

Optimizing Inventory Replenishment and Shelf Space Management in Retail Stores

Alyaa Abouali, Nermine Harraz, M. Nashat Fors

Abstract - *The retail stores put up for sale multiple items while the spaces in the backroom and display areas constitute a scarce resource. Availability, volume, and location of the product displayed in the showroom influence the customer's demand. Managing these operations individually will result in sub-optimal overall retail store's profit; therefore, a non-linear integer programming model (NLIP) is developed to determine the inventory replenishment and shelf space allocation decisions that together maximize the retailer's profit under shelf space and backroom storage constraints taking into consideration that the demand rate is positively dependent on the amount and location of items displayed in the showroom. The developed model is solved using LINGO® software. The NLIP model is implemented in a real world case study in a large retail outlet providing a large variety of products. The proposed model is validated and shows logical results when using the experimental data collected from the market.*

Keywords - Retailing management, inventory replenishment, shelf space allocation, showroom, and backroom

I. Introduction

In retail stores demand is influenced by the availability, volume, and location of the product displayed. Noteworthy developments in retail industries need to foster customer service and product availability. Product availability is a critical performance measure in the retail industries therefore retailers are obliged to provide high product availability to achieve satisfaction and loyalty among customers. The retail stores sell multiple items and have limited spaces in the backroom and display areas. The ordering quantities and the allocated shelf space in each display area are critical retailing operations having major impact on the financial performance of retail stores. Managing the ordering quantities and the allocated shelf space in each display area individually will result in sub-optimal overall retail store's profit.

Therefore the retailers' aims are to globally determine the optimum inventory replenishment and shelf space allocation decisions. The objective of this paper is to maximize the retailer's profit by optimizing the ordering quantities and quantities to be displayed jointly. In order to achieve an optimum solution a non-linear integer programming model will be developed.

II. Review of Literature

The retail industry represents a highly competitive industry characterized by low margins and high sales volumes [1]. The core of the retail business operates by breaking up bulk products into smaller lot-size quantities.

Holding substantial inventory carries the risk of a decline in value if the buying habits of customers change and results in significant fixed capital [2]. However, low operation cost and high product availability provide good customer service and competitive profitability. [3].

The retail store layout usually describes two main areas, a backroom to store the products and a showroom to display them. It is a well-established phenomenon that high product availability positively influences the sales of many items [4]. Due to shelf space limitations, efficient shelf space allocation management and product assortment are critical retail operations. Many manufacturers are willing to pay premiums in order to obtain preferred [7]. In return, manufacturers expect retailers to efficiently manage on-shelf inventory to provide high availability of the manufacturer's products.

Shelf space is one of the most important resources to attract more consumers in logistic decisions. Managing shelf space well can not only decrease inventory level but also have stronger vendor relationship and higher customer satisfaction as well [9]. Shelf space allocation problem arises when retailer has large number of products to display on limited shelf space available [10]. Shelf space allocation in a retail store is considered one of the most crucial areas where retailers have the opportunity to increase their sales, as previous research shows that unplanned purchases make about one third of all transactions in many retail stores [11].

Facing is a very important variable for shelf space allocation, and is one of the variables that influence the demand of the product [10], the allocated shelf space may

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have a different impact on sales from one product to another. Space elasticity is a retail parameter that is usually used to measure the responsiveness of the sales with regards to the change of allocated space [11], it is defined as “the ratio of relative change in unit sales to relative change in shelf space” [11].

Location is another variable which can influence the demand of a product. It is generally believed that “eye-level is buy-level”, shelves at the end of aisles, and at the store entrance are better positions, while top and bottom shelves are less important. However, some research shows that the shelves at both ends of the aisles are better than the middle positions, while others believe that customers prefer middle locations as opposed to the ends of the aisles [7]. These findings are based on the fact that some customers prefer to take the first item once they enter an aisle whilst others take time to “acclimatize” themselves and so ignore the first few items.

The present paper focuses on the facings (including the depth and stacking) and location variables only; nevertheless there are other variables that influence the sales, such as advertising, promotion, discounting, and seasonal products.

III. Mathematical Model Development

a. Problem Definition

The retail stores sell multiple items and have limited spaces in the backroom and display areas. The ordering quantities and the allocated shelf space in each display area are critical retailing operations having major impact on the financial performance of retail stores. Moreover the store’s inventory management system carries an overall quantity that combines the backroom and showroom together. The traditional inventory management system fails to separate between the two inventories. As a result, it was found that many items that run out-of stock on a retail shelf can still be found at another location in the retail store, such as the backroom.

Due to the recent competition in the retailing industry, retailers are striving to improve their operations. Managing these operations individually will obviously result in sub-optimal overall retail store’s profit. Therefore, a decision-making process is proposed to integrate these operations to reach a global optimal profitability.

b. Objective

The objective of this paper is to maximize the retailer’s profit by optimizing the ordering

quantities and quantities to be displayed jointly. In order to achieve an optimum solution a non-linear integer programming model is developed. The demand function for each item incorporates the main effects of shelf space as well as the location effects. In the model formulation, the inventory investment costs are included, which are proportional to the average inventory, and storage and display costs as components of the inventory costs and make a clear distinction between showroom and backroom inventories. The effect of the display area location on the item demand is also considered.

c. Model Structure and Formulation

Notations

- Indices
 - i: item number (1,2, ..., n), where n: set of items
 - k: product location (1,2, ..., m), where m: set of locations
- Model Parameters
 - F_k : display capacity of shelf (k)
 - F_b : backroom capacity
 - α_i : space scale parameter for item (i)
 - β_{ik} : shape elasticity for item (i) at location (k)
 - P_i : selling price of item (i)
 - C_i : purchasing cost of item (i)
 - A_i : ordering cost of item (i)
 - h_i : inventory investment cost of item (i)
 - h_{bi} : storage cost of item (i) in backroom
 - h_{sik} : display cost of item (i) on shelf (k)
 - f_i : the shelf space required per unit of item (i)
 - y_i : Product Assortment (i)
 - y_{ik} : Product Assortment (i) in location (k)
 - k: Percentage of lost items
- Decision Variables
 - q_{ik} : Total quantity of item (i) displayed at location (k)
 - q_i : Total quantity of item (i) replenished
 - s_i : Quantity of item (i) displayed in showroom
 - s_{ik} : Quantity of item (i) displayed in showroom at location (k)
- Other Notations
 - T_i : total inventory cycle time of item (i)
 - T_{ik} : total inventory cycle time of item (i) displayed on shelf (k)
 - τ_{ik} : Time to consume quantity ($q_{ik} - s_{ik}$) at rate of $D_{ik}(0)$ per unit of time
 - IIC_{ik} : Investment Inventory Cost of item (i) displayed at location (k)

$D_{ik}(t)$: Demand function for item (i) in location (k) at time (t)
 $D_{ik}(0)$: Demand function for item (i) in location (k) at time (0)
 $I_{s_{ik}}(t)$: quantity of item (i) displayed on shelf (k) at time (t)
 $\bar{I}_{s_{ik}}$: Average inventory at showroom of product (i) at location (k) over cycle time T_{ik}
 $\bar{I}_{b_{ik}}$: Average inventory at backroom of product (i) at location (k) over cycle time T_{ik}
 TNP: Total Net Profit for all products

d. Model Assumptions

- The inventory system involves a set of items, $N = \{1, 2, \dots, n\}$, m locations within the store each with a display capacity F_k ($k = 1, 2, \dots, m$), and a backroom facility with limited storage capacity F_b .
- Replenishments to the system are independent for each item (no joint replenishments).
- Replenishments are sent directly to the backroom inventory, and are instantaneous with a known and constant lead time.
- The display areas are partitioned into dedicated areas for each product.
- Showroom is replenished continuously from the backroom inventory.
- The demand rate of the item is deterministic and is a known function of its displayed inventory level.
- The products are independent and not correlated.
- The products are pre-assorted to their assigned shelves
- Shortages are not allowed.

IV. Model Formulation

In order to follow the variation of the displayed quantity of item i on shelf k over time, the retail store operates after the following assumption. After receiving quantity q_i in the backroom area and reserving a quantity q_{ik} for shelf k, s_{ik} (maximum display space allocated to item i on shelf k) units are moved to the k^{th} display area.

Since then, the allocated shelf space is continuously replenished from the backroom to maintain it fully stocked items until the time at which the backroom's inventory is completely depleted at time (τ_{ik}), as shown in Figure 1. During this time period the displayed quantity is kept constant at level s_{ik} . Thereafter, the allocated shelf is no longer refilled from the backroom until all displayed quantity is sold. At that time, a new inventory cycle will start again. It can be observed from Figure 2 that since $D_{ik}(t)$ is a function of the displayed inventory levels of all items included in the assortment, the demand rate of an

individual item will change for every instance when the displayed quantity in any location of any other item falls below its shelf space allocation (s_{ik}). Generally, as commonly used from previous references the demand rate for any product i displayed on shelf k is the result of the space scale parameter (α_i) multiplied by the quantity of product i displayed on shelf k at time t ($I_{s_{ik}}(t)$) to the power of the shape space elasticity (β_{ik}) [6, 9, 10]. Therefore, the demand for any product i displayed on shelf k can be estimated by

