

# Physics of Gyroscope Motion About One Axis

Ryspek Usubamatov

**Abstract** - The known gyroscope theory based on principle of the rate change in the angular momentum of a spinning rotor. Practice demonstrates this mathematical model cannot describe other gyroscope effects and shows that there are other unsolved problems, which is spawned steady myth of the gyroscope mystery. These phenomena attract new researchers to solve gyroscope problems. The nature of gyroscope effects is more complex that represented in publications and known mathematical models do not match the actual motions and forces of the gyroscope effects. Latest studies of forces that involved in motions of a gyroscope demonstrate that there are four dynamical components, which act simultaneously and interrelated with others. The gyroscope with the external torque applied is experienced the resistance torque generated by the centrifugal and Coriolis forces and the precession torque generated by the inertial forces and the rate change of the angular momentum of the spinning rotor. These four dynamical components is represented the fundamental principles of gyroscope theory. Interrelation of this torques demonstrates the new unknown effects of the gyroscope. In case of the blocking the action of the precession torques resistance torques are deactivated. The mathematical model of forces acting in the gyroscope practically tested and the results validated the theoretical approach.

*Keywords* – gyroscope theory

## I. Introduction

Gyroscope was invented by different civilizations of people thousand years ago and still attracts attention by the astonishing behavior and amazing by unusual gyroscope properties. Simple gyroscopes are represented as the top toys and were not use as instruments yet scientists realize to study and better understand of spin and then to develop of a vast number of practical applications in aviation, space, ship and other industries. Numerous publications in area a gyroscope studies represent it as a device for measuring or maintaining orientation, based on the principles of angular momentum of the spinning rotor [1-4]. The gyroscope properties are relayed in many engineering calculations of rotating parts [5-8].<sup>1</sup>

This fundamental principle of the gyroscope theory derived by famous mathematician L. Euler in 1765 in his work on the dynamics of rigid bodies. Later, Sir Isaac

Newton and many other famous and outstanding scientists developed and added new interpretations for the gyroscope phenomena, which display in the rotor's persistence of maintaining its plane of rotation.

However, known gyroscope theory and its modifications do not match practice of the gyroscope forces and motions, i.e., no true gyroscope theory more than 200 years. This is unusual phenomenon in classical mechanics and probably the authority of the famous scientists dominated over other researchers, which did not try refuting settled formulations of gyroscope theory.

The known gyroscope theory cannot give true information about gyroscope motions and forces that called artificially gyroscope effects and phenomenon. All mathematical models content numerous assumptions and simplifications and explain the gyroscope effect by the rate change in the angular momentum only that generates the precession torque [9-14]. However, the nature of the gyroscope physics is more complex than presented in encyclopaedias, textbooks and numerous publications.

Analyses of the motions of gyroscope devices demonstrate that the load torque applied to the gyroscope generates the four fundamental components that are pseudo centrifugal, Coriolis and inertial forces of the spinning rotor and the rate change in the angular momentum. These fundamental components formulate the gyroscope effects. The centrifugal and Coriolis forces generate the resistance torque in change the rotor's location. The inertial force of the spinning rotor generates the net of precession torques. The rate change in the angular momentum of the spinning rotor involves in resistance and precession torques. The simultaneous action of this group of forces and their interrelations has not been described in the physics of gyroscope effects. Based on new fundamental approaches the gyroscope effects are represented by new mathematical model [15-19]. This model is well-matched with practical results that conducted on the Super Precision Gyroscope model "Brightfusion Ltd". This paper represents unknown interrelations of the gyroscope dynamical components which are new properties of gyroscope and action of the described above forces. New gyroscope properties practically tested and validated new mathematical models of acting forces and motions.

## II. Methodology

New studies of the gyroscope motions and forces demonstrate that gyroscope devices generate the pseudo centrifugal, Coriolis and inertial forces of the spinning

<sup>1</sup> R. Usubamatov  
University Malaysia Perlis  
Malaysia

rotor and the rate change of the angular momentum of the spinning rotor. Several publications about these forces are represented by the articles [15-19].

