

# Effect of Tapered Housing on the Axial Stress Distribution in a Stuffing Box Packing

Mehdi Kazeminia and Abdel-Hakim Bouzid

**Abstract** — The sealing performance of a packed stuffing-box used in pumps and valves is strongly dependent on the distributions of the axial stress and the contact pressure between the packing rings and the side walls. In particular the stress state of the bottom packing ring that is in contact with the internal fluid which is at its minimum is the leak controlling parameter. This minimum axial stress and its resultant radial stress control the pore size and therefore dictate the quantity of leak that passes through the packing rings.

This paper presents a new design feature that improves the stress distributions. The introduction of a variable gap between the packing and the side walls helps producing a more uniform distribution of the stresses in the packing rings while increasing the minimum stress. An analytical model that takes into account the effect a tapered housing on the distribution of the axial stress in a stuffed packing box is also developed. The analytical model is validated by comparison with numerical finite element simulations.

**Keywords**—Packed stuffing-box, analytical modelling, sealing performance, axial stress, tapered housing

## I. Introduction

Valves are used to control fluid circulation in the nuclear, chemical and petrochemical process plants. In general, valves are designed to have two sealing functions. The first one is related to one of the extreme condition of fluid flow control which is the shut-off state and is referred to as internal sealing. The second one is related to the control the fluid leakage to the outside boundary of the valve through the stem and is referred to as external sealing. Packed stuffing-boxes are the preferred sealing system that is widely used in valves to carry out the second described sealing task.

There is an increasing concern about the sealing performance of packed stuffing-boxes. A recent study shows that 40% of all leaking equipment comes from packed stuffing-boxes [1]. Several standards have recently been developed or improved in order to reduce valves fugitive emissions [2-5] and comply with the new strict regulations worldwide. However all standards are related to the qualification of the packing material or valves through special testing procedures and quality control programs. Currently there is no standard that provides design rules or design calculation procedure for stuffed packing boxes. Nevertheless, there are few models to evaluate the axial stress distribution

along with the recommendations by manufacturers to select suitable packing materials. Although several analytical, numerical and experimental investigations have been carried out, the characterization and modelling of packed stuffing-box has not matured enough to reach the level of standardization.

There exist few parameters that can improve the sealing performance of packed stuffing-boxes but not treated in the literature. The introduction of tapered housing that generates a variable gap between the packing and side walls is a design concept that can have a significant effect on the distribution of radial contact pressure depending on the packing material. This paper aims at testing this concept by simulating both analytically and numerically a staffed packing box with such a configuration to evaluate the axial stress distribution. This concept can be used to improve the sealing performance of packed stuffing-box in valve applications.

## II. Theoretical Background

Although the use of packed stuffing-boxes as a sealing system in valves is a relatively old concept that was used in the early steam engines, the analytical and theoretical modeling of its mechanical and sealing behaviors is rather limited. The very few existing models are limited to some particular applications and unfortunately they do not reproduce the wide range of existing experimental data. Several researchers have studied the behavior of packed stuffing boxes both analytically and experimentally. In 1960, Deny [6] studied the ratio of the radial contact pressure to the axial gland stress known as the lateral pressure coefficient. He found that after some minimum gland pressure this ratio is constant and independent of the magnitude of gland axial pressure. Ochonski [7] proposed an equation of axial stress distribution in soft packed stuffing-boxes by considering the equilibrium of forces on a packing ring such that:

$$\sigma(z) = \sigma_D e^{-\beta z} \quad (1)$$

where

$$\beta = 4 \frac{\mu_i K_i d + \mu_o K_o D}{D^2 - d^2} \quad (2)$$

He supported the proposed analytical model with experimental investigation using two test apparatus.

The constants  $K_i$  and  $K_o$  are lateral pressure coefficients described as the ratio between the radial contact stress with the stem at the packing internal side wall and, the housing, at the packing external side wall and the axial stress respectively. These constants depend on the tribological properties at the interface and can be determined experimentally [8,9].

