

# Seismic Vulnerability Assessment of Tall Buildings: Spectrum-based Nonlinear Dynamic Analysis

Don Y. B. Ho and J. S. Kuang

**Abstract**—In this paper, the development of a spectrum-based nonlinear dynamic analysis, named as incremental substitution procedure (ISP), is presented for the seismic vulnerability assessment of tall buildings. The proposed procedure is derived from a special case of nonlinear time-history analysis, where the seismic responses of a structure can be captured from a series of equivalent linear systems and ordinary response spectra. Case studies are conducted and the results show very good agreement with the field observations. Conclusions are drawn and recommendations are made for the reconstruction work of earthquake-stricken areas and the vulnerability assessment of buildings in the nearby regions.

**Keywords**—seismic vulnerability assessment, tall buildings

## I. Introduction

In strong earthquakes, the design practice on mixed construction types of buildings and the soft storey mechanism are suspected to be the major deficiencies responsible to the extensive destruction during earthquake attacks. Historically, the high vulnerability of structures with soft storeys are well demonstrated by severe damage of the Olive View Hospital building during the San Fernando earthquake in 1971 and collapse of a 7-storey apartment house during the Hygoken-Nanbu earthquake in 1995. However, there is still lack of theoretical models for the design of high-rise buildings with such special arrangement. Thus, there is an urgent need for the development of practical methods of analysis to assess the vulnerability of buildings, in particular those incorporating special arrangements, such as soft storeys.

In this paper, a spectrum-based nonlinear dynamic analysis, named as incremental substitution procedure (ISP), is presented as an improved alternative over the conventional strength-based and nonlinear static approaches [1] and proposed as a suitable method of analysis for practical seismic evaluation of tall buildings. The simplicity and robustness of the proposed procedure enable the future codification for practical use of seismic vulnerability assessment. The proposed ISP may ease the urgent need for the development of a comprehensive and robust seismic vulnerability and integrity assessment guidance for practising engineers.

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## II. Theory

### A. Rule of hysteresis

In nonlinear time-history analysis of reinforced concrete structures, the origin-oriented hysteresis rule is often assumed [2], as shown in Figure 1, where the stiffness degradation is taken into account. This hysteresis also provides a simple rule to simulate the pinching effect, so that less energy can be dissipated through post-yield oscillation. The restoring force  $\mathbf{F}$  in a member, by the origin-oriented rule, should be modelled as a linear function of the deformation when the force state is inside the yield surface, expressed by

$$\mathbf{F} = \bar{\mathbf{F}} + \bar{\mathbf{K}}\mathbf{x} \quad (1)$$

where  $\bar{\mathbf{F}}$  is the mean force vector,  $\bar{\mathbf{K}}$  is the modified stiffness, and  $\mathbf{x}$  is the displacement.

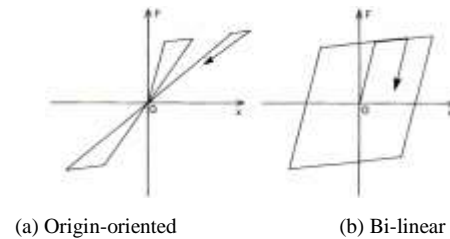


Figure 1. Simple hysteresis functions.

For multi-axial coupled members, best estimate of the force-displacement response inside the yield surface may be a scaled initial stiffness, which is scaled according to the strain energy in a member. In the case of a force-displacement system, the modified stiffness is given by

$$\bar{\mathbf{K}} = \mathbf{K}_{\text{ini}} \times \frac{\mathbf{x}^T \mathbf{F}}{\mathbf{x}^T \mathbf{K}_{\text{ini}} \mathbf{x}} \quad (2)$$

where  $\mathbf{K}_{\text{ini}}$  is the undamaged initial stiffness. The use of a scalar stiffness modification is equivalent to assuming a scalar damage variable in isotropic damage mechanics, where different material behaviours are considered to be affected in the same way by changing the surface density. To determine if yielding condition is reached, an internal variable, the ductility measure  $\mu^*$  should be introduced,

$$\mu^* = \int_V \boldsymbol{\varepsilon}^T \mathbf{E}_{\text{ini}} \boldsymbol{\varepsilon} dv / \int_V \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}_n dv \quad (3)$$

where  $E_{ini}$  is the undamaged initial material elastic modulus,  $\epsilon$  is the material strain,  $\sigma_n$  is the nominal rigid plastic strength, and  $V$  is the volume of the member. By assuming a uniform isotropic scalar damage variable for all degrees of freedom, the yield condition can approximately be indicated as the internal variable reaches its historical maximum. More importantly, the quotient in the equation takes a physical measure of the displacement ductility, as shown in Figure 2.

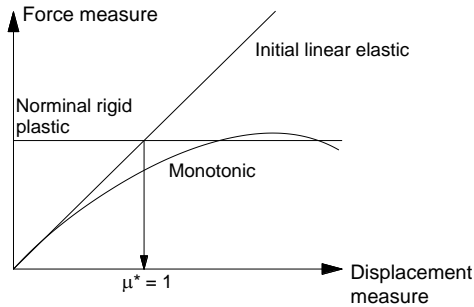


Figure 2. Physical meaning of internal variable  $\mu^*$ .

When yield condition occurs, the restoring force in the member may be calculated as that in the case under monotonic loading. Thus, the required information in the hysteresis rule is not more than that required in a monotonic-load analysis, but allowing for simulating the effect of pinching.

### B. Loading history

In inelastic analysis, the load history is always important. The continuous adjustment of the equivalent linear system should be conducted in time domain. When a stronger shock is encountered, the equivalent linear system is updated and then the analysis continues to the next time step. For practical consideration, the excitation is assumed as a repeating and increasing, hypothetical load history, as shown in Figure 3. The basic combined hypothetical load history shown in Figure 3 shares an identical response spectrum as the natural ground excitation time-history, which can be found at the last segment of the hypothetical load history. In other words, the response spectrum at any time instant shares the same shape.

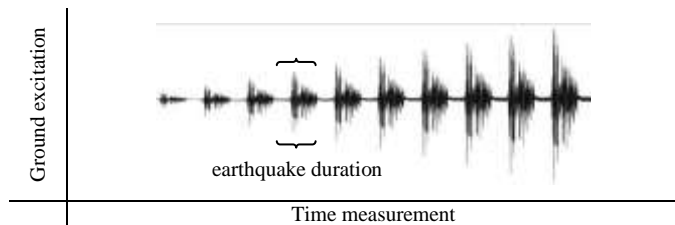


Figure 3. Basic hypothetical loading history in ISP.

Taking into account of the non-steady state nature of earthquake excitations, damage evolution spectra can be introduced. As the damage is considered as irrecoverable, the absolute maximum spectral displacement within a specific elapsed time gives a measure of damage encountered at a particular instant. With the concept of the internal variable  $\mu^*$ , the normalised damage measure is defined by

$$R_d = r_t / r_{max} \quad (4)$$

where  $r_t$  and  $r_{max}$  are the maximum absolute seismic responses of a SDOF system within a given elapsed time  $t$  and the duration of the earthquake, respectively, as shown in Figure 4.

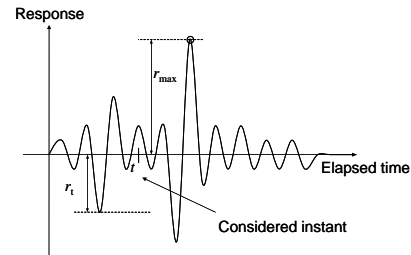


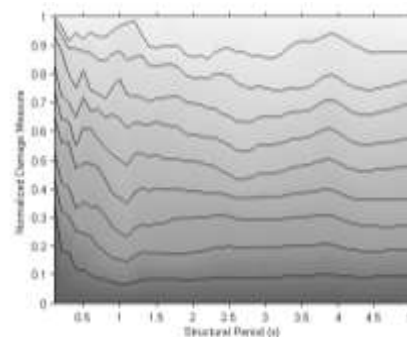
Figure 4. Maximum seismic responses within a given elapsed time and duration of an earthquake.

The sequence of damage (i.e. the sequence of plastic hinge formation) in a tall building under seismic excitation is important, while the exact times for different levels of the damage need not to be known. Hence, for a direct comparison among different ground motions, the elapsed time axis can be rescaled to match with the mean normalised damage within a range of the structural periods. This normalised damage can be expressed by the arithmetic mean of seismic responses with different natural periods at the same time instant,

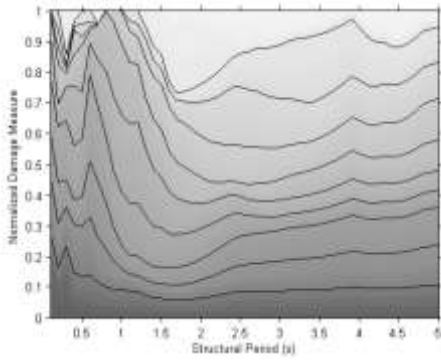
$$\bar{R}_d = \frac{1}{T_{spm}} \int_0^{T_{spm}} \frac{r_t}{r_{max}} dT \quad (5)$$

where  $T_{spm}$  is the maximum considered structural period. Thus, the mean normalised damage measure may be considered as the seismic load level during an earthquake.

The damage evolutions are then plotted in a conventional two-dimensional plane and presented by a set of damage evolution spectra relating to different time instants, as shown in Figure 5, where the time instant difference between two adjacent spectra corresponds to a 10% change of the damage level defined by Equation 5. In Figure 5, zones enclosed by the damage evolution spectra have dark and light colours to represent the damage at early and later stages of the seismic duration, respectively.



(a) For 14 selected ground motions



(b) For 4 near-fault ground motions

Figure 5. Averaged seismic damage evolution spectra.

The averaged damage evolution spectra of 14 selected ground motions shown in Table 1 are presented in Figure 5(a). It is seen that seismic responses, thus the damage levels, of structures with different periods are approximately the same at the same time instant, except those with a very short period, say less than 0.5s, where severe damage may come early. The early growth of damage of short-period structures shows an in-phase response to short-period vibrations under earthquakes, which is often ignored in most of the modal superposition rules. Figure 5(b) shows the averaged damage evolution spectra from 4 near-fault earthquakes given in Table 1, all of which have a distance to strike-slip fault rupture of about 1 km. It is seen from Figure 5(b) that the average damage evolution spectra contains the approximate L shapes, and the damage growth rates are different when structural periods are shorter or longer than 1.5s. The damage growth rate of structures with a natural period shorter than 1.5s is generally higher than that for those with a natural period longer than 1.5s.

It can be shown that the basic hypothetical load history also represents an averaged damage evolution process shared in most earthquake records unless near-fault effects are necessary to be considered.

TABLE I. SELECTED EARTHQUAKE EXCITATIONS [PEER 2000]

Earthquake	Record/ Component	Distance to rupture (km)	PGA(g)
Loma Prieta 1989	LOMAP/A07000	46.5	0.156
Kern County 1952	KERN/TAF111	42.0	0.178
Imperial Valley 1979	IMPVALL/H- CPE237	23.5	0.157
Landers 1992	LANDERS/DSP090	22.5	0.154
Duzce, Turkey 1999	DUZCE/BOL000	17.6	0.728
Duzce, Turkey 1999	DUZCE/BOL090	17.6	0.822
Northridge 1994	NORTHR/BVA285	16.3	0.165
Victoria, Mexico 1980	VICT/CPE045	14.4	0.621
Imperial Valley 1979	IMPVALL/H- BCR230	2.6	0.775
Kobe 1995	KOBE/TAZ000	1.2	0.693
Kobe 1995	KOBE/TAZ090	1.2	0.694
Landers 1992	LANDERS/LCN000	1.1	0.785
Kobe 1995	KOBE/KJM000	0.6	0.821
Morgan Hill 1984	MORGAN/CYC195	0.1	0.711

### C. Spectrum-based approximation

The basic ISP makes the assumption of steady state vibration and eventually fine-tunes the analysis results through correction on the hysteresis damping. In the adjustment of the effective stiffness, the energy equation may be used in the analysis with response spectrum. But minor modification should be incorporated as an average value of different states of response, given by

$$\bar{\mathbf{K}} = \mathbf{K}_{ini} \frac{\mathbf{x}_+^T \mathbf{F}_{mon}^+ + \mathbf{x}_-^T \mathbf{F}_{mon}^-}{\mathbf{x}_+^T \mathbf{K}_{ini} \mathbf{x}_+ + \mathbf{x}_-^T \mathbf{K}_{ini} \mathbf{x}_-} \quad (6)$$

where  $\mathbf{x}_+$  and  $\mathbf{x}_-$  are the positive and negative displacement maxima pair on the displacement envelop, as illustrated in Figure 6, and  $\mathbf{F}_{mon}^+$  and  $\mathbf{F}_{mon}^-$  are the restoring forces at the displacements of  $\mathbf{x}_+$  and  $\mathbf{x}_-$ , respectively, on the monotonic force envelop (i.e. the backbone curve/surface), which approximates the yield surface of a member. This proposed modification aims to seek for a physically reasonable equivalent stiffness.

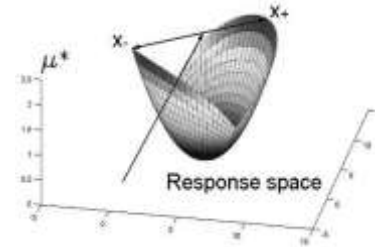


Figure 6. Ductility maxima pair.

The resulted linear function can be used to estimate the correct restoring force on the response envelop. After obtaining the modified stiffness, the mean force vector may be defined as the average deviation between the monotonic restoring and the substituted linear force pair at the displacement ductility maxima pair,

$$\bar{\mathbf{F}} = (\mathbf{F}_{mon}^+ + \mathbf{F}_{mon}^-) / 2 - \bar{\mathbf{K}} \bar{\mathbf{x}} \quad (7)$$

where  $\bar{\mathbf{x}}$  is the displacement resulted from the pseudo-static force. Such modification on the hysteresis is applicable at any levels. For practical applications, it is suitable for applying the modification to the level of structural members, and the displacement envelop can be obtained through modal superposition rules. In this application, a scaled sum method is adopted, which is often used in directional combination for elastic analysis. The discrete displacement envelop can be presented as a set of displacement vectors, defined by

$$\mathbf{D} = \left\{ \pm q_i \phi_i + 0.3 \sum_{j \neq i} (\pm q_j \phi_j) + \bar{\mathbf{x}} \right\} \quad (8)$$

where  $\phi$  and  $q$  are the mode shape and modal displacement, and  $i$  and  $j$  are the modes considered. For the searching of displacement maxima pair, the objective function is defined as the average value of the displacement ductility measure,  $\mu^*$ , of a particular displacement pair on the displacement envelope,

$$\bar{\mu}^* = \frac{\mu_p^* + \mu_n^*}{2} \quad (9)$$

where  $\mu_p^*$  and  $\mu_n^*$  are the ductility measures at displacement pair  $\mathbf{x}_+$  and  $\mathbf{x}_-$  respectively. Thus  $\pm q_i \phi_i + \bar{x}$  will always be the displacement maxima pair for fundamental mode dominated structures for example. If the maximum value of the average internal variable  $\bar{\mu}^*$  in iteration is larger than the corresponding historical maximum, the member is considered to have further damage, and the effective stiffness will be modified.

With a suitable modification on the linear force displacement relationship, the equation of motion comes

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \bar{\mathbf{F}} + \bar{\mathbf{K}}\mathbf{x} = -\mathbf{M}\mathbf{r}\ddot{x}_g + \mathbf{F}_{\text{ext}} \quad (10)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{r}$  are the mass, damping and directional vector, respectively, and  $\mathbf{F}_{\text{ext}}$  is the external static load, such as the gravity load. With sufficient number of segments, the equation of motion within each segment can essentially be solved by conventional multi-modal linear spectral analysis.

On the other hand, the determination of convergence and modal damping may follow the method given in the original substitution procedure [3]. Energy dissipation of each mode is assumed to be independent of each other and proportional to ductility demand of each member. If the equivalent viscous damping  $\xi_{is}$  from each member is assumed to be a function of the ductility measure,  $\bar{\mu}_{is}^*$ , the modal member ductility measure and modal damping are computed by

$$\bar{\mu}_{is}^* = \frac{\mu_{isp}^* + \mu_{isn}^*}{2} \quad (11a)$$

$$\xi_i = \frac{\sum_s \xi_{is} \phi_{is}^T \bar{\mathbf{K}}_s \phi_{is}}{\phi_i^T \bar{\mathbf{K}} \phi_i} \quad (11b)$$

where  $i$  and  $s$  are the mode and member number respectively. The relationship between member damping and ductility measure should be calibrated directly from quasi-static tests [4] or be standardised.

A flow chart for summarising the proposed procedure is presented in Figure 7. Based on the proposed incremental substitution procedure, a computer programme has been successfully developed and linked with the commercial structural analysis software SAP 2000. The current version supports material yielding and damage on frame and shell fibre elements, brittle shear failure, P- $\Delta$  effects,

multidirectional excitations, near-fault effects, coupled vibration, automatic step size correction and multiprocessor optimisation. The proposed method is readily available for tall buildings that are suspected to be vulnerable under unexpected seismic attacks.

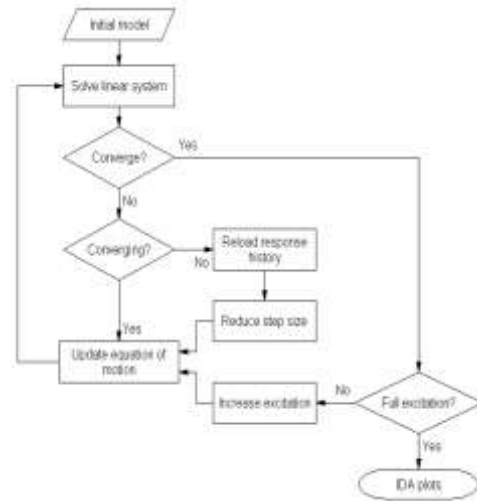


Figure 7. A flow chart of ISP.

### III. Case Studies

Cases studies are conducted on two simplified 7-storey with 27m(W)×11m(B)×21m(H), confined masonry buildings supported by RC frames on ground floor, as shown in Figure 8. The two buildings are mostly identical except that four pieces of RC shear walls are introduced on the ground floor in one of the buildings. To best represent the real situation, floor beams, non-structural partition walls and stair-cases are also included into the model. Both buildings are designed to resist earthquakes with spectral acceleration of 0.08g (Intensity VII) elastically. The response spectrum in major directions of the structures are simplified and given in Figure 9.



Figure 8. FEM model of example buildings.

The undamaged fundamental periods of the buildings with and without RC shear walls on the ground floor are 0.31s and 0.33s, respectively. Both buildings are survived from earthquakes with PGA = 0.22g, which is considered as the PGA value under a major earthquake event with return period of 2500 years for Intensity 7 regions. However, the failure modes are significantly different.

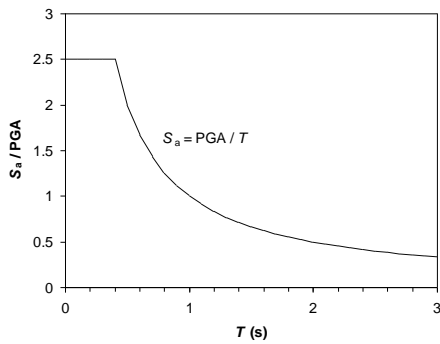


Figure 9. Spectral acceleration.

Figure 10 shows the maximum inter-storey drift profiles of the two buildings at  $PGA = 0.36g$ . With such intensity, both buildings experience a column sway mechanism with maximum inter-storey drift of higher than 2%; thus the buildings would be close to collapse. This agrees with the actual situation that most masonry buildings are heavily damaged or collapsed in a column sway mechanism, in regions where the  $PGA$  level is about  $0.4g$  to  $0.6g$ . However, for the building with RC shear walls on the ground floor, the maximum inter-storey drift is found on the third floor, instead of the ground floor in the case without shear walls.

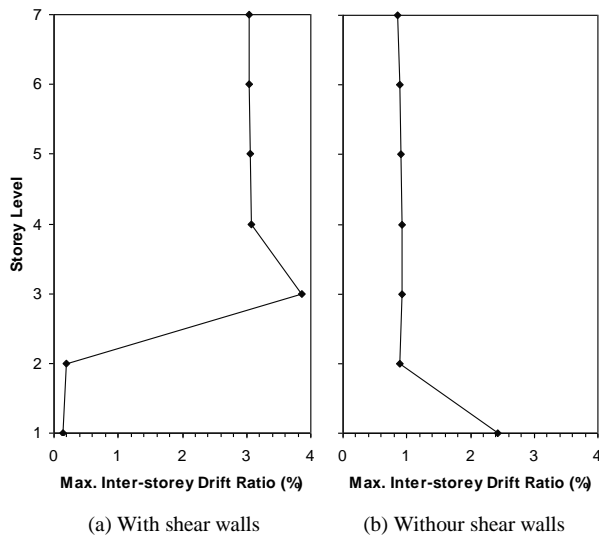


Figure 10. Maximum inter-storey drift at  $0.36g$ .

## iv. Conclusions

In this paper, an incremental substitution procedure (ISP) is developed based on the spectrum-based nonlinear dynamic analysis, where seismic responses of a structure can be captured from a series of equivalent linear systems and ordinary response spectra. The proposed ISP is proposed as a suitable technical tool for practical seismic vulnerability assessment of tall buildings. In the analysis, the effects of change in vibration patterns, contributions of higher vibration mode, interaction of damage resulting from multi-directional

ground excitation, and non-steady state characteristics on seismic responses of the structures are considered. With introducing an internal variable and hypothetical load history, convergence is guaranteed with any excitation.

Case studies are conducted using the ISP. The analysis results show very good agreement with the field observations. The proposed procedure can conveniently be implemented with existing commercial finite element structural analysis software. With its low demand on required information, high flexibility, transparency, accuracy and computation efficiency, the ISP provides a practical and effective, yet accurate, mean of conducting seismic design and vulnerability assessment for tall and special buildings.

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