

Richards Model to Determine the Height Growth Rate of Mung Bean Plant

Wayan Surya Wardhani and Muslikhah

Abstract— The aim of this research is applying sigmoid growth model to determine the mung bean height growth pattern, in order to find out the important period during the plant growth. The data used was secondary data of mung bean plant with nitrogen fertilizer treatment. Data were collected for three months in a week interval. The analysis result showed that Richards model is suitable in describing the plant growth pattern, as the adjusted coefficient of determination is more than 99%. The Richards growth curve and the trend of the growth rate were also drawn. Analyzing the trend, it was found that the maximum growth rate happens at about five weeks after planting and at that time the plant height is about 33 cm. It means that the important period is at about or just before week five after planting.

Keywords— Growth analysis, Mung bean, Richards model

Introduction

There are several research to forecast response variable and growth analysis using deterministic time series growth models. For example, a research about determining the best growth pattern and forecasting of chick pea and mung bean pulses production in Bangladesh [8]. It is also said that the models are appropriate if the annual growth rate is constant overtime. Otherwise, the models are not suitable to fit the time series data. For that kind of data, forecasting is usually done by using a model called Autoregressive Moving Average (ARIMA). Other research to determine the trend of plant growth are done using sigmoid growth models. As plant growth can not be studied and illustrated easily as the elements are very complex, researchers try to explain what happen in growth process. One way to study plant growth is to establish a model of growth whose parameters could be interpreted from biological point of view such as Richard growth model; but it is not used as a prediction tool.

It was said that the quartic regression is the best fit model describing the growth of mung bean with and without fertilizer [1]. Other researcher in plant breeding field, it is common to use analysis of variance for the data analysis.

Such a research [7] about the effect of nitrogen on the yield of mung bean at varying plant growth; the result showed that nitrogen level are correlated with plant height. The mung bean plant showed better growth when treated with Nitrogen of 60 and 80 kg ha⁻¹.

It was also said that with the height at about 60 cm. Other analysis result showed that higher nitrogen yield higher crop growth rate.

Using anova, it was proved that the treatment of the fertilizer significantly affect the growth of mung bean plants. Other researcher also states that fertilization significantly affect plant height, number of green bean stalks, pods per stem, number of seeds per pod and long pods; this agrees with the research results states that higher plants formed many branches that affect the branches then the pod crop production was higher than that of short crops or have a little branch.

Richards (1969) in [6] stated that the growth model is an appropriate tool in characterizing changes in plant height with increasing time due to the growth model; it is also may provide a satisfactory description of the shape of the growth curve and the process of growth. In general, more growth model is mechanistic rather than empirical. A mechanistic model is usually developed from assumptions about the type of growth, by differential equations and then solve the differential equation to obtain growth models. On the other hand, an empirical model was chosen to approach empirically a mechanistic model of the unknown. Typically, the empirical model in the form of a polynomial with a certain order [9].

Richards growth model would be applied to model the height growth of mung bean plant and determine the height growth rate of the plant, under an assumption that there is no interplant competition as it was measured individually. According to [4], the model could be expressed as:

$$h(t) = \frac{A}{(1 + be^{-kt})^{\frac{1}{v}}} \quad (1)$$

where $h(t)$ is the plant height at the age of all t , A is the asymptote value that means maximum height to be achieved by the plant, b is the scale of growth, v is allometric constant parameters and k is the growth rate parameters that describes the constant rate when the plant reaches its maximum, t is age. As the model is non linear, Levenberg-Marquardt method is used to estimate the parameters of the Richards model [9]

The absolute growth rate could be found by the integration of the model that is expressed by:

$$\frac{dh(t)}{dt} = \frac{1}{v} Akbe^{-kt} (1 + be^{-kt})^{-(\frac{1}{v}+1)} \quad (2)$$

while the growth rate is expressed as

$$\begin{aligned} \frac{dh(t)}{dt} \frac{1}{h(t)} &= \frac{1}{v} Akbe^{-kt} (1 + be^{-kt})^{-(\frac{1}{v}+1)} \frac{(1+be^{-kt})^{\frac{1}{v}}}{A} \\ &= \frac{kb e^{-kt}}{v(1+be^{-kt})} \quad (3) \end{aligned}$$

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The inflection point of the model is very important from biological point of view as it showed the fastest growth rate during the growth period; the maximum growth rate is reached when $\frac{d^2 h(t)}{dt^2} = 0$. Finding out when the inflection point happens namely abscis $t^* = -\frac{1}{k} \ln\left(\frac{v}{b}\right)$, then substitute to (1), we'll find the ordinate showing the plant height that is $h(t)^* = \frac{A}{(1+v)^{1/v}}$; then the coordinate is $(t^*, h(t)^*)$.

