# Interval Type-2 Fuzzy Logic Systems for CO<sub>2</sub> Emissions Forecasting: A Performance Comparison

Lazim Abdullah and Herrini Mohd Pauzi

Abstract—This paper presents an application of fuzzy logic systems to forecast carbon dioxide  $(CO_2)$  emissions. Interval type-2 TSK fuzzy logic system and Interval type-2 Mamdani fuzzy logic system were employed using a set of secondary data. Data of  $CO_2$  emissions in Malaysia and its related causal variables were collected over the period of 1971-2011. Six variables were considered as inputs to the models. Performances of the models were compared using error analysis. The analysis shows that  $CO_2$  emissions forecasting based on interval type-2 TSK fuzzy logic system is more reliable compared to Mamdani fuzzy logic system.

*Keywords*— carbon dioxide, error analysis, interval type-2 fuzzy logic system, forecasting

### I. Introduction

Environmental sustainability is one of the key components to sustain well-balanced development of a country. Analyses and forecast of gas emissions could be a significant step to create awareness among people on the importance of environmental sustainability. Carbon dioxide (CO<sub>2</sub>) is one of the gas emissions that give a great impact towards environment. In a nutshell, forecasting of CO<sub>2</sub> is important to provide information for a well planned development. Forecasting of CO<sub>2</sub> are generally associated with high level of uncertainties and complexity of data. Several methods have been proposed to find best model and forecasting systems in CO<sub>2</sub> emissions. The methods vary from traditional statistical methods to many types of intelligent systems. During the last decade, computational intelligence method has been attracted many researchers to develop the method in CO<sub>2</sub> forecasting. A considerable amount of literature have discussed the application of neural network, grey model, support vector machines and others in forecasting CO<sub>2</sub> emissions. Radojevic et al. [1] and Liu et al. [2] developed artificial neural network (ANN) for estimating CO2 emissions in Serbia and China respectively. They found that ANNs can be applied for modelling and simulating the greenhouse gas emissions as one of the environmental parameters of sustainable development. Other researchers also investigated the ability of the intelligent networks on forecasting CO2 and they discovered that the results supported the network [3]-[6]. Application of grey model also showed that the model is applicable and able to provide good forecasting result on  $CO_2$  emissions [7]-[10].

In an effort to deal with uncertain behaviour of CO<sub>2</sub> emissions, other fuzzy based methods are also applied in the forecasting CO<sub>2</sub> emissions. Rodrigues et al. [11] proposed a Neuro-Fuzzy Intelligent System – Adaptive Network based Fuzzy Inference System (ANFIS) for the annual forecast of greenhouse gases emissions (GHG) into the atmosphere. Hossain et al., [12] applied a hybrid approach of hidden Markov model (HMM) with fuzzy logic (HMM-fuzzy) to model hourly air pollution at a location related to its traffic volume and meteorological variable. Pauzi and Abdullah [13] proposed fuzzy rules based methods to forecast CO<sub>2</sub> emissions. All of these researches banked on the knowledge of type-1 fuzzy sets in development of the models. These models, however, have their own drawbacks owing to the tremendous uncertainties of environmental data. In addition, different types of causal variables data may exhibits complex relationship among them. Nonlinearity of the data give impact to the data as the data can be very difficult to model. The fuzzy model based on type-1 fuzzy sets is able to handle uncertainties and vagueness. However, the model has its own limitation in terms of how good it can handle uncertainties and vague is still disputable. The membership functions of type-1 fuzzy sets are often overly precise, requiring each element of the universal set be assigned a particular real number [14]. As to deal with these issues, many researchers now pay their attention in development of fuzzy model based on type-2

Type-2 fuzzy set that was introduced by Zadeh [15] strongly anticipated to provide additional design degrees of freedom in fuzzy logic systems (FLSs). Utilization of the systems can be very useful in situations where lots of uncertainties are present Type-2 fuzzy logic inference systems have been proposed as a new system for both forecasting and classification to handle all forms of uncertainties. The examples of the systems can be seen from Liang, Karnik and Mendel [16], Mendel [17], [18], and Mendel and John [19] and Olatunji et al. [20]. It has been widely applied in many range of areas, not only in forecasting area [21]-[27] but also in decision making [28]-[30], recognition [31],[32] and etc. General model of the type-2 fuzzy logic is computationally cost. It is due to its type-reduction that is very intensive [33]. This is why most literatures suggested interval type-2 FLS which is simpler than its general model to compute. It happened to be called interval type-2 FLS because its secondary membership functions are interval sets (either zero or one). Based on a given set of training data, we present an



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application of type-2 FLSs to forecast  $CO_2$  emissions. Two types of interval type-2 FLSs are applied in this study: interval type-2 singleton Mamdani and interval type-2 Takagi-Sugeno-Kang (TSK). In order to validate the forecasting results, the same set of data were simulated using type-1 FLS.

This paper is organized as follows. Section II describes the type-2 fuzzy set. Section III generally describes the type-2 fuzzy logic systems. Section IV discusses the design of CO<sub>2</sub> emission forecasting model by interval type-2 FLS. Variables in the forecasting and data collection are described in the next section. Section VI discusses the three error analyses in this paper. Comparative result of the forecasting performance between the two type of type-2 FLSs and type-1 FLS are presented in Section VII. Finally conclusion and suggestion for future work are made in Section VIII.

## II. Type-2 Fuzzy Set

The definitions of type-2 fuzzy set, interval type-2 fuzzy set and its operations are defined as follows.

#### A. Type-2 Fuzzy Set

Unlike type-1 fuzzy logic, type-2 fuzzy logic gives different definitions for membership functions and consists of its own set of operators. The properties of type-2 fuzzy logic can be derived from type-1 fuzzy logic by using these operators and extension principle.

Definition of type-1 fuzzy set is given by,

$$A = \{(x), \, \mu_A(x)\} \mid \forall x \in X \}, \qquad 0 \le \mu_A(x) \le 1$$
 (1)

The type-1 fuzzy set, for example, when x=x' has a crisp membership value  $\mu_A(x)$  and this is can be described by Fig. 1.

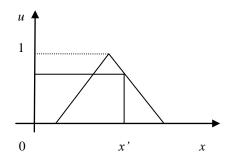


Fig. 1 Type-1 membership function

Meanwhile, the type-2 fuzzy set is defined as

$$\tilde{A} = \left\{ (x,u), \mu_{\tilde{A}}(x,u) \right\} \mid \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\},$$
 where  $0 \le \mu_{\tilde{A}}(x,u) \le 1$ 

As can be seen from the definition, a type 2 fuzzy set has an extra dimension, u, that associated with the membership value,  $\mu_{\widetilde{A}}(x)$ . Hence, unlike the type-1 model, the type-2 model's membership function is a fuzzy set. Example of type-2 fuzzy set is shown in Fig. 2.

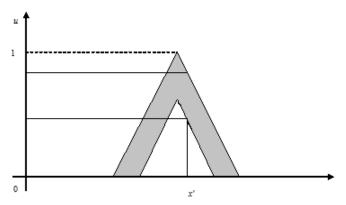


Fig.2.Type-2 fuzzy set membership function

This is means that u is a type-1 fuzzy set, with the membership function  $J_x$  in three-dimensional space. The  $J_x$  is also known as secondary membership of  $\tilde{A}$  is actually a vertical slice of  $\mu_{\tilde{A}}(x,u)$ . It expressed as

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x') = \int_{u \in Jx} f_{x}(u) / u \qquad \text{where} \\
0 \le f_{x}(u) \le 1$$
(3)

 $\int\limits_{u\in Jx} f_{x\cdot}(u) \,/\, u \text{ in (3) means that the type-2 fuzzy set has a}$  membership u associated with secondary grade  $f_{x\cdot}(u)$  for

x=x'. It is important to note that, the notation of  $\int$  is not an

integration operator but a symbol that represents the collection of all points of u in  $J_x$ . In addition, instead of division operator,  $f_x$ . (u) / u is actually defining that the grade corresponding to the membership value u is  $f_x$ . (u).

### B. Interval Type-2 Fuzzy Set

For interval type-2 fuzzy set, its secondary membership functions are defined by  $f_x(u)=1, \ \forall u \in J_x \subseteq [0,1]$ . For x=x', the primary membership value u can be represented as an interval [l,r].

As  $X' \in X$ , the prime notation can be dropped and then referring to  $\mu_{\tilde{A}}$  as a secondary membership function, the type-2 fuzzy set can be defined as :

$$\tilde{A} = \{(x), \, \mu_{\tilde{A}}(x) \mid \forall x \in X \}$$

$$\tag{4}$$

or

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x) / x = \int_{x \in X} \left[ \int_{u \in J_x} f_x(u) / u \right] / x \qquad J_x \subseteq [0,1]$$
 (5)



(2)

Primary membership of x is the domain of secondary membership function. In (5),  $J_x$  is the primary membership of x, where  $J_x \subseteq [0,1]$  for  $\forall x \in X$ .

