

Modeling and structure parameters design of the silicon resonant Pressure sensor

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Abstract— Based on the sensing mechanism of microsensor, a simulation model of a practical silicon beam resonator attached to an E-type round diaphragm and used for measuring pressure is established. The relationship between the basic natural frequency of the beam resonator and the pressure is calculated, analyzed and investigated. As a microsensor FEM is used to study some important simulation results on the vibration features of the beam resonators. Based on the differential output signals, a set of optimum parameters of the proposed sensing unit is determined.

Keywords: Microsensor, beam, E-type round diaphragm, pressure, finite-element method.

I. Introduction

During the past decade, many experts in sensor technology have noticed the rapid development in silicon resonant sensors. The silicon microstructure resonant sensors are noted for the advantages of a generalized resonant sensor, such as direct digital output (without A/D conversion), long-term stability, low hysteric and high repeatability. Other advantages of silicon material are excellent mechanical properties; high strength; free from mechanical hysteric; suit ability to batch processing at low cost; and the compatibility of mechanical and electrical properties. Meanwhile, the dynamic characteristics of silicon resonant sensors are much better than those of conventional ones, due to their high working frequency[1].

In addition, the temperature characteristics of silicon resonant sensors are much better than those of another important silicon sensor, the piezoresistive sensor. It is much easier to interface them with a microprocessor to develop smart or 'intelligent' sensors or sensor systems due to their unique operating principle based on the relationship between the natural frequency and measured signal instead of the silicon piezoresistive effect. Recently, it has been reported that some prototypes, with different structure and configurations, and even few batch products of silicon resonant sensors have come into being for measuring pressure, differential pressure or force. They have been developed and manufactured in Japan, USA, Netherlands and Britain. Although the properties of these silicon resonant sensors have not reached the designed

criteria, they could be used widely in many technical fields in the near future.

II. Sensor Structure and Sensing Mechanism

Fig. 1 shows the structure of a silicon resonant sensor for measuring pressure. The preliminary sensing unit is an E-type round diaphragm. The pressure being measured acts perpendicularly on the lower plane of the diaphragm and yields radial and circumferential stresses. The final sensing unit is a beam attached to the upper plane of the diaphragm. Thickness of the beam h should be much less than the thickness of the diaphragm H .

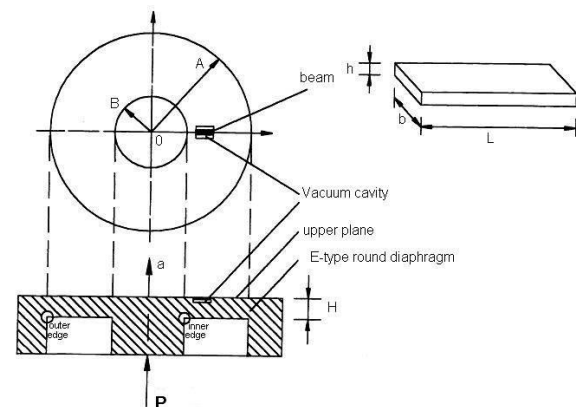


Fig. 1 Sensing structure of the sensor

Based on the above structural features, an appropriate initial stress is applied along the axial direction of the beam, which is identical with the radial stress of the E-type round diaphragm at the same place. Because the principal vibration of the beam is along its normal direction (direction of the beam's thickness) and the above initial stress is applied along the axial direction of the beam (direction of the beam's length), thus the natural frequency of the beam is varied with the radial stress on upper plane of the diaphragm.

Therefore, the applied pressure will be measured via the change of the natural frequency of the beam. The advantage of the above sensing structure is that the beam resonator is of very high Q-factor because it can be packaged within a vacuum housing, thereby, the sensor's quality will be much better[2].