---

Mehdi Kazeminia/ Abdel-Hakim Bouzid  
Mechanical Engineering/ École de Technologie Supérieure  
Canada

Another analytical model has been proposed by Pengyun and et al. [10] by considering moment equilibrium. The main difference between Pengyun and Ochonski models is the assumption of the location of the packing axial force. By solving the moment equilibrium equation around the center of the packing, the latter proposed a similar analytical equation that describes the axial gland stress in a packed stuffing-box having also an exponential form but with a new  $\beta$  factor such that:

$$\beta = 16 \frac{\mu_o K_o D - \mu_i K_i d}{(D - d)^2} \quad (3)$$

Using a more rigorous analysis, Diany and Bouzid [11] proposed also an exponential form for the distribution of the axial stress in soft packed stuffing-boxes but with a different  $\beta$  factor such that:

$$\beta = 24 \frac{\mu_o K_o D - \mu_i K_i d}{(D - d)^2} \quad (4)$$

The above described models do not consider the case of a variable gap between the packing and the side walls.

### III. Analytical Model

The distribution of the axial compressive stress is the most important factor that has a direct effect on the sealing performance of the packed stuffing-box. This stress is transferred to the side walls due to the packing ring lateral transmission capacity which induces contact stresses in the radial direction. The lack of contact stress uniformity makes the packed stuffing box difficult to seal with time. Although applying higher gland stress might seem to be attractive to control leakage from the valve, it increases relaxation of the packing material with time [12], and gradually reduces its sealing performance. Hence the presence of a variable gap due to a tapered housing could provide a more uniform distribution of the contact stress distribution. Consequently, there is a sealing advantage to increase the axial stress in the far most packing ring that is in direct contact with the high pressure fluid inside the valve.

The introduction of a variable gap between the packing and the side walls produces a more uniform distribution of the stresses in the packing rings. Such a design helps reduce the difference between axial gland stress applied at the top packing ring in the stuffing-box, and the axial stress of the last packing ring in contact with the operating fluid. The stress at this location is important in the determination of the gland stress and is supposed to be at least, equal to the pressure of the ambient fluid. Improving the capability of the packing rings to transfer the gland stress to the bottom packing can also be made possible by reducing wall friction gradually throughout the packing length or creating a gradual contact between the wall surfaces. This is a costly technique since it involves a sophisticated machining procedure to alter the tribological properties of the side walls. In addition, the surfaces can increase the torque during operation while introducing disassembly difficulties.

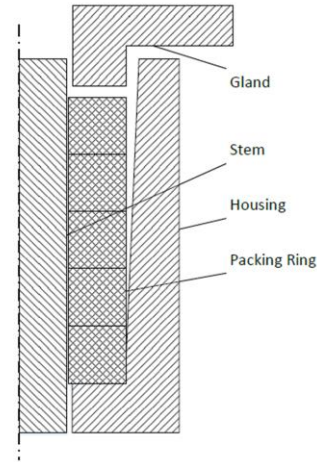


Figure 1: Sectional view of the packed stuffing-box configuration with a linearly varying gap

Following this concept, a sample design of the stuffing-box with a linearly varying gap between the packing ring and the housing walls is considered, as shown in Fig. 1.

Before developing the analytical model, few assumptions are made:

- The packing material is isotropic and obeys to Hooke's law. Its compression modulus and Poisson's ratio are constants.
- The value of the lateral pressure coefficient is depended on the gland stress,  $K = \sigma_r / \sigma_x$ .
- The stem and housing are considered to be rigid.

The process of loading the packing is treated analytically

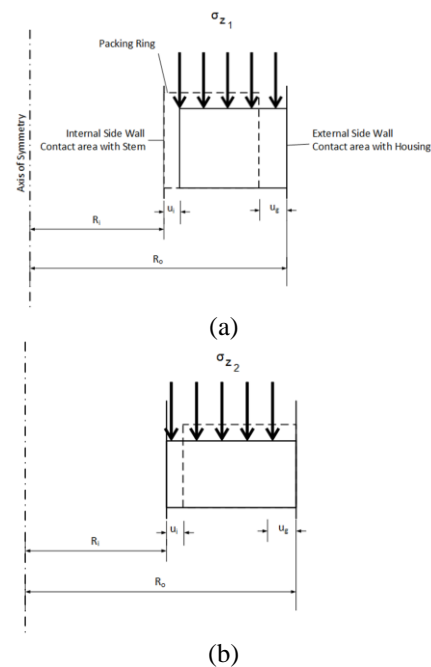


Figure 2. Process of gap filling by expansion of the packing ring due to axial compression; a) application of stress to fill in the gap at the external wall, while creating one at the inside diameter and b) additional stress to produce contact at the internal wall.

