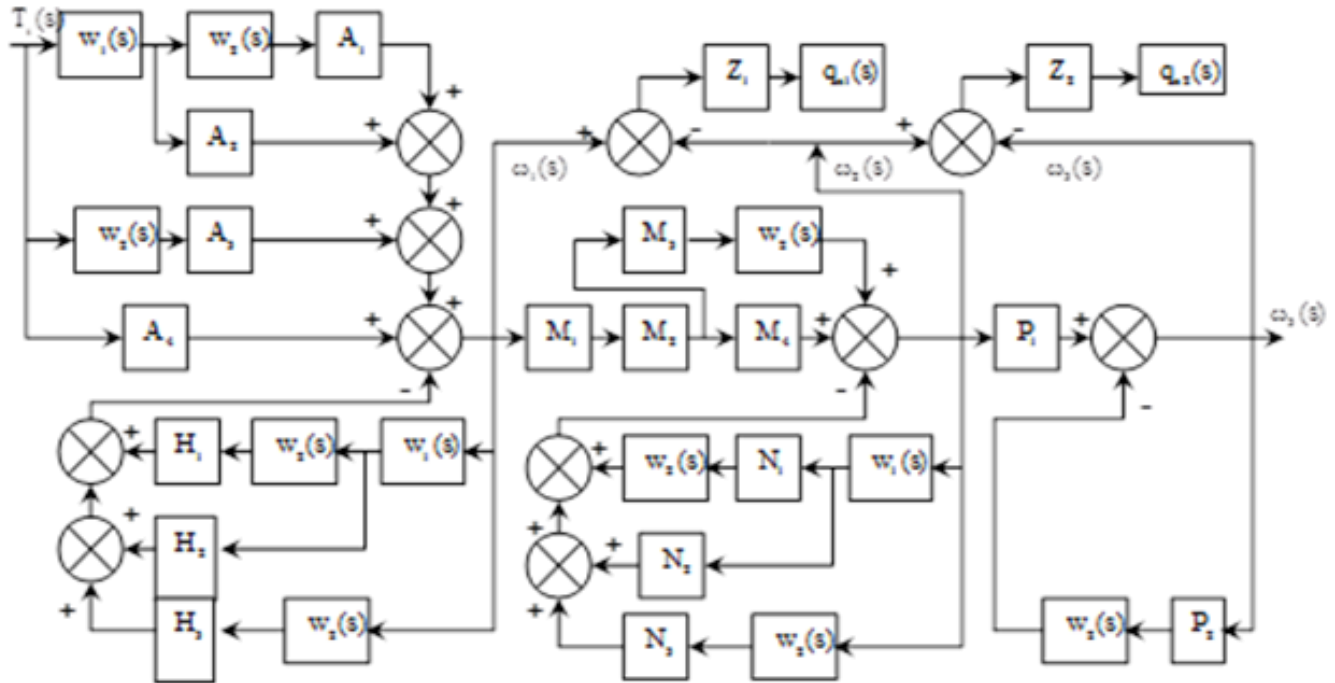
$$r_1 = 0.025, r_2 = 0.025 \text{m}, G_1 = G_2 = 80 \times 10^9 \text{ N/m}^2,$$

$$\rho_1 = \rho_2 = 7800 \text{ kg/m}^3, J_{s1} = 6.1359 \times 10^{-7}, J_{s2} = 6.1359 \times 10^{-7} \text{ m}^4$$