III. Stresses on the Upper Plane of the E-type Round Diaphragm

The differential equation of the round diaphragm loaded applied by a Pressure at center, can be written as follows [3]

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dW}{dr} \right) \right] = \frac{Q}{D} \quad (1)$$

$$D = \frac{EH^3}{12(1-\mu^2)}, \quad Q = \frac{rP}{2}$$

where W is the normal displacement of the E-type round diaphragm, D is the flexural rigidity of the diaphragm, Q is the shearing force at the circumference of radius r of the diaphragm, P is the applied Pressure (≥ 0), and E, μ are Young's modulus and poisson ratio of the sensing structure, respectively.

The boundary conditions at the inner and outer edges of the E-type diaphragm are as follows

$$\begin{cases} r=B & \frac{dW}{dr} = 0 \\ r=A & W \frac{dw}{dr} = 0 \end{cases} \quad (2)$$

where r radial coordinate of the E-type round diaphragm in polar.

From Eqs. (1) and (2), the normal displacement of the E-type round diaphragm W can be obtained. Then the radial stress $\sigma_r(r)$ and circumferential stress $\sigma_\theta(r)$ of the E-type round diaphragm can be obtained as follows [4]

$$\begin{cases} \sigma_r(r) = \frac{-3PA^2}{8H^2} \left[(3+\mu)R^2 - (1+\mu)(K^2+1) - \frac{(1-\mu)K^2}{R^2} \right] \\ \sigma_\theta(r) = \frac{-3PA^2}{8H^2} \left[(1+3\mu)R^2 - (1+\mu)(K^2+1) + \frac{(1-\mu)K^2}{R^2} \right] \end{cases} \quad (3)$$

$$R = \frac{r}{A}, \quad K = \frac{B}{A}$$

IV. Finite Element Model of the Beam

Fig. 2 shows the mathematical model of the beam. The axial and normal vibrating displacements $u(s, z, t)$ and $w(s, z, t)$ of the beam at an arbitrary point in Cartesian coordinates can be written as

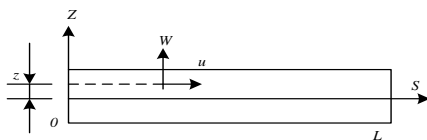


Fig. 2 The mathematical model of the beam

$$\begin{cases} u(s, z, t) = -z \frac{dw(s)}{ds} \cos \omega t \\ w(s, t) = w(s) \cos \omega t \end{cases} \quad (4)$$

where s, z are axial and normal coordinate of the beam in Cartesian coordinates, ω [rad/s], $w(s)$ are natural frequency

and its corresponding vibrating shape along the axial direction of the beam.

Then the finite element equation is as follows

$$(K - \omega^2 M)a = 0 \quad (5)$$

where K-assembly stiffness Matrix, M-assembly Mass Matrix, a the assembly nodal vector, consisting of all a_j .

For the actual structural features of Fig. 2, the boundary conditions of the beam are as follows

$$\begin{cases} s=0: & w(s) = w'(s) = 0 \\ s=L: & w(s) = w'(s) = 0 \end{cases} \quad (6)$$

Eq.(5) is the FEM model for the above microstructure silicon resonant Pressure sensor shown in Fig. 1. From Eqs. (5) and (6) the natural frequencies and their corresponding vibrating shapes can be obtained.

V. Calculations and Simulations

In this work, the main investigations are the varying laws of the Pressure -frequency relationship for the beam resonator as the thickness H, inner radius B for the E-type round diaphragm and the corresponding length of the beam L are varied. In order to get some generalized results of the pressure -frequency relationship for the beam resonator, some related parameters are selected as follows:

Reference value for the outer radius of the E-type round diaphragm is $A = 3.4 \times 10^{-3}m$. Reference values for the width and thickness of the beam are $b = 10 \times 10^{-5}m$ and $h = 1 \times 10^{-5}m$. The sensor is made of silicon, $E = 1.3 \times 10^{11}Pa$, $\rho = 2.33 \times 10^3 kg/m^3$, $\mu = 0.17$. Moreover, the total element number of the beam N is 17 for FEM calculation in this work.