The mass elements of the spinning rotor experience a radial acceleration and the pseudo centrifugal forces which a plane locates strictly perpendicular to the axis of the spinning rotor. Centrifugal forces naturally counteract on the action of the forces that are directed to changing the location of the spinning rotor plane. The load torque applied to the gyroscope leads to change the angular location of spinning rotor plane and generates a pseudo contracting moment of the centrifugal forces' components. The mathematical model of the resistance torque generated by the centrifugal forces for a spinning rotor is represented by the following equation [15].

$$T_{cr} = 2(\pi/3)^2 J\omega\omega_p \quad (1)$$

where  $T_{cr}$  is a resistance torque generated by the centrifugal forces of the spinning mass elements,  $J$  is a rotor's mass moment of inertia,  $\omega$  is an angular velocity of the spinning rotor,  $\omega_p$  is an angular velocity of precession of a spinning rotor.

The analysis of Eq. (1) demonstrates that the centrifugal forces' resistance on the action of the external torque applied to a gyroscope depends on the angular momentum of rotating mass, the angular velocity  $\omega$  of the spinning rotor and on the angular velocity of the forced precession  $\omega_p$ . The angular velocity of the precession  $\omega_p$  is causing the action of the external torque applying  $T$  to the spinning rotor.

The inclination of the disc of the spinning rotor will change the axial tangential velocity's direction of mass elements. This change in an axial velocity generates the axial acceleration as well as the inertial forces of mass elements. The action of the inertial forces of the mass elements is perpendicular to the plane of the spinning rotor. These inertial forces are causing the angular torque and the angular velocity of the rotor precession. This torque acts in the plane that perpendicular to the plane of the resistance torque action, which generated by the centrifugal forces. The mathematical model of the gyroscope precession torque generated by the inertial forces of the spinning mass elements is formulated and represented by the following equation [15]:

$$T_{in} = 2(\pi/3)^2 J\omega\omega_p \quad (2)$$

where  $T_{in}$  is a precession torque generated by the inertial forces, other parameters are as specified above.

The analysis of Eq. (2) demonstrates the precession torque generated by the axial inertial forces of the spinning mass elements of the rotor is the same as Eq. (1). The precession and resistance torques are generated by the same rotating masses, which accelerations are directed perpendicular to each other.

The rotating mass element of the spinning rotor and its forced precession that leads to inclination of the rotor's disc generates the Coriolis accelerations as well as the

Coriolis forces of mass elements. These Coriolis forces of the mass elements are acting perpendicular to the plane of the spinning rotor and are causing the resistance torque action as the centrifugal forces. The mathematical model of the resistance torque generated by the Coriolis forces is formulated and represented by the following equation [15]:

$$T_{cr} = (8/9)J\omega\omega_p \quad (3)$$

where  $T_{cr}$  is a resistance torque created by the Coriolis forces, other parameters are as specified above.

The analysis of Eq. (3) demonstrates the external torque applied to the spinning rotor causes its angular velocity of forced precession around the axis which is perpendicular to the rotor's axis and generates the resistance torque of the Coriolis forces of the spinning rotor. The resistance torque of the gyroscope is generated by the centrifugal and Coriolis forces.

The rate change in the angular momentum of the spinning rotor that well-described in classical mechanics, generates the torque, which acts as precession torque and represented by the well known equation [1-4]

$$T_{a.m} = J\omega\omega_p \quad (4)$$

where  $T_{a.m}$  is a torque generated by the change of angular momentum, other parameters are as specified above.

The total precession torque of the gyroscope is generated by the inertial forces and by the change of angular momentum of the spinning rotor. The defined torques based on the action of the centrifugal, inertial, Coriolis forces and the rate change in the angular momentum are acting simultaneously on the spinning rotor and represent the artificial term of gyroscope effects. The equations of the resistance and precession torques and percentage of acting in the gyroscope is represented in Table I.