### C. Fuzzy Set Operators

The definition of type-2 fuzzy sets paves the way to to build a FLS model. It is based on the fuzzy set and the choice of operators for its operations [34]. Operators for the operations on fuzzy sets are the basis of a type-2 FLS. Several set theoretic operation on the system can be view in [35]. Combination of two or more fuzzy sets by the operators will produce another fuzzy set. Three basics operators are join, meet and negation.

# III. type-2 fuzzy logic systems

Fuzzy inference system can be defined as a rule base system that implements fuzzy logic in data analysis. It is based on if-then rules, such that we can obtain the relation between input and output variables by these rules. IF (antecedent) and THEN (consequent) expression of fuzzy relation is determined by a fuzzy implication operator. The fuzzy relation defines the membership function of the rule. A group of fuzzy logic expression connected with fuzzy operators defines the antecedent meanwhile; consequent is an expression that assigns fuzzy values to output variables. Basically, the fuzzy inference system consists of a "rule base" containing fuzzy rules, a "database" defining membership functions of the fuzzy sets, and a reasoning mechanism which performs the inference procedure.

Generally, components of a type-2 FLS is just the same as the type-1; fuzzifier, fuzzy rules, inference, and defuzzifier. However, the type-2 FLS has one more addition on type-reducer. There are two types of fuzzifier; singleton and non-singleton. The singleton is the most commonly used in literatures because of its simplicity in computation. There are also two types of fuzzy inference systems. The Mamdani-type and Takagi-Sugeno-Kang (TSK) type are the types of fuzzy inference systems. The only significant difference between these two types is, the Mamdani uses fuzzy set in its consequent part and the TSK uses a function of the input variables in its consequent part.

Output set corresponding to each rule will be computed by the inference block when an input is applied to a type-1 FLS. After that, the defuzzifier will compute a crisp output from these rule output sets. The same concept applied to the type-2 FLS. Fig. 3 briefly describes the steps from each component of a type-2 rule based system. However, as mentioned before the type-2 model required a type-reducer. The type-reducer plays its function to reduce its model from type-2 to type-1 before it will be defuzzified to a crisp output.

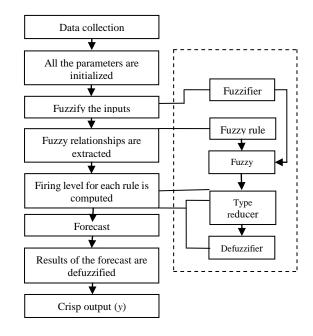


Fig. 3. Framework of the developed type-2 fuzzy system

There are a few type of membership function in interval type-2 FLS. For example, Gaussian, trapezoidal and triangular membership functions. The Gaussian is the most commonly applied as it only consider two parameters in its function. In our paper, Gaussian primary function with fixed standard deviation is considered. The membership function can be expressed as below:

$$\mu_{A}(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^{2}\right] \qquad m \in [m_{1}, m_{2}]$$
 (6)

An interval type-2 FLS has an upper and lower membership function. Corresponding to each value of  $m_1$  there will be different membership curve. The value of  $m_1$  and  $m_2$  is depending on historical data information. As

$$\exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right]$$
 in (6) is denoted by  $N(m,\sigma;x)$ , the upper membership function

$$\overline{\mu}_{\widetilde{A}}(x) = \begin{cases} N(m_1, \sigma; x) & x \le m1 \\ 1 & m_1 \le x \le m_2 \\ N(m_2, \sigma, x) & x \ge m_2 \end{cases}$$
(7)

And the lower membership function can be expressed as below,

$$\underline{\mu}_{\widetilde{A}}(x) = \begin{cases} N(m_2, \sigma, x) & x \le \frac{m_1 + m_2}{2} \\ N(m_1, \sigma, x) & x \ge \frac{m_1 + m_2}{2} \end{cases}$$
(8)

According to Klir and Yuan [36], there are many methods to construct membership functions. The method can be classified as direct approach and indirect approach. Direct approach is totally base on experts opinion by answering to some questions that are pertaining to the membership function. In other words, direct approach tends to initialize the model by arbitrary values. For indirect approach, experts are



required to answer simpler questions and implicitly related to the membership functions which the approach is able to reduce the arbitrariness. However, in this study, direct approach is applied where the values to construct membership functions are initialized from the scratch.

# IV. Design of CO<sub>2</sub> Emission Forecasting Model

Several assumptions have been proposed in order to develop the type-2 FLS to forecast CO<sub>2</sub> emissions. In this paper, our framework is initialized from numerical datasets. The training data set are created from available data. First of all, an assumption is made that all the type-2 fuzzy sets are interval type-2 fuzzy sets. Next, the proposed model required that the antecedent and consequent membership function are considered as type-2 Gaussian primary membership. Meanwhile, the input membership functions are Gaussian primary membership functions, with uncertain mean as shown in previous section.

Construction of fuzzy rules process is similar to training process in which historical data of CO<sub>2</sub> emissions are utilised one by one. The centre of the fuzzy set in antecedents and consequents will be established as soon as the process finished. After that, fuzzy inference engine will play its roles by calculating 'firing level' for each rule and then applies the firing levels to the consequents fuzzy sets. Since we are using an interval type-2 fuzzy set, hence our firing level will be in an interval set. Product implication and t-norm are used for the fuzzy operations. Outputs from the inference engine will be processed by a type reducer. Centre-of-sets method has been used in this paper as the type reduction.

Meanwhile, extended weighted average was used as the defuzzification method. The defuzzification process is the last process whereby the result from type-reducer will be defuzzified into a crisp value which presented the forecasted values of  $CO_2$  emissions. These assumptions are made to avoid noisy data and to simplify computation in the model.

### V. Data

The data used in this paper were retrieved from the website of World Bank's World Development Indicators [37]. The models were generated using the collected data. The data were divided into two sets: 80% of the data has been used for training data and 20% for testing data. Inputs that were assigned as  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$ , and  $x_6$  represented gross domestic product per capita (GDP) with the unit constant 2000 US\$, energy use in kg of oil equivalent per capita, population density (people per sq. km of land area), combustible renewable and waste (% of total energy), CO<sub>2</sub> intensity (kg per kg of oil equivalent energy use) and CO<sub>2</sub> emissions from transport respectively. The annual historical data for Malaysia were ranged from 1971 to 2009. CO<sub>2</sub> emissions (metric tons per capita) are the output of the model denoted as y.

# VI. Error Analysis

Models' performances are measured using error analysis. Small error indicates a precise and accurate model. Three statistical evaluation criteria are chosen as indicator to evaluate the performances of the type-2 FLS. The three

commonly used criteria are mean absolute percentage error (MAPE), root mean square error (RMSE), mean absolute error (MAE). The measures are defined as follows:

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{\left(x - \hat{x}\right)^2}{n}}$$

$$MAE = \sum_{l=1}^{n} \frac{|\chi - \hat{\chi}|}{n}$$
(9)

$$MAPE = \frac{100}{n} * \left| \frac{x - \hat{x}}{\hat{x}} \right|$$

where x = actual value,  $\hat{x} =$  predicted value of x, n = total number of points.

### VII. Results and Discussion

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TABLE I: RESULTS OF THE MODELS APPLIED IN FORECASTING  $\mathrm{CO}_2\,\mathrm{EMISSIONS}$ 

Performance Criteria	Type-1 Singleton FLS	Interval Type-2 Mamdani Singleton FLS	Interval Type-2 TSK FLS
RMSE	1.8723	0.6538	0.5338
MAE	1.7662	0.5438	0.5112
MAPE	24.3349	8.0676	7.2597

The experimental results show that interval type-2 TSK FLS outperforms other models. It has the smallest values of RMSE, MAE and MAPE which are 0.5338, 0.5112 and 7.2597 respectively. There are a huge difference between the values of the performance criteria from type-1 singleton FLS and the other interval type-2 FLS models. The difference means that there is a significant improvement between type-1 FLS model and interval type-FLS.

### VIII. Conclusion

Two types of interval type-2 FLS, Mamdani and TSK have been presented to forecast  $CO_2$  emissions. The same design was applied to the two models. The same data were also simulated using type-1 fuzzy logic system. The interval type-2 TSK model shows its superiority over the other two models with respect to error minimization. Hence, interval type-2 TSK FLS should be suggested as a promising and reliable tool in  $CO_2$  emissions forecasting.

There are a few aspects that can be improved in this study. For example, consideration of others environment quality indicators as our input variables in a long period should be made. However, there is limitation of available data that has



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made us to consider short-term forecasting. Furthermore, there are many other computational intelligence method could be proposed to compare and validate the results. The interval type-2 fuzzy logic could be improved from predefined initial values of antecedents and consequences, approach to construct membership function, its learning algorithm and hybridization of the model.

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