This study aims to model the mung bean plant height in mathematic formula. Moreover, it aims to determine when the maximum growth rate, the effective time of growth and how high the maximum attainable mung beans is. Through this research it is expected to know whether this model fit the data well and it can be used as mung bean database and a reference for the breeder to make efforts to maximize the plant growth.

Material and method

A. Data source

This research used a secondary height data of mung bean plant individually that obtained from the experiment in a glass house. The treatments given are of 0, 5, 10 and 15 grams nitrogen per six kilograms standard growing media with three replications. Based on the result of analysis of variance, the treatment of 10 grams Nitrogen fertilizer yields the highest plant, and we use it to determine the trend of its growth.

B. Data analysis

Richards function has four parameter to be estimated. There are methods to estimate the unknown parameters; in this case we use the iterative method of Levenberg-Marquardt that widely use in computing a non-linear least square. The next step was testing the assumptions underlying the analysis. The accuracy of the model could be verified by using adjusted R^2 .

Result and Discussion

As the research has three replication in each treatment, then the scatter diagram of mung bean plant height (cm) over time (Week After Planting=WAP) is described as follows :

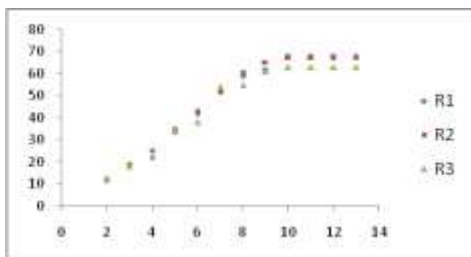


Figure.1. Plant height (cm) against time (WAP)

The plant growth tended to increase at the age of 4-9 weeks after planting (WAP) and tend to be slow and constant in the last week, at about age 10-13 WAP. To estimate the parameter by iterative method yield Richards model as:

$$h(t) = \frac{67.807}{[1 + 11.122e^{-0.496t}]^{1/0.933}}$$

The maximum plant height estimated to be 67.807 cm; the change in plant height is 11.122 cm WAP⁻¹, the parameters of the growth rates are 0.496 and 0.933 for allometric constant. It is stated that the parameter v in Richards model varies; Richards growth model would be similar to that of Monomolecular model if v = -1; if v = 0 the Gompertz model is more alike. When v = 1 it similar to Logistic growth model and Von Bertalanffy when $-1/3 \leq v < 0$ (France and Thornley, 2007). Based on the analysis result it is seen that the v value is 0.933 that is close to 1, which means the growth of mung bean plants could be explained by Logistic growth model.

Assumptions of homogeneity of variance has a very important role in the estimation of parameters and could be tested by J. Szroeter method [3]; the test statistic is 0.52 that is less than the critical value; it means that the assumption is fulfilled. The assumption about residual normality is tested by Kolmogorov-Smirnov method and it is fulfilled as p-value is 0.95. The adjusted R^2 of 99.23% indicated that Richard model fit the data well.

It was stated that the absolute growth rate generally increases rapidly at early time; from the first derivative of the model, the absolute growth rate is seen as the curve obtained below. It is inline with the research result of [7] which shows that the mung bean crop growth rate in certain level of nitrogen fertilizer increased up to 30 days after emergence and decreased afterwards.

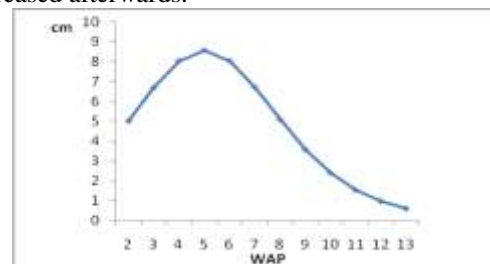


Figure 2. Absolute growth rate versus time

Based on Figure 2, the absolute growth rate increased at the beginning of the week to achieve maximum growth rate then continued to decrease with increasing age of the plant. The absolute growth rate illustrated the change in the increase of plant height of each observation. Based on the second derivative, the maximum growth rate occurred at about week five and the growth rate was 8.59 cm/WAP. After week five, the growth rate was decreasing to a constant. Mathematically, the coordinate was found to be (4.997, 33.46); it means that maximum growth rate occurs at about the end of week four and the plant height at that time is about 33.46 cm.

The relative growth rate was different from absolute growth rate as it was relatively high values at the beginning and then continued to decrease with increasing plant age. Before the

decreased growth rate, the growth rate had a constant height in the early growth and was accompanied by a rapid decline with age. Relative growth rate the curve obtained as follows.

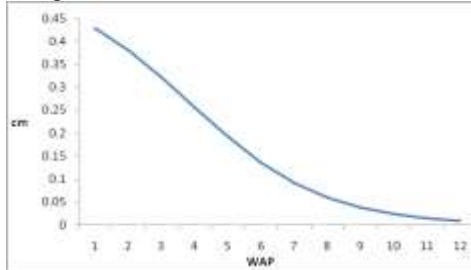


Figure 3. Relative Growth Rate versus time

Relative growth rate illustrates the changes in the rate of plant growth; it declined to near zero point as the plant goes to maturity. Time around the maximum growth rate occurs is called effective time of growth where the special treatment is needed.

Conclusion

Based on the statistical approach, the Richards model is suitable in describing the growth of mung bean plant as it gives a high adjusted determination coefficient. Moreover, the graph of absolute growth rate is quadratic and right-skewed. The maximum height growth rate occurs at about five weeks after planting. At that time the plant height is about half of the maximum height potential, inline with that of Logistic model theoretically.

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References

[1] Ansari, F. 2011. Mung bean susan (1) <http://www.slideshare.net/faisal93/mung-bean-susan-1> . Access April 2014 .

[2] Chong, E.K.P and S.H. Zak. 2001. *An Introduction to Optimization*. Second Edition. John Wiley and Sons, Inc.: New York. http://kia.ac.ir/~optimization/index_files/An_Introduction_to_Optimization_Chong_and_Zak.pdf Access Januari 2014

[3] Dielman, T.E. 1991. *Applied Regression Analysis for Business and Econometrics*. PWS-KENT Publishing Company. Boston

[4] Fekedulegn, D., M. Siurain and J.J Colbert. 1999. *Parameter Estimation of Nonlinear Growth Models in Forestry*. *Silva Fennica* 33(4) : 327-336. <http://www.metla.fi/silvafennica/full/sf33/sf334327.pdf> Tanggal. Access: December 2013

[5] France, J. and J.H.M. Thornley. 2007. *Mathematical Models in Agriculture Quantitative Methods for the Plant, Animal and Ecological Sciences*. Second Edition. CABI North American Office Publishing. Cambridge. USA

[6] Kakar, K. M., Tariq, M., Tareen, M. R., and Ullah, W. 2002. *Shoot Growth Curve of Wheat (Triticum aestivum L.) Receiving Different Levels of Boron and Iron*. *Pakistan Journal of Agronomy* 1(1) : 47-48, 2002 <http://docsdrive.com/pdfs/ansinet/ja/2002/47-48.pdf>. Access: December 2013

[7] Mian, M.A.K and J. Hossain, 2014. *Nitrogen Level and Physiological Basis of Yield of Mungbean at Varying Plant Population in High Ganges River Flood Plain Soil of Bangladesh*. *Pakistan Journal of Biological Sciences*, 17: 925-930. Access June 2014

[8] Rahman, N.M.R; M.A. Aziz; M.M Rahman; N Muhammad. 2013. *Modeling on Grass Pea and Mung Bean Pulse Production in Bangladesh Using ARIMA Model*. *IOSR Journal of Agriculture and Veterinary Science*. Volume 6 issue-1. Access May 2014

[9] Rawlings, J. O., Pantula, S. G. and D. A. Dickey. 1998. *Applied regression analysis*. 2nd Ed. Spring-Verlag New York, Inc. New York, NY. 657p. http://web.nchu.edu.tw/~numerical/course992/ra/Applied_Regression_Analysis_A_Research_Tool.pdf. Access :January 2013