### III. Gyroscope Tests

The action of the pseudo forces which are centrifugal, inertial and Coriolis and generated by the rate change of the angular momentum of the spinning rotor is validated by practical tests of the gyroscope with one side support. The action of mentioned internal forces is displayed on the motion of the gyroscope about axis  $ox$ . The test on the angular velocities of gyroscope precessions was conducted on the base of the Supper Precision Gyroscope "Brightfusion LTD" (Fig. 1), which the technical data are represented in Table III. The velocity of the spinning rotor measured by the Optical Multimeter Tachoprobe Model 2108/LSR Compact Instrument Ltd. with range of measurement 0 – 60,000.00 rpm. The change of the revolution velocity for the spinning rotor per time is represented in Fig. 1. For simplicity of calculations, the tests of the gyroscope motions are conducted for the location of the gyroscope axis that close horizontal with deviations  $\pm 10^0$ . The angular measurements of the

location for the gyroscope axis conducted optically several times by visualization and the angular template with accuracy  $\pm 1.0^0$ .

The Supper Precision Gyroscope “Brightfusion LTD” designed with spinning rotor of the complex form with components that mounted in the spherical ribbed frame

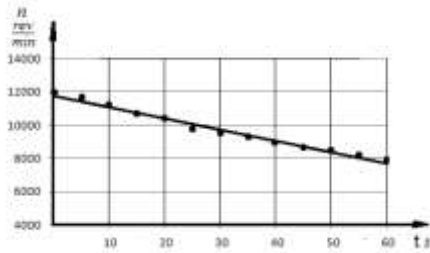


Fig. 1. Velocity for the spinning rotor versus time.

TABLE I. EQUATIONS OF TORQUES ACTING IN GYROSCOPE

Type of the torque	Equation	Percentage of action
The torque generated by the centrifugal forces	$T_{ct} = T_{in} = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_x$	34.95
The torque generated by the inertial forces		34.95
The torque generated by the Coriolis forces	$T_{cr} = (8/9)J\omega\omega_x$	14.16
The torque generated by the change in the angular momentum	$T_{am} = J\omega\omega_x$	15.94
Total		100

Type of the torque	Equation	Percentage of action
Resistance torque $T_r = T_{cr} + T_{cr}$	$T_r = \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right] J\omega\omega_x$	48.51
Precession torque $T_p = T_{in} + T_{am}$	$T_p = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right] J\omega\omega_x$	51.49
Total		100

TABLE II. DATA OF GYROSCOPE “BRIGHTFUSION LTD”.

		Parameters and components	Numerical data
Weight, W	Spinning components		0.1159 kg
	Aluminium frame		0.0294 kg
	Gyroscope,		0.1453 kg
	Total gyroscope		0.146 kg
Mass moment of inertia, J kgm <sup>2</sup>	About axis oz	Spinning components	$0.5543873 \times 10^{-4}$
		Aluminium frame	$1.3981904 \times 10^{-4}$
	About axis ox	Spinning components	$0.70871974 \times 10^{-4}$
		Total	$2.1069097 \times 10^{-4}$



Fig. 2. Supper Precision Gyroscope “Brightfusion LTD”

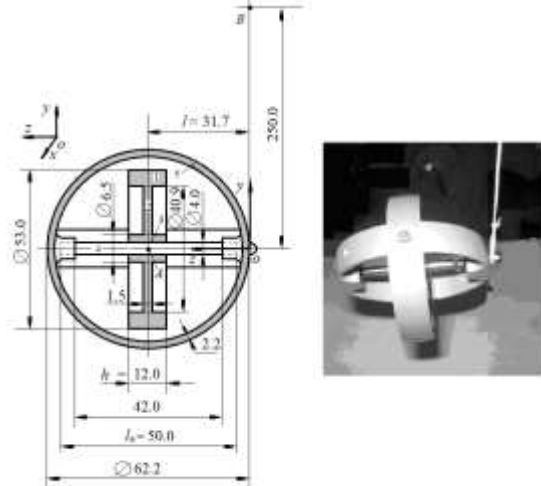


Fig. 3. Geometrical parameters of the gyroscope components

(Fig. 2). The technical parameters of the gyroscope components were calculated with aim to use for the equations of the gyroscope torques (Eqs. (1) -(4)). The percentage of action of the resistance and precession torques in the gyroscope has around three percentages of differences.

The gyroscope with one side free support is suspended on the cord and its weight generates the external torque  $T$  that results the gyroscope precessions about the point  $o$  of the support, which is the centre of coordinate system  $\Sigma oxyz$  (Fig. 3). This type of the tests for the gyroscope with one side free support enables to rotate the gyroscope about two axes  $ox$  and  $oy$ . The defined gyroscope parameters is represented in Table II and enable to calculate the torques by the equations represented in Table II.

Based on the initial data of the gyroscope is calculated the magnitudes of internal torques. The resistance and precession torques that generated by the centrifugal and inertial forces is represented by the following result:

$$T_{cti} = T_{inri} = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_{p,i} = 2\left(\frac{\pi}{3}\right)^2 \times 0.5543873 \times 10^{-4} \omega\omega_{p,i} = 1.2159074 \times 10^{-4} \omega\omega_{p,i}$$

The resistance torque that generated by the Coriolis forces is represented the following result:

$$T_{cri} = \frac{8}{9} J \omega \omega_{p,i} = \frac{8}{9} \times 5.543873 \times 10^{-5} \omega \omega_{p,i} = 0.4927887 * 10^{-4} \omega \omega_{p,i}$$

where all parameters are as specified above.

Gyroscope parameters	Tests, gr	Theoretical, gr
Reaction of the weight with screw and cord	146.0	146.0
Reaction of forces of resistance torque		7.21
Reaction of forces of precession torque		442.0
Total reaction of the free support	146.0	595.21

The tests of the gyroscope with one side free support on motions are conducted at the horizontal location of the gyroscope axis. This condition simplifies measurements and gives more accurate result in tests and calculations. The weight of the gyroscope and torques generated by centrifugal, inertial and Coriolis pseudo forces and generated by the rate change of the angular momentum of the spinning rotor should react on the one side free support of the gyroscope. The reactions all these force components are represented in Fig. 4 according to the rules of classical mechanics.

The formulas of reactive forces at the free support by axis  $oy$  based on action of the weight resistance and precession torques ( $F_i = T_i / l$ ) of the gyroscope are represented by the following equations:

$$F_w = Wg \tag{5}$$

$$F_{rx} = \left[ 2 \left( \frac{\pi}{3} \right)^2 + \frac{8}{9} \right] \frac{J \omega \omega_x}{l}$$

$$F_{py} = \left[ 2 \left( \frac{\pi}{3} \right)^2 + 1 \right] \frac{J \omega \omega_y}{l}$$

where all parameters are as specified and calculated above.

The tests of the gyroscope with one side free support demonstrate the gyroscope precession velocities about axis  $ox$ ,  $\omega_x = 0.0125386$  rad/s and about axis  $oy$ ,  $\omega_y = 0.741588$  rad/s. Substituting defined parameters that calculated and represented in Table III into Eqs. (5) and transforming yield the following results:

$$F_w = Wg = 0.146 \times 9.81 = 1.43226N$$

$$F_{rx} = \left[ 2 \left( \frac{\pi}{3} \right)^2 + \frac{8}{9} \right] \frac{J \omega \omega_x}{l} =$$

$$\left[ 2 \left( \frac{\pi}{3} \right)^2 + \frac{8}{9} \right] \times \frac{0.5543873 \times 10^{-4}}{0.0317} \times \frac{10000 \times 2\pi}{60} \times 0.0125386 = 0.0707755N = 0.00721kg$$

$$F_{py} = \left[ 2 \left( \frac{\pi}{3} \right)^2 + 1 \right] \frac{J \omega \omega_y}{l} =$$

$$\left[ 2 \left( \frac{\pi}{3} \right)^2 + \frac{8}{9} \right] \times \frac{0.5543873 \times 10^{-4}}{0.0317} \times \frac{10000 \times 2\pi}{60} \times 0.741588 = 4.336881N = 0.442087kg$$

Total reaction of the free support by axis  $oy$  on action of the active forces of the gyroscope is as follows:

$$F_o = F_w + F_{rx} + F_{py} = 1.43226 + 0.0707755 + 4.336881 = 5.839916 N = 0.595 kg$$

TABLE III. TEST RESULTS OF FORCES ACTING IN GYROSCOPE

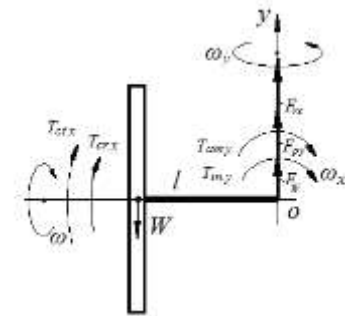


Fig. 4. Reactions of the one side free support of the gyroscope.

The theoretical reaction of the forces acting on the cord does not correspond definitely to the actual magnitude. This disparity leads to practical investigation and validation of the real magnitude of reaction on the cord. Practical test of the gyroscope with one side free support and measurement of the forces action on the cord (along the axis  $oy$ ) was conducted by the stand that represented in Fig. 5. The forces were measured by the Compact Digital Scale of the model Taylor TE10FT 5.0 kg with increments 1.0 gr. Figure 5 (a) and (b) represent the photography and the sketch of the stand with the digital scale and the gyroscope with one side support. The gyroscope with one side free support is suspended on the flexible cord that connected with the platform of the scale. The tests conducted for the functioning gyroscope which axis of the spinning rotor was at horizontal location. The results of theoretical calculations by Eq. (5) and measurements of practical tests of the acting forces are represented in Table III. Results of practical tests and

measurement of the forces acting on the cord of free support are unusual and demonstrate that the force weight of the gyroscope is equal to the reactive force of the support of the functioning gyroscope with horizontal location of its axis. The reactive forces that generated by the centrifugal, inertial, Coriolis forces and by the rate change of the angular momentum do not act on the free support of the cord or along the axis  $oy$ . At the first sight this phenomenon contradicts to the rules of the classical

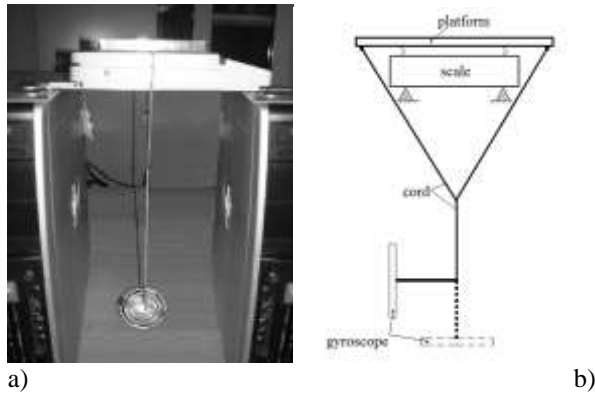


Fig. 5. Measurement of the acting forces on the gyroscope with one side free support.

mechanics. However, these gyroscope phenomena can be explained by the following reasons:

- The internal pseudo resistance torques generated by the centrifugal, inertial, Coriolis forces and by the rate change of the angular momentum are the reaction on action of the applied or load torque.
- The magnitude of the pseudo resistance torques are acting about axis  $ox$  cannot exceed the magnitude of the load torque that can be external or internal one. It means this resistance torques is restraint torques only and do not generate the reactive force on the support.

Described phenomena of the action the gyroscope internal resistance torques is new unknown property of the gyroscope that should be taken into account for the calculations of the gyroscope forces and motions at different mechanisms and devices. Gyroscope internal resistance torques that generated by pseudo forciers do not cause the reaction on the support along the axis  $oy$ . This statement is contradicted to the rules of the classical mechanics that deal with real forces and moments. However, in the gyroscope are acting the pseudo forces that appearing only in process of motions and disappearing in case of absent one. The nature of these phenomena of the action the pseudo forces in mechanical systems did not describe in science and need detailed investigation and mathematical modelling of their physics.

The obtained results, which representing new phenomena, can be spread on the action of the torques are acting about axis  $oy$  that are load torques and resistance torques. In this case, the resistance torques is restraint torques also that does not act on the support. It is proven

by the magnitudes of the load torques and resistance torques. Substituting the magnitudes of the angular velocities into equations of this torques, the magnitude of the resistance torque will be many times larger than the magnitude of the load torque. Theoretically, it leads to rotation of the gyroscope in opposite direction that acts the load torque. Practically, the gyroscope rotates in direction of the load torque. This is proving that resistance torque in the gyroscope is restraint torque. As a result, the gyroscope gets high angular velocity about the axis  $o$  under action of the inertial torque and the rate change of the angular momentum that represent the load torque.

The action of the pseudo torques acting about the axis  $oy$  for the gyroscope suspended on the flexible cord cannot be measured due to free motion of the cord at the horizontal plane  $xoz$ . However, the action of these torques are measured for the gyroscope with fixed supports.

The test of the gyroscope with fixed support about axis  $ox$  when the gyroscope rotation about axis  $oy$  is blocked or inhibiting demonstrates other phenomena. These phenomena are represented in the gyroscope rotation about the axis  $ox$  that is carrying out with high angular velocity, which larger when the gyroscope rotation is free about two axes. Analysis of acting torques on the gyroscope enables to explain this phenomenon by the following reason.

In case of the blocking the gyroscope rotation about axis  $oy$  the following torques is involved in process:

- The torque generated by the inertial forces and the rate change of the angular momentum of the spinning rotor with the precession about axis  $ox$ , activates the reactive counter torque from the support.
- Reactive counter torque about axis  $oy$  leads to deactivation of the resistance torque generated by the centrifugal and Coriolis forces about axis  $ox$  and to deactivation of the resistance torque generated by the inertial and rate change of the angular momentum about axis  $oy$ .
- Deactivation of all resistance torques about axis  $ox$  leads to increasing the angular velocity of the gyroscope precession about the axis  $ox$ .
- Inhibiting the gyroscope rotation about axis  $oy$  leads to proportional decreasing of the action the resistance torques that are involved in processes and to proportional increasing the angular velocity of gyroscope precession about axis  $ox$ .

## iv. Results and Discussion

The load torque applied to the gyroscope leads to an angular velocity of precessions and generates the torques based on action the centrifugal, inertial, Coriolis forces and the rate changes in the angular momentum of the spinning rotor. Based on the action of these torques is formulated the mathematical models for the resistant and precession torques about axis  $ox$  an  $oy$  at accepted systems of coordinates. These torques generated by the gyroscope weight which suspended by one side support demonstrate that torques depend on the mass moment of

inertia and angular velocity of the spinning rotor, as well as on the angular velocity of its precession. The new analytical approach to gyroscopic problems demonstrates that centrifugal, inertial and Coriolis forces of the mass elements for the spinning rotor are really active physical components as its the rate change in the angular momentum. These forces generate the torques that act simultaneously and result in the resistance and precession torques, respectively. Experimental tests and results of the gyroscope torques that conducted for the gyroscope suspended on side support demonstrates that resistance torque does not generate the reactive force of the support. The magnitude of the resistance torque does not exceed the magnitude of the load torque and inertial torque. Blocking motions of the gyroscope about one axis deactivate the resistance torques.

## v. Conclusion

The gyