# Robust active queue management for uncertain TCP network congestion control in Internet

Chao-Jung Cheng

Abstract—This paper presents an active queue management (AQM) method of uncertain TCP network using the robust control. A numerical simulation for AQM of uncertain TCP network is conducted to justify the validity and feasibility of the proposed approach which is robust to high load variations in steady-state behavior.

Keywords—Active queue management, Robust control, Congestion control

## I. Introduction

Congestion control is surely a mean to enhance network performances. Active queue management (AQM), which is a proactive approach, has been proposed as a solution to these problems [1]. One of the most well-known AQM policies is random early detection (RED). Although RED is an effective TCP congestion control [2], it can induce network instability and major traffic disruption if not properly configured. One of RED's main weaknesses is that the average queue size varies with the level of congestion and with the parameter settings. Moreover, the average queuing delay from RED is sensitive to the traffic load and also to parameters setting and is therefore not predictable in advance. Delay is a major component of the quality of service delivered to the customers, so network operators would naturally like to roughly estimate the average delays in their congested routers.

The main difference between the different AQMs proposed in the literature is the method used to compute the probability of dropping/marking. Some AQMs use the queue length as information, and the dropping/marking probability is computed either as a function of its average value as in RED, or depending on its instantiated value as with the controllers [3].

Our concern is to synthesize an AQM which explicitly takes account of both model uncertainties and time-varying delay. A fluid-based model of the dynamics of the TCP and

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RED was developed by the theory of stochastic equations [4]. Hence, different congestion control strategies have been developed to raise the performance of Internet [5-6]. This paper is to design a feedback controller which guarantees the queue length stability and robustness against uncertain parameters. Based on the Lyapunov-Krasovskii functional approach, the robust controller is designed such that the TCP/AQM model can be stabilized asymptotically.

The rest of the paper is organized as follows. In Section II, our robust TCP/AQM congestion control scheme is derived for TCP/AQM model. Section III shows the parameter settings and simulation results. The conclusions are given in Section IV.

## II. Robust Congestion Control Scheme

The TCP/AQM fluid model is described by a set of nonlinear ordinary differential equations (ODEs) as follows:

$$\dot{w}(t) = \frac{1}{\tau(t)} - \frac{w(t)}{2} \frac{w(t - \tau(t))}{\tau(t - \tau(t))} p(t - \tau(t))$$

$$\dot{q}(t) = \begin{cases} -C(t) + \frac{N(t)}{\tau(t)} w(t), & q(t) > 0 \\ \max\left\{0, -C(t) + \frac{N(t)}{\tau(t)} w(t)\right\}, & q(t) = 0 \end{cases}$$
(1)

where w(t) is the TCP/AQM window size (packets), q(t) is the instantaneous queue length (in packets), C(t) is the link capacity (packets/s),  $\tau(t) = q(t)/C(t) + T_p$  is the round trip time (RTT),  $T_p$  is the propagation delay, p(t) is the probability of packet dropout, N(t) is the number of TCP sessions. To analyze its local stability around an operating point  $(w_0, q_0, p_0)$ , the transmission RTT and the number of TCP sessions are assumed constants, i.e.  $\tau(t) = \tau_0$  and N(t) = N. Furthermore, the operating point  $(w_0, q_0, p_0)$  satisfies  $\dot{w}(t) = 0$ and  $\dot{q}(t) = 0$ , i.e.  $w_0 = \tau_0 C / N$  and  $p_0 = 2 / w_0^2$ . Additionally, the probability of packet mark or drop p(t) is taken as system input in TCP/AQM and modeled as a stochastic process, which implies the dynamics of congestion window size and queue length are stochastic processes. Therefore, the linearization of Eq.(1) at the equilibrium is derived as the following stochastic linear time-delayed equation with parameter uncertainties:



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We use a state feedback to make the closed-loop system asymptotically stable, i.e., the control signal is given by u(t) = Kx(t) where *K* is the control gain matrix to be designed. The control system with structured uncertainties can be transform into the following model without structured uncertainties through some manipulations.

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)) + B_d u(t - \tau(t)) + J\hat{\phi}(t)$$

$$\bar{y}(t) = H_1 x(t) + H_2 x(t - \tau(t)) + H_3 u(t - \tau(t))$$
(3)

where  $x(t) = \begin{bmatrix} \delta w(t) & \delta q(t) \end{bmatrix}^T$  and A,  $A_d$ ,  $B_d$  are system parameters.  $H = \begin{bmatrix} 0 & 1 \end{bmatrix}$ ,  $0 \le \tau(t) < \infty$ ,  $\dot{\tau}(t) \le \tau_m < 1$ ,  $J = \gamma \lambda M$ ,  $\overline{y}(t) = \begin{bmatrix} y(t) & \hat{y}(t) \end{bmatrix}^T$ ,  $H_1 = \begin{bmatrix} H & N_1 / \lambda \end{bmatrix}^T$ ,  $H_2 = \begin{bmatrix} 0 & N_2 / \lambda \end{bmatrix}^T$ ,  $H_3 = \begin{bmatrix} 0 & N_3 / \lambda \end{bmatrix}^T$ ,  $\lambda$  is a positive real number,  $\hat{\phi}(t)$  and  $\hat{y}(t)$  are the additional

disturbance input and controlled signal output, respectively.

The congestion control scheme design is stated in the following theorem.

**Theorem 1.** Consider the TCP/AQM model as shown in (1), if there exist positive definite matrices P,  $R_1$ ,  $R_2$  and matrix K such that the following matrix inequality

$$\begin{bmatrix} \tilde{A} + H_1^T H_1 & PA_d + H_1^T H_2 & PB_d + H_1^T H_3 & PJ \\ * & H_2^T H_2 - \tilde{R}_1 & H_2^T H_3 & 0 \\ * & * & H_3^T H_3 - \tilde{R}_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0$$
(4)

holds, then the TCP/AQM model is asymptotically stable by a state feedback controller u(t) = Kx(t) and meet the disturbance rejection performance index:

 $\Psi(\gamma) = \int_0^\infty \left( \overline{y}^T(t) \overline{y}(t) - \gamma^2 \hat{\phi}^T(t) \hat{\phi}(t) \right) dt < 0 \text{ for a given scalar } \gamma > 0,$ where  $\widetilde{A} = A^T P + PA + R_1 + K^T R_2 K$ ,  $\widetilde{R}_1 = (1 - \tau_m) R_1$ ,  $\widetilde{R}_2 = (1 - \tau_m) R_2$ . The symbol \* is used in some matrix expressions to denote the transposed elements in the symmetric positions of a matrix. **Proof:** 

One can consider a Lyapunov function of the form

$$V = x(t)^{T} P x(t) + \int_{t-\tau(t)}^{t} x(s)^{T} (R_{1} + K^{T} R_{2} K) x(s) ds$$
(5)

Then calculating the derivative of V yields  

$$\dot{V} = \dot{x}(t)^T Px(t) + x(t)^T P\dot{x}(t) + x(t)^T R_1 x(t) + x(t)^T K^T R_2 K x(t)$$
  
 $-(1-\dot{\tau}(t))x(t-\tau(t))^T R_1 x(t-\tau(t)) - (1-\dot{\tau}(t))x(t-\tau(t))^T K^T R_2 K x(t-\tau(t))$   
 $\leq \dot{x}(t)^T Px(t) + x(t)^T P\dot{x}(t) + x(t)^T R_1 x(t) + x(t)^T K^T R_2 K x(t)$   
 $-(1-\tau_m)x(t-\tau(t))^T R_1 x(t-\tau(t)) - (1-\tau_m)x(t-\tau(t))^T K^T R_2 K x(t-\tau(t))$   
 $= \dot{x}(t)^T Px(t) + x(t)^T P\dot{x}(t) + x(t)^T R_1 x(t) + x(t)^T K^T R_2 K x(t)$   
 $-x(t-\tau(t))^T \tilde{R}_1 x(t-\tau(t)) - x(t-\tau(t))^T K^T \tilde{R}_2 K x(t-\tau(t))$ 

$$= \begin{bmatrix} X(t) \\ x(t-\tau(t)) \\ Kx(t-\tau(t)) \end{bmatrix} \begin{bmatrix} A & TA_d & TB_d \\ A_d^T P & -\tilde{R}_1 & 0 \\ B_d^T P & 0 & -\tilde{R}_2 \end{bmatrix} \begin{bmatrix} X(t) \\ x(t-\tau(t)) \\ Kx(t-\tau(t)) \end{bmatrix}$$
$$+ \left(J\hat{\phi}(t)\right)^T Px(t) + x(t)^T P\left(J\hat{\phi}(t)\right) = \dot{V}_a + \left(J\hat{\phi}(t)\right)^T Px(t) + x(t)^T P\left(J\hat{\phi}(t)\right)$$

where 
$$\tilde{A} = A^T P + PA + R_1 + K^T R_2 K$$
 (7)  
If system (5) is stable and meet the disturbance rejection  
performance index, then  
 $\Phi(\gamma) \leq \int_0^\infty (\bar{y}^T(t)\bar{y}(t) - \gamma^2 \hat{\phi}^T(t)\hat{\phi}(t) + \dot{V}) dt$   
 $\leq \int_0^\infty (\bar{y}^T(t)\bar{y}(t) - \gamma^2 \hat{\phi}^T(t)\hat{\phi}(t) + \dot{V}_a + (J\hat{\phi}(t))^T Px(t) + x(t)^T P(J\hat{\phi}(t))) dt$ 
(8)

$$\overline{y}^{T}(t)\overline{y}(t)$$

 $\left( H_1 x(t) + H_2 x(t-\tau(t)) + H_3 u(t-\tau(t)) \right)^T \left( H_1 x(t) + H_2 x(t-\tau(t)) + H_3 u(t-\tau(t)) \right)$ 

then we have

$$\Phi(\gamma) \leq \int_0^\infty \left[ \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ Kx(t-\tau(t)) \\ \hat{\phi}(t) \end{bmatrix}^T \Omega \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ Kx(t-\tau(t)) \\ \hat{\phi}(t) \end{bmatrix} \right] dt < 0$$
(10)

(9)

where

$$\Omega = \begin{bmatrix} \tilde{A} + H_1^T H_1 & PA_d + H_1^T H_2 & PB_d + H_1^T H_3 & PJ \\ * & H_2^T H_2 - \tilde{R}_1 & H_2^T H_3 & 0 \\ * & * & H_3^T H_3 - \tilde{R}_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$
(11)

Hence Theorem 1 is proofed.

#### **III.** Simulation Results

For comparison purposes, the choosing of the parameters is based on the existing result [7] and the dumbbell simulation topology [8] as shown in Fig. 1 with multiple TCP connections, N=100. Connections share a single bottleneck link with a capacity of 10Mbps, i.e.,  $C_0=1250$ (packets/second), the RTT was 0.2, and TCP-Reno is used as the transport agent. Suppose the desired queue size is  $q_d = q_0 = 150$ , the desired window size is  $w_0 = 2.5$  packets. Considering system uncertainties as follow:  $J = \gamma \lambda M$ ,  $\hat{\phi}(t) = 0.5 * rand(t) , \quad \hat{y}(t) = (N_1 / \lambda)x(t) + (N_2 / \lambda + N_3 K / \lambda)x(t - \tau(t)) ,$ where  $\gamma = 2, \lambda = 1, M = \begin{bmatrix} 2 & 60 \end{bmatrix}^T$ ,  $N_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, N_2 = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, N_3 = 40$ , The function rand(t) generates random number between 0 and 1. There exist positive definite matrices  $P, R_1, R_2$  and matrix K such that the matrix inequality (4) holds, furthermore, fig. 2 shows the Queue length response of the corresponding closeloop system with the feedback gain of the controller K=[0.05]0.004] which is obtained from Theorem 1 by LMI toolbox. It can be observed that the controller successfully maintained the desired queue size and stabilized the network at the operating point. Fig. 3 shows the corresponding queue evolutions obtained under the various AQM schemes compared to the PIP controller [8] in various networks conditions, even with a larger uncertainty and input disturbance. Although the transient responses of the PIP controller can maintain the desired queue size, serious overshoots occur in this case.



(6)

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## **IV.** Conclusions

Although numerous AQM schemes have been proposed to regulate a queue size close to a reference level, TCP's nonlinearity and time-varying stochastic properties result that most of them are incapable of adequately adapting to TCP network dynamics. Our paper presents the design of robust AQM control schemes in order to address the issue of variations of net-work parameters unavoidable in real networks, e.g. norm bounded uncertainties and time-varying time delay appearing both in the state and the control signal. The proposed robust controller achieves both good queue regulation and high link utilization and enhances TCP network control with uncertainty. A numerical simulation for AQM of uncertain TCP network is conducted to justify the validity and feasibility of the proposed approach. Moreover, the results show that it has fast response and also is robust to high load variations in steady-state behavior.

#### Acknowledgment

This research was supported by the National Science Council, Republic of China, under Grant NSC 102-2221-E-168-036.

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Fig.3. Queue length responses (in packets) using PIP, and the proposed AQM scheme with long propagation delay time.



Fig. 1. Dumbbell simulation topology.