From the theoretical analysis, for these parameters:

$$\zeta_1 = \zeta_2 = 15.3275, \Gamma_1 = \Gamma_2 = 3.1225 \times 10^{-4} \text{ s},$$

$$w_1(s) = w_2(s) = \frac{1 + e^{-0.00093675s}}{1 - e^{-0.00093675s}}$$



$$n_1(s) = \gamma_1(s)\gamma_2(s)\gamma_3(s) + \gamma_1(s)\zeta_2^2 + \gamma_3(s)\zeta_1^2, n_2(s) = \gamma_2(s)\gamma_3(s) + \zeta_2^2, n_3(s) = \gamma_3(s)$$

$$A_1(s) = \frac{\zeta_1\zeta_2}{n_1(s)}, A_3(s) = \frac{\zeta_2(\gamma_2(s) + \gamma_3(s))}{n_1(s)}, A_2(s) = \frac{\zeta_1\gamma_3(s)}{n_1(s)}, A_4(s) = \frac{\gamma_2(s)\gamma_3(s) + \zeta_2^2}{n_1(s)}$$

$$H_1(s) = \frac{\zeta_1\zeta_2(\gamma_1(s) + \gamma_2(s) + \gamma_3(s))}{n_1(s)}, H_2(s) = \frac{\zeta_1(\gamma_1(s)\gamma_3(s) + \gamma_2(s)\gamma_3(s) + \zeta_2^2)}{n_1(s)}, H_3(s) = \frac{\zeta_2(\gamma_1(s)\gamma_3(s) + \gamma_2(s)\gamma_1(s) + \zeta_1^2)}{n_1(s)}$$

$$N_1 = \frac{\zeta_1\zeta_2}{n_2(s)}, N_2 = \frac{\zeta_1\gamma_3(s)}{n_2(s)}, N_3 = \frac{\zeta_2(\gamma_2(s) + \gamma_3(s))}{n_2(s)}, M_1 = \zeta_1, M_2 = (w_1^2(s) - 1)^{1/2}, M_3 = \frac{\zeta_2}{n_2(s)}, M_4 = \frac{\gamma_3(s)}{n_2(s)}$$

$$P_1 = \zeta_2(w_2^2(s) - 1)^{1/2}, P_2 = \frac{\zeta_2}{n_3(s)}, Z_1 = \frac{Gr_1}{l_1s}, Z_2 = \frac{Gr_2}{l_2s}$$

Figure 3, Simulation Block Diagram for 3 Rotor Machine Drive System

The series model for this this system will be employed which results in the simulation block diagram shown in figure 3. Following a step input, torque change of 2119.8 Nm the angular velocity response and the dynamic shear stress in the motor and load shafts are as shown in figure 4a and figure 4b, respectively.

Figure 4a shows the maximum angular velocity overshoot increases to approximately 150% on the gearbox end rotor. The speed oscillations now persist for > 0.03 seconds. Figure 4b shows the transient stress in the motor shaft to exceed the steady state stress by over $1.5 \times 10^8 \text{ N/m}^2$. There is also a stress reversal in the motor drive shaft which would result in fatigue problems. The peak stress in the load shaft advances to

$1.75 \times 10^8 \text{ N/m}^2$ although there is no stress reversal problem associated with this element. With a the load shaft length of 3.0m there is a further reduction in the frequency of the shaft speed oscillations, as shown in Fig. 4a. Again the speed transient increases in amplitude with persistence for $t \gg 0.03$ seconds showing the noise and harshness problems often encountered originate from these internal vibrations.

VI. Conclusion

A distributed-lumped, hybrid modelling approach was employed in this contribution to accurately compute the response of a machine drive system. This enabled the transient stress levels in the drive shafts to be determined following any arbitrary, input torque change, shaft length and diameter, frictional damping and for any inertia loads.

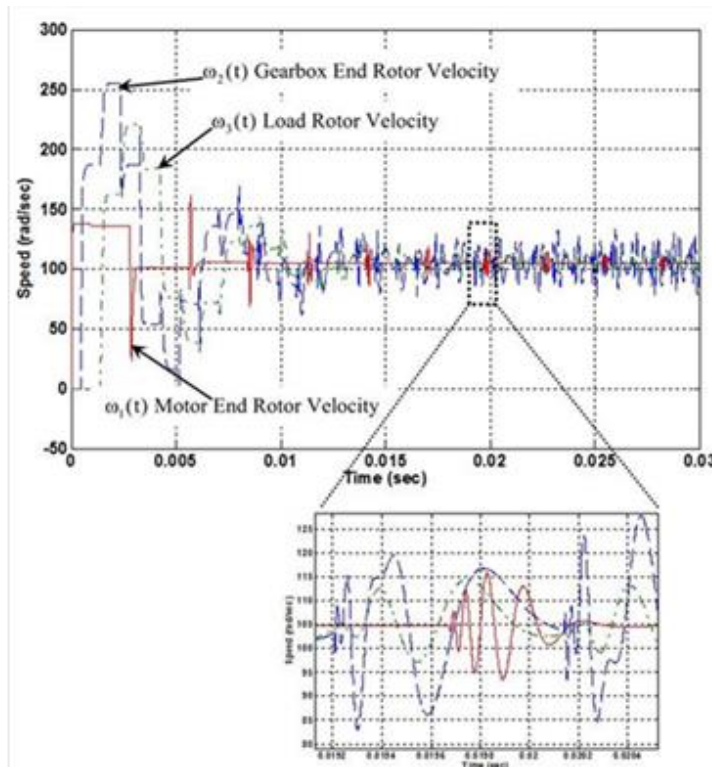


Figure 4a. 3 Rotor-Shaft System Responses Following a Step Input Torque Change of 2119.8 Nm, $l_1 = 1.5$, $l_2 = 3.0$ m

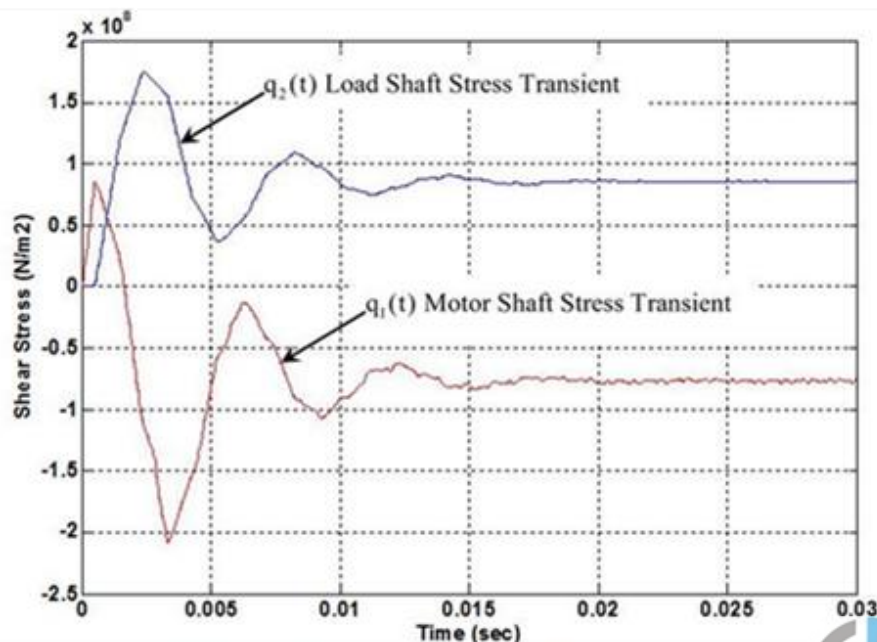


Figure 4b. 3 Rotor-Shaft System Shear Stress Transient Following a Step Input Torque Change of 2119.8 Nm, $l_1 = 1.5$, $l_2 = 3.0$ m

The analysis procedures outlined allowed the formulation of the relatively lumped rotor and bearing models for distributed shaft representations. Multiple shaft, bearing and rotor arrangements can also be easily accommodated using this approach.

Detailed procedures were provided for two and three rotor models enabling simple simulation studies to be undertaken.

To illustrate the versatility of the analysis method, changes in the peak dynamic stress levels were investigated for the rotor systems, considered.

The simulation studies show the speed transient problems in terms of the amplitude, frequency and the decay rates to be expected, in mechanical drives, where input and load disturbances are continuously imposed on the system. Equally, the problem of the dynamic stress changes occurring can be easily determined using hybrid modelling and simple simulation techniques.

Although the complexity of the system increases, as further shaft and rotor sectors are included in the modelling process, the important issue is that these representations can be easily adapted to suit any multiple rotor system, once established. Moreover, the inclusion of non-linear elements such as bearing characteristics with coulomb friction and/or hysteresis effects can be easily accommodated as can the non – linearity of the material properties, using for example, Popov’s procedure outlined in [15].

Acknowledgment

Dr. A. Abdul-Ameer “the author” wishes to acknowledge the support and encouragement for this research provided by Professor Abdullah Alshamsi “the Vice Chancellor”, The British University in Dubai-UAE.

References

- [1] E. O. Doebelin, “System Dynamics Modelling, Simulation, Design”, Marcel Dekker, NY, 1998.
- [2] H. Rahnejat, “Multibody Dynamics, Modelling and Simulation”, IMechE, PEP Publishing, 1997.
- [3] J. Hannah and R. C. Stephens, “Mechanics of Machines”, Edward Arnold, London, 1997.
- [4] R. D. Cook, “Finite Element Modelling for Stress Analysis”, J. Wiley, NY, 1995.
- [5] K.E. Rouch and J.S. Kao, “Dynamic Reduction in Rotor Dynamics by the Finite Element Method”, Trans ASME J Mech-Des., Vol. 102, 1980, pp 360-367.
- [6] Y. D. Kim and C. W. Lee, “Finite Element Analysis of Rotor – Bearing Systems using a Model Transformation Matrix”, Journal of Sound and Vibration, 111 (3), 1986, pp 441-456.
- [7] G. Ganta, “Dynamics of Rotating Systems”, Springer, New York, 2005.
- [8] H. Bartlett and R Whalley, “Analogue Solution to the Modelling and Simulation of D-L Parameter System”, Proc. IMechE, pt I, Vol. 212, 1998, pp 99-114.
- [9] H. Bartlett and R Whalley, Power Transmission System Modelling”, Proc. IMechE, pt C, Vol. 212, 1998, pp 497-508.
- [10] S.S.I. Rizivi, H. Bartlett and R Whalley, “Distributed-Lumped Modelling and Simulation of Marine Propulsion Plant Dynamics”, The Application of Multivariable System Techniques, MEP Ltd., London, 1998.

- [11] R Whalley and H. Bartlett, “The Rotor Dynamics of Reeling Machinery”, App. Math. Modelling, Vol. 22, 1999, pp 1-10.
- [12] R Whalley, “Interconnected Spatially Distributed Systems”, Trans.Inst.M.C., Vol. 12, No. 5, 1990, pp 262-270.
- [13] R Whalley, “The Response of Distributed-Lumped Parameter Systems”, Proc. IMechE pt C6, Vol. 202, 1988, pp 421-427.
- [14] R Whalley, M. Ebrahimi, A. Abdul-Ameer and S. Shalabi, “Optimum Machine Tool Axis Traverse Regulation”, International Journal of Machine Tools and Manufacture, No. 46, 2006, pp 1835-1853.
- [15] H. K. Khali, “Non-linear Systems”, McMillan Press, NY, 1992.



Alaa Abdul-Ameer is Associate Professor in system dynamics and control. His research studies encompassed large-scale engineering systems, modelling and simulation. He was awarded the Degrees of Bachelor of Science (BSc) in Mechanical Engineering in 1979 and Master of Science (MSc) in Applied Mechanics (Tribology) in 1983 from the University of Technology in Baghdad. He has worked in the Automotive Industries in Germany-Ulm and for Scania – France and in the oil refinery industry in the Middle East. He was appointed to a lectureship at the University of Technology in Baghdad in 1986. In 2001, he gained his PhD in Engineering Systems Modelling, Simulation and Control from the University of Bradford-UK, Faculty of Engineering. He continued as a Research Fellow thereafter being appointed as Lecturer in the School of Engineering, Design and Engineering. In 2007, he joined the British University in Dubai as an Associate Professor in the Faculty of Engineering.