based on deformation. Referring to Fig. 2, it is divided into three steps;

a) In the first step the applied load from the gland expands the packing materials to fill the gap between packing and housing side walls without generating lateral forces. During this process a gap between the stem and the packing is created as a result of the compression.

b) In the second step, the additional gland force generates a side pressure with the housing wall till the packing inside surface is in contact with the stem.

c) In the third step, the packing being in contact with the side walls, the normal load generates contact pressure that decreases due to friction forces acting on the side walls. This is considered in an analysis that can be describe by Eq.(1).

Let  $\sigma_{z_1}$  be the gland axial load to achieve the first step, and  $\sigma_{z_2}$  be the additional load to achieve the second step. The required total axial pressure to fill the gap is obtained by

$$\sigma_g = \sigma_{z_1} + \sigma_{z_2} \quad (5)$$

Based on the assumption that the packing material behavior is linear, the Hooke's stress-strain relationship can be applied to an axisymmetric configuration to give

$$\begin{aligned} \epsilon_z &= \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_\theta)] \\ \epsilon_r &= \frac{1}{E} [\sigma_r - \nu(\sigma_\theta + \sigma_z)] \\ \epsilon_\theta &= \frac{1}{E} [\sigma_\theta - \nu(\sigma_r + \sigma_z)] \end{aligned} \quad (6)$$

Treating the stem and housing as rigid bodies and the packing ring as a thick cylinder subjected to only one stress in the axial direction, the strain-displacement relations are,

$$\begin{aligned} \epsilon_r &= \frac{\partial u}{\partial r} \\ \epsilon_\theta &= \frac{u}{r} \end{aligned} \quad (7)$$

The resulting axial stress to initiate contact between the packing ring and the housing wall, as presented in the Fig. (2-a) is given by

$$\sigma_{z_1} = -\frac{2Eu_g(z)}{\nu D} \quad (8)$$

This axial compression force causes a displacement at the internal radius of the packing ring as

$$u_i(z) = -\frac{\nu d \sigma_{z_1}}{2E} \quad (9)$$

Substitution of Eq. (8) into Eq. (9) gives

$$u_i(z) = \frac{d}{D} u_g(z) \quad (10)$$

The next step is to obtain the additional axial load required to bring the packing to the inside surface to initiate the contact with the surface of the stem. In this step and referring to Fig.(2-b), a radial contact pressure,  $P_c$  is generated at the housing to packing interface. Considering Hoop's strain, the radial displacement at the inside diameter of packing ring is given by

$$u_i(z) = \frac{d}{2E} (\sigma_{\theta_i} - \nu \sigma_{z_2}) \quad (11)$$

From the theory of the thick-walled cylinder subjected to external pressure, the hoop stress at this location is given by Lamé [13] as:

$$\sigma_{\theta_i} = -\frac{2Y^2}{Y^2 - 1} P_c \quad (12)$$

The contact pressure,  $P_c$  is obtained by considering the hoop strain equal to zero at the packing ring outside diameter since the contact had already been established at the end of step one. Therefore using Hooke's law at the outside diameter of the packing ring gives:

$$\sigma_{\theta_o} = \nu(\sigma_{z_2} - P_c) \quad (13)$$

Where the hoop stress can be obtained based on the theory of thick-walled cylinder. Consequently, the relation between contact pressure and the additional axial stress at the end of this second step is:

$$P_c = -\frac{\nu \sigma_{z_2}}{(Y^2 + 1)/(Y^2 - 1) - \nu} \quad (14)$$

Substituting Eq.(14) into Eq. (12) and then into Eq.(11) gives

$$u_i(z) = -\frac{\nu d}{2E} \left( \frac{2Y^2}{(1 - \nu)Y^2 + 1 + \nu} + 1 \right) \sigma_{z_2} \quad (15)$$

Finally substituting Eq. (15) into Eq.(10) gives the additional axial stress required to close the induced internal gap

$$\sigma_{z_2} = \frac{2E}{\nu d N} u_g(z) \quad (16)$$

where

$$N = \frac{2Y^2}{Y^2(1 - \nu) + 1 + \nu} - 1 \quad (17)$$

The total axial gland stress needed to close the initial gap is

$$\sigma_g = \sigma_{z_1} + \sigma_{z_2} = B u_g(z) \quad (18)$$

where

$$B = -\frac{2E}{\nu} \left( 1 + \frac{1}{N} \right) \quad (19)$$

The effective axial stress that contributes to create the contact stress required to seal the joint is therefore given by:

$$\sigma_e = \sigma_z - \sigma_g \quad (20)$$

Where  $\sigma_z$  is the gland applied stress. Therefore, the lateral pressure coefficient is given by

$$K = \frac{\sigma_r}{\sigma_e} \quad (21)$$

Considering the equilibrium of an element of the packing ring in the axial direction gives

$$\frac{\partial \sigma_z}{\partial z} + \beta \sigma_e = 0 \quad (22)$$

Where  $\beta$  can be defined by Eq. (2).

Table 1 : Mechanical and geometrical properties used in the numerical and analytical simulations

$E_p(MPa) FG$	36	$d(mm)$	28.576
$E_p(MPa) PTFE$	98	$D(mm)$	47.624
$\nu_p FG$	0.46	$d_h(mm)$	47.625
$\nu_p PTFE$	0.37	$D_h(mm)$	57.625
$E(GPa) Steel$	200	$D_s(mm)$	28.575
$\nu Steel$	0.3	$H_p(mm)$	9.525

Substituting Eq. (20) into Eq. (22) gives

$$\frac{\partial \sigma_z}{\partial z} + \beta \sigma_z = \beta \sigma_g \quad (23)$$

The solution of this first order non-homogeneous differential equation is

$$\sigma_z = e^{-\beta z} \left[ A + \int \beta e^{\beta z} \sigma_g dz \right] \quad (24)$$

Where  $A$  is a constant that can be obtained by considering the following boundary condition:

$$\sigma_z = -\sigma_D, \text{ at } z = 0 \quad (25)$$

A linearly varying gap between the packing and the housing is introduced such that

$$u_g(z) = \delta - \alpha z \quad (26)$$

where  $\delta$  is the initial gap at the top side of the packing rings inside the stuffing-box. Substitution of Eq.(26) into Eq. (24) and evaluating constant  $A$  with Eq.(25) in gives the axial stress in a packed stuffing-box with linear gap between the packing and housing.

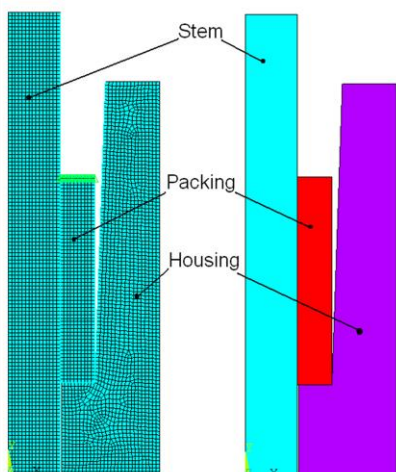


Figure 3. FE model of a packed stuffing box with a tapered housing

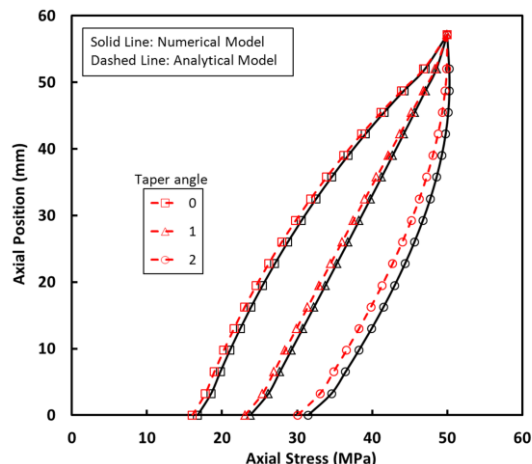


Figure 4. Effect of tapered housing on the axial stress distribution with 6 PTFE packing rings

$$\sigma_z = -e^{-\beta z} \left( \sigma_D + B \left( \delta + \frac{\alpha}{\beta} \right) \right) + B \left( \delta + \alpha \left( \frac{1}{\beta} - z \right) \right) \quad (27)$$

#### IV. FEM Simulations

The numerical simulation is conducted using finite element method with ANSYS software [15] on a stuffed yearn box having a 1-1/8 in. diameter stem used with two sets of 3/8 in. packing rings made of Teflon (PTFE) and flexible graphite (FG). The finite element model shown in Fig. 3 was generated to simulate the effect of gap on the distribution of axial stress in the packing rings. Based on the fact that the configuration of packed stuffing-box and the applied loads and boundary conditions are all axisymmetric, hence the simulation can be reduced to a 2D model. For this model, 4-node axisymmetric elements were used to generate the mesh of the three components of the packed stuffing-box namely stem, housing and packing rings. Contact elements are used to simulate the relative movement between the packing ring and the stem and

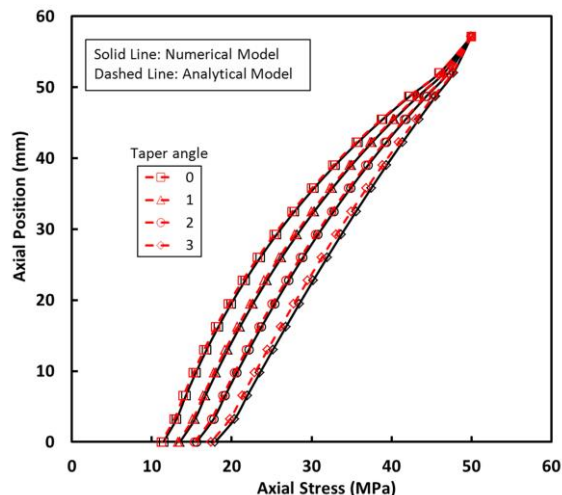


Figure 5. Effect of tapered housing on the axial stress distribution with 6 FG packing rings



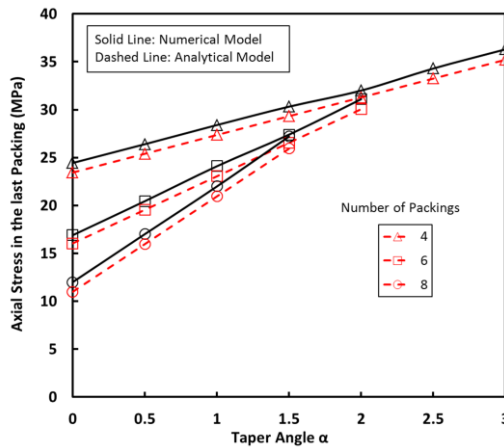


Figure 6. Minimum axial stress in the bottom PTFE packing ring for a 50 MPa gland stress vs housing taper angle

the packing ring and the housing. The geometry and the physical properties used by both the analytical and numerical approaches are presented in the Table 1. The values regarding to the mechanical properties of packing rings are cited from [9]. To simplify the analysis all the materials have a linear elastic behavior.

## v. Results and Discussion

In order to demonstrate the effect of the tapered housing on the distribution of the packing axial stress, four simulations with taper angles ranging from 0 to 2 degrees were conducted using the two set of packing rings. The distributions of the axial stress through the packing length of the stuffed yarn box used with PTFE and FG packing materials are given in Figs. 4 and 5 respectively. It can be said that a good agreement exist between the results of the FE numerical simulations and the analytical model which gives an indication on the accuracy of the developed analytical model. In addition, it can be deduced that by increasing the taper angle, there is a significant change in the magnitude of the axial stress transferred from the gland to the last packing which is in direct contact with confined fluid. This is particularly true with the PTFE packing material with a 2 degrees angle where the stress variation is less pronounced.

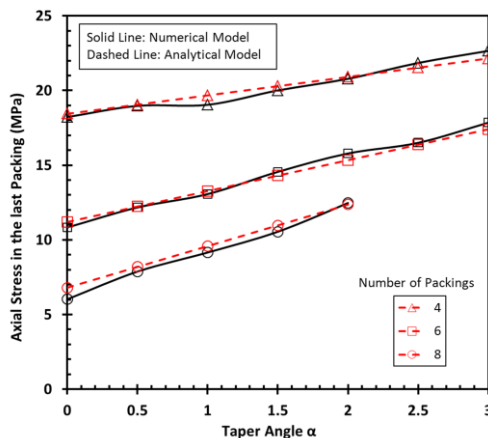


Figure 7. Minimum axial stress in the bottom FG packing ring for a 50 MPa gland stress vs housing taper angle

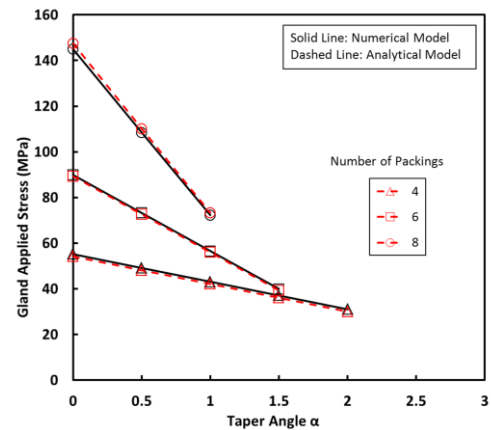


Figure 8 Required gland stress to achieve a threshold axial stress of 20 MPa in the bottom PTFE packing ring

In a packed stuffing-box, the axial stress on the bottom packing ring that is directly in contact with the pressurized fluid determines the leakage level. A common practice used as a design criterion is to adjust the gland stress so that the minimum stress on the bottom packing equals a multiple of the fluid pressure inside the valve usually between 1 and 2 times the pressure. In Figs. 6 and 7, the variation of the axial contact stress of the bottom packing ring as a function of the housing taper angle is given for a set of 4, 6 and 8 packing rings subjected to 50 MPa gland stress. All figures show that the minimum stress is increased when the taper angle increases. The increase in the minimum axial stress is more important in the case of PTFE as compared to FG indicating that former requires less gland stress to seal. However it should be noted that there is a limit on the amount of the taper angle that can be introduced. Depending on the packing material, the gland stress and the number of packing the tapered angle produces a gap at the top packing ring that is in contact with the gland that cannot be closed. This is way the plots of Figs. 6 and 7 stops after a certain taper angle.

Figures 8 and 9 show a required gland stress to produce a minimum axial stress of 20 MPa for the two packing materials. Depending on the gland stress the taper angle can be selected to achieve this 20 MPa minimum stress criteria; a

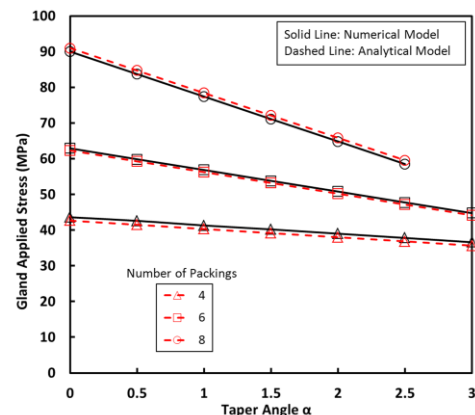


Figure 9. Required gland stress to achieve a threshold axial stress of 25 MPa in the bottom FG packing ring

value taken arbitrarily for illustration. However such graphs can be reproduced for different minimum stresses depending on the application tightness criteria. These are important graphs based on 3 parameters gland stress, threshold axial stresses and taper angle. From the design stand point selecting two parameters can lead to the third one.

### Nomenclature

$d$	Internal diameter of the packing
$d_h$	Internal diameter of the housing
$D$	External diameter of the packing
$D_h$	External diameter of the housing
$E$	Young's modulus of housing and stem
$E_p$	Young's modulus of the Packing rings
$H$	Height of the one packing ring
$H_h$	Height of the housing
$K$	Lateral pressure coefficient
$K_i$	Internal lateral pressure coefficient
$K_o$	External lateral pressure coefficient
$\sigma_D$	Applied axial load by gland
$\sigma_r$	Radial stress
$\sigma_z$	Axial stress
$\sigma_\theta$	Circumferential stress
$\mu_i$	Friction between packing and stem
$\mu_o$	Friction between packing and housing
$\nu$	Poisson ratio of the packing
$\nu_h$	Poisson ratio of the housing
$\nu_s$	Poisson ratio of the stem

### References

[1] Hoyes, J.R. and Thrope, L.C., 1995, "The influence of constructional details on the performance of valve stem seals," 1st European Conference on Controlling fugitive Emissions from Valves, Pumps and Flanges, European Sealing Association (FSA), Antwerp, Belgium.

[2] ANSI/ISA-93.00.0., 1999, Standard Method for the Evaluation of External leakage of Manual and Automated On-Off Valves. American National Standards Institute.

[3] API-622, 2011, Type testing of process valve packing for fugitive emissions. American Petroleum Industries.

[4] API-624,2011, Type Testing of Rising Stem Valves Equipped with Flexible Graphite Packing for Fugitive Emissions, American Petroleum Industries.

[5] ISO-15848-1, 2006, Industrial Valves-measurement, test and qualification procedure for fugitive emissions-Part:1 Classification System and qualification Procedures for type testing of valves. International Standards Organization.

[6] Denny, D. F., 1957, "A Force Analyse of the Stuffing-Box Seal," BHRA Research Report No. 550, the Fluid Engineering Centre, Harlow, England.

[7] Ochonski, W., 1988, "Radial Stress Distribution and Friction Forces in a Soft-Packed Stuffing-Box Seal," Tribol. Int., **21**(1), pp. 31–38.

[8] Klenk, T., Kockelmann, H., Roos, E., Bartonicek, J., and Schoeckle, F., 1999, "Characteristics and Testing Techniques for Stuffing Box Packings," ASME, Pressure Vessels and Piping Conference, Vol. 382, pp. 135–143.

[9] Diany M. and Bouzid A., 2011, "An experimental-Numerical Procedure for Stuffing Box Packing Characterisation and Leak Tests," ASME Journal of Tribology, **133** (1), 012201-8

[10] Pengyun S, Kuangmin C and Zongyun, D., "Theoretical analysis of the lateral pressure coefficients in a soft-packed stuffing-box seal," Tribology International, **30**(10), pp. 759–65.

[11] Diany M. and Bouzid A., 2009, "Analytical Evaluation of Stresses and Displacements of Stuffing-Box Packing Based on a Flexibility Analysis," Tribology International, **42** (6), pp. 980-986.

[12] Diany M. and Bouzid A., 2009, "Short Term Relaxation Modelling of Valve Stem Packings," ASME Journal of Tribology, **131** (3), 032201-6.

[13] Lamé G., 1837, "Sur les surfaces isothermes dans les corps homogènes en équilibre de température," J. Math. Pures Appl. , **2**, pp. 147–188.

#### About Author (s):



Mehdi Kazeminia is a research assistant and PhD student at the École de Technologie Supérieure of Montreal. He has BSc and MSc degrees in Mechanical Engineering from Iran. After working for a few years in industry he is now pursuing his Ph.D studies in joint sealing at the Static and dynamic Sealing Laboratory of ÉTS in Canada.



Hakim A. Bouzid is a full professor at the École de Technologie Supérieure of Montreal. He is a fellow of ASME; he is an expert in pressure vessels, stress analysis, and gasketed and non gasketed joints. Dr Bouzid is the director of the Static and dynamic Sealing Laboratory. He is a member of the ASME SWG on Bolted flange connection and a member of the ASTM F03 comity on Gaskets. He has authored over 200 papers on bolted joints and seals.