$$D_{ik}(t) = \alpha_i I_{s_{ik}}^{\beta_{ik}}(t)$$

From Figure 1 and Figure 2, it could be deduced that the at time zero, the demand can be given by

$$D_{ik}(0) = \alpha_i s_{ik}^{\beta_{ik}}$$

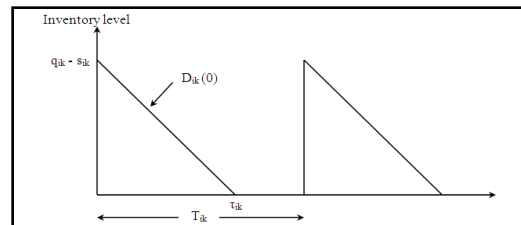


Figure 1: Variation of the Inventory Level Over Time in the Backroom Facility.

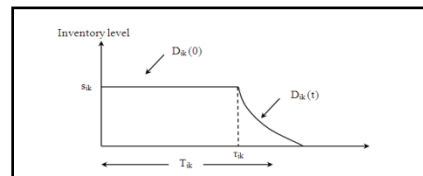


Figure 2: Variation of the Inventory Level Over Time in the Display Facility.

From the previous figures the cycle time is given by

$$T_{ik} = \frac{q_{ik} - s_{ik} + \frac{s_{ik}}{1 - \beta_{ik}}}{1 - y_{ik} + (y_{ik} * D_{ik}(0))}$$

Now an approximate expression of the demand function over time has been developed, the next step is to develop an expression for the total profit per unit of time. The different components of the net profit realized by product i in location k are as follows:

1. Gross profit per cycle.
 $(P_i - C_i)q_{ik}$
 Other revenue sources are neglected such as prizes or entrance fees.
2. Ordering cost = A_i .
3. Inventory investment cost per cycle for the i^{th} product in the k^{th} location.

$$IIC_{ik} = h_i(\bar{I}_{b_{ik}} + \bar{I}_{s_{ik}})$$

$$IIC_{ik}$$

$$= h_i \left(\frac{\frac{1}{2}((q_{ik} - s_{ik})^2 + s_{ik}(q_{ik} - s_{ik}) + s_{ik}T_{ik}D_{ik}(0))}{1 - y_{ik} + y_{ik}D_{ik}(0)} \right)$$

4. Backroom storage cost per cycle in the k^{th} location
 $h_{bi}T_{ik}(q_{ik} - s_{ik})$
5. Display cost per cycle in the k^{th} location
 $h_{sik}T_{ik}s_{ik}$
6. Lost Items
 kP_iq_i
7. Cost of Review C_rq_i

By letting $T_i = \text{Max} \{T_{ik}: k = 1, 2, \dots, m\}$

Hence, the integer non-linear programming formulation for shelf space allocation and inventory replenishments can be stated as:

a. Objective Function

Maximize

$$TNP = \frac{\sum_{i=1}^n \sum_{k=1}^m \{(P_i - C_i)q_{ik} - HC_{ik} - h_{bi}T_{ik}(q_{ik} - s_{ik}) - h_{sik}T_{ik}s_{ik}\} - 0.015P_iq_i - C_rq_i - A_iy_i}{1 - y_i + (T_i)y_i}$$

b. Subject to

$$\sum_{i=1}^n f_i s_{ik} \leq F_k \quad \forall k \quad (1)$$

$$\sum_{i=1}^n f_i q_i \leq F_b \quad (2)$$

$$q_i = \sum_{k=1}^m y_{ik} q_{ik} \quad \forall i \quad (3)$$

$$s_i = \sum_{k=1}^m y_{ik} s_{ik} \quad \forall i \quad (4)$$

$$q_i^{\min} \leq q_i \leq q_i^{\max} \quad \forall i \quad (5)$$

$$s_i^{\min} \leq s_i \leq s_i^{\max} \quad \forall i \quad (6)$$

$$s_{ik} \leq q_{ik} \quad \forall i, k \quad (7)$$

$$0 \leq T_{ik} \leq y_{ik} T_i \quad \forall i, k \quad (8)$$

$$q_i, q_{ik}, s_i, \text{ and } s_{ik} \text{ are integers} \quad (9)$$

The objective function maximizes the total net profit for all products. The total net profit components include the gross profit generated over an inventory cycle which is equal to the purchasing cost subtracted from the selling price for all the quantity of products i displayed on shelf k , the ordering cost, the inventory investment cost per cycle for item i on shelf k , the backroom storage cost per cycle on shelf k , the display cost per cycle on shelf k , the cost of error that can be due to thefts, misplacement, or lost items, and the cost of periodic review to keep track of the

quantity replenished and quantity displayed on the shelves. Constraint sets (1) and (2) ensure that the amount of shelf space required does not exceed the display space capacity (F_k), and that the total storage required for the quantity replenished does not exceed the backroom storage space capacity (F_b) respectively, where f_i is the amount of shelf space required per unit of item (i). Constraint sets (3) and (4) state that the sum of products ready to be displayed on shelf k equals the total quantity replenished and the sum of products (i) displayed on shelf (k) equals the total number of products (i) displayed in the showroom respectively. Constraint sets (5) and (6) limit the maximum and minimum number of stored and displayed products respectively and impose the upper and lower bounds for the quantity stored. Constraint set (7) is the balance constraint for the quantities displayed in the showroom on shelf (k) and the total quantity waiting to be displayed on shelf k . In constraint set (8) T_i is the maximum value of the T_{ik} s and puts T_i equal to zero when item (i) is not included in the assortment. Finally, constraints set (9) are the integer constraints.

V. Case Study

A case study was conducted in a large retail store in Alexandria, Egypt. This store does not take into consideration that the demand rate depends on the quantity of stock displayed on the shelf, the location of the product on the shelf, the location of the shelf, and the type of product, where some products may be under stocked or over stocked causing shortage or excess inventory, increasing costs and customer dissatisfaction. Besides, achieving maximum shelf utilization is one of the most difficult challenges that may face this store or any retail store. This paper focuses its research on 48 products to be assigned on 5 shelves. The quantities displayed on the shelf are decided according to the sales of the previous month, and is updated each month. In addition, the cycle time of the product is one week. The products arrive at the backroom and instantaneously go to the showroom, to be displayed in their location on the shelf.

The proposed model assumptions were inspired by the previous research Gajjar and Adil 2008, Hariga et al. 2007, Bai 2005, Yang and Chen 1999, and Urban 1988 [4, 6, 8, 9, 12]. These five articles and the unpublished thesis considered that the demand rate is stock dependent, hence, taking into consideration the retail parameters (space scale parameter- α_i , and shape space elasticity- β_{ik}). As the products are not the same size, the shelf space required for the product (f_i) is considered. Besides, previous research showed that the depth and stacking of the product is ignored, and only the facing is taken into consideration, this paper included the stacking and depth

of the product on the shelves. Therefore, the minimum number to be displayed on the shelf is 12 products. The retail parameters were deduced data from the supermarket's conditions and the previous articles, as they showed that the retail parameters should be $\alpha_i > 1$ and $0 < \beta_{ik} < 1$.

Another scenario is introduced to demonstrate another approach to the proposed model; which is to determine the optimum quantity to be replenished and the optimum quantity to be displayed that jointly maximize the customers' satisfaction, by maximizing the demand rate which depends on the stock level in the showroom. Therefore the new objective function will become

$$\text{Maximize Demand Rate} = \sum_{i=1}^n \sum_{k=1}^m \alpha_i \beta_{ik} S_{ik}$$

VI. Results and Discussion

The net profit of the proposed model was better than the current situation of the supermarket. This proves the importance of taking the inventory level on shelves and the retail parameters into consideration, and assures the important advantage of including the stock level to the demand rate.

The introduced scenario is concerned about the customer satisfaction rather than maximizing the profit, by comparing the proposed model with the suggested scenario, it is observed that the profit per day decreased, and the quantities to be replenished in the backroom reached its maximum capacity, and that the products with high space scale parameters (unmodified demand). However, the total quantities replenished did not change and are immediately displayed on the shelves in the showroom to ensure that the shelves are efficiently utilized with the products, and hence customer satisfaction.

VII. Conclusions

In this paper, an NLIP deterministic Stock-Dependant Demand model in the Retail Supply Chain has been developed. The model can be used in deciding crucial retail industry decisions. The NLIP model attempts to cover important issues identified from the comprehensive review of literature. The model has been tested, validated, and then applied in a large scale case study. The developed model determines the inventory replenishment and shelf space allocation decisions that jointly maximized the retailer's profit. The model could be adapted to model different scenarios, including changes in the retail store, such as increasing or decreasing the number of products, number of shelves, capacity of shelves, capacity of showroom, and capacity

of the backroom. Moreover the model has the advantage of stock-dependant demand rate. Where the demand rate depends on the amount of products displayed on the shelves. It would be useful to consider the probabilistic nature of some parameters in the future as well as introducing the issue of perishability of the products.

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