A) Qualitative Investigation

A positive and a negative stress ranges are available along the radial direction on the upper surface of the E-type round diaphragm. As the beam is located at the positive stress range, its frequency increases with the applied Pressure due to the beam in stretching state; and as the beam is located at the negative stress range, the frequency decreases with the applied Pressure due to the beam in compression state. As one part of the beam is located at the positive stress range of the diaphragm, while another at the negative stress range, then one part of the beam is in stretching state, the other is in compression state. Thus the natural frequency characteristics of the beam depend on the overall effects of stretching and compression, which is much worse for the beam resonator's operating state. Based on the above feature, the differential output scheme can be realized. In addition, as the beam 2 is located at the inner edge $r \in (B \sim B+L)$ (Fig. 3) in the positive stress range, while the beam 1 is located at the outer edge $r \in (A-L \sim L)$ in the negative stress range, the sensitivity of the output frequency to the applied Pressure reaches the highest obtainable value.

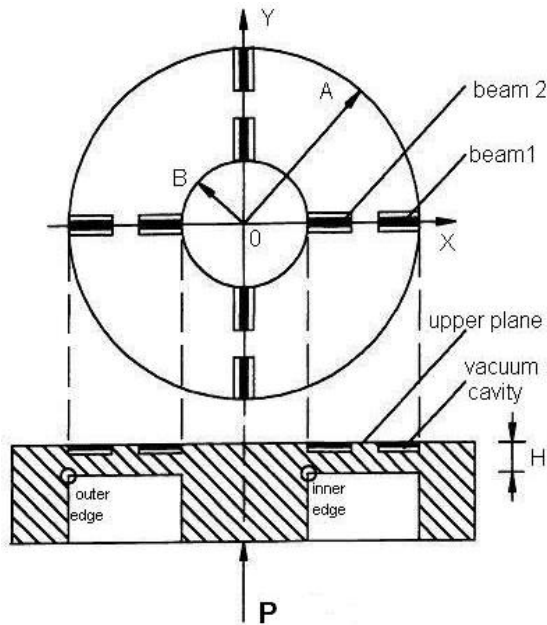


Fig. 3 The differential output scheme (four pairs of beams)

Moreover, frequency changes of the above two beams are opposite, one is positive, the other is negative. Therefore, the differential output scheme can be realized, and the measured Pressure can be determined by the frequency difference between beam 1 and beam 2. On the other hand, the influence of the temperature on the frequency difference can be reduced, because its influence on frequency of beam 1 and beam 2 are almost the same.

In order to eliminate mechanical coupling and mutual interference, the thickness of the beam h should be much less than the thickness of the diaphragm H , and the distance $A-B-2L$ between beam 1 and beam 2 could not be selected too small compared with length of the beam L . On the other hand, in order to make use of the effective annular sensing area of the diaphragm, the distance $A-B-2L$ could not be selected too big. In this paper, the relationship for A , B and L are selected as

$$A-B=2.5L \quad (7)$$

In such a case, the length of the beam L is dependent on the inner radius of the E-type round diaphragm B .

B) Investigations of the Frequency Pressure Relationship

Define $f(P)$, $f(0)$ as the basic natural frequency of the beam for pressure P and for pressure $P=0$; $\Delta f = f(P) - f(0)$ as the variation of the basic natural frequency of the beam within $(0, P)$ and $\beta = [f(P) - f(0)] / f(0) [\%]$ as the relative variation or the sensitivity of the basic natural frequency for the beam within $(0, P)$ interval.

1- Location of Beam on the Upper Plane of the E-Type Diaphragm.

Table I gives the beam's relationships between frequency and measured Pressure within $(0, 10^5)$ Pa, as the beam being located at different positions.

Table II gives the frequency variation and sensitivity of the beam corresponding to Table 1.

TABLE I THE FREQUENCY OF BEAM WITHIN $(0, 10^5)$ PA AS $B=1.4$ MM, $L=1.04$ MM, $H=0.15$ MM

Pressure $\times 10^5$ Pa	A. Location of the beam				
	(1.4,2.44)	(1.79,2.83)	(2.18,3.22)	(2.57,3.61)	(2.96,4)
0.0	709.813	709.813	709.813	709.813	709.813
0.1	733.529	722.733	711.723	700.025	687.046
0.2	756.510	735.237	713.616	689.866	663.181
0.3	778.435	747.544	715.391	679.650	638.349
0.4	799.793	759.480	717.054	669.018	612.187
0.5	820.569	771.291	718.821	658.229	584.803
0.6	840.645	782.763	720.408	646.989	555.753
0.7	860.180	794.075	721.932	635.694	524.982
0.8	879.247	805.204	723.436	623.998	491.937
0.9	897.836	816.081	724.813	611.944	456.055
1.0	916.049	826.739	726.177	599.546	417.015

TABLE II THE VARIATION FREQUENCY AND SENSITIVITY OF BEAM WITHIN $(0, 10^5)$ PA AS $B=1.4$ MM & $L=1.04$ MM, $H=0.15$ MM

POSITION	(1.4,2.44)	(1.79,2.83)	(2.18,3.22)	(2.57,3.61)	(2.96,4)
RELATIVE FREQ. VARIATION	29.1 %	16.4 %	2.3 %	-15.5 %	-41.3 %
VARIATION (KHZ)	206.23	116.92	16.36	-110.26	-292.97

From above Tables, some results can be obtained as follows:

As the beam is located at different positions of the upper plane of the E-type diaphragm along its radial direction, the sensitivity of the beam resonator is different. The best locations for the beam resonator is at the inner edge and the outer edge for the E-type round diaphragm, where the beam's sensitivity reaches the biggest.

The beam resonator is defined as beam1 as it is located at the outer edge, and defined as beam 2 as located at the inner edge.

2-THE THICKNESS OF THE E-TYPE DIAPHRAGM

Fig. 4 shows the relationships between the frequency of beam1 and the pressure for the different thicknesses of the diaphragm H . Fig. 5 show the relationships between the Frequency of beam 2 and the pressure for the different thicknesses of the diaphragm H .

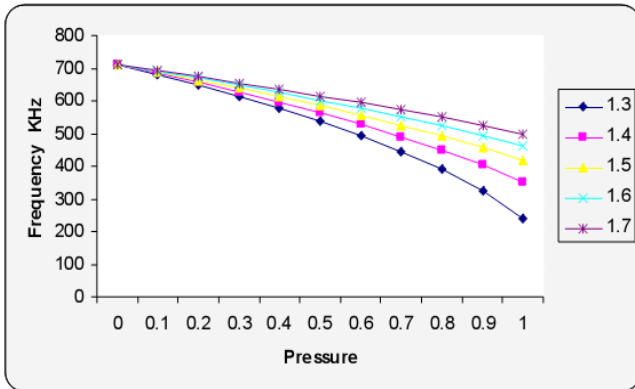


Fig. 4 The relationships between the frequency of beam 1 and the pressure

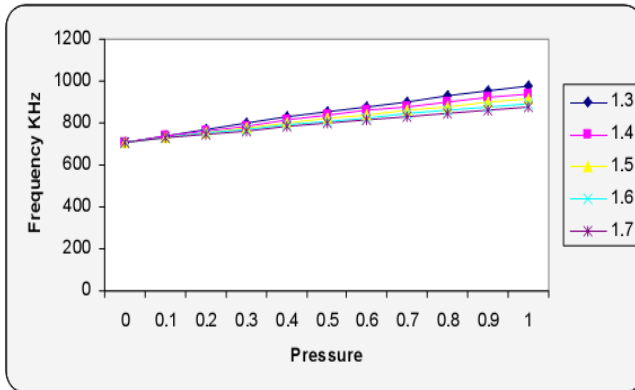


Fig. 5 The relationships between the frequency of beam 2 and the pressure

TABLE III The variation frequency(kHz) of Beam1 and Beam2 within $(0,10^5)$ Pa for the different H,B,L

H ($\times 10^{-4}$ m)	$(B \times 10^{-3}$ m, $L \times 10^{-3}$ m)			
	Beam 1 (1.35,1.06)		Beam 2 (1.35,1.06)	
1.3	-500.206	-469.157	263.574	263.531
1.4	-372.354	-359.624	232.437	232.297
1.5	-300.317	-292.789	206.508	206.236
1.6	-251.103	-245.825	184.463	184.179
1.7	-214.453	-210.66	165.818	165.431

Table IV The Sensitivity of Beam1 and Beam 2 within $(0,10^5)$ Pa for different H, B & L

H ($\times 10^{-4}$ m)	$(B \times 10^{-3}$ m, $L \times 10^{-3}$ m)			
	Beam 1 (1.35,1.06)		Beam 2 (1.35,1.06)	
1.3	-73.2 %	-66.1 %	38.6 %	37.1 %
1.4	-54.5 %	-50.7 %	34.0 %	32.7 %
1.5	-43.9 %	-41.3 %	30.2 %	29.1 %
1.6	-36.9 %	-34.6 %	25.9 %	25.9 %
1.7	-31.4 %	-29.7 %	24.3 %	23.3 %

From the above Tables and Figures, we can get the following results:

The relative frequency variation (sensitivity) is increased as the thickness of the E-type round diaphragm H decreasing for beam1 and beam 2.

The relative frequency variation (sensitivity) is increased as the inner radius of the E-type round diaphragm B decreasing or the length of the beam increasing for beam 1 and beam 2.

Then, the length of the beam L should be increased or the inner radius of the diaphragm B should be decreased or the diaphragm thickness H should be decreased if you want to increase the relative frequency variation (sensitivity) of the beam resonator within a measurement range of pressure. Moreover, as the measurement range is narrow, the relative frequency variation (sensitivity) of the beam resonator should be high; as the measurement range is wider, the relative frequency variation (sensitivity) of the beam resonator should be narrower.

VI. Conclusions

The modeling and simulation for a novel resonant silicon microsensor are carried out in this work. The elementary sensing component of the microsensor is the E-type round diaphragm, and its final sensing component is the beam resonator which is attached to the E-type round diaphragm. The main results obtained here are as follows:

- (1) The sensitivity of the basic natural frequency to measure Pressure for the beam resonator will be increased as:
 - *-the thickness of the diaphragm H being decreased.
 - *-the inner radius of the diaphragm B being decreased, as the outer radius keeping constant.
 - *-the length of the beam being increased.
- (2) It is best selection for a pair of beams (beam 1 and beam 2) to be located at the outer and inner edges of the E-type round diaphragm, respectively. The output of the sensor is the differential frequency between beam 1 and beam 2. Thus the temperature influence can be reduced.
- (3) Based on the differential output scheme of the resonant sensor and some related criteria, a set of appropriate parameters for the above sensing structure of the sensor is determined for measuring the Pressure within $(0,10^5)$ Pa.
 - *-The outer radius of the E-type round diaphragm A= 3.4 mm;
 - *-The inner radius of the E-type round diaphragm B=1.4 mm;
 - *-The thickness of the E-type round diaphragm H=0.17 mm;
 - *-The length of the beam L=1.04 mm;
 - *-The width of the beam b=0.1mm;
 - *-The thickness of the beam h=0.01 mm.

In the above case, the frequency range is (709.813~350.189) kHz for Beam 1, which is located at the outer edge; and (709.813~942.110) kHz for Beam 2, which is located at the inner edge. The differential frequency output

between Beam 2 and Beam 1, related to the basic natural frequency as the zero Pressure is 53.0% [23.3% - (-29.7%)].

Some results obtained here are of significance for designing and developing the microstructure silicon resonant Pressure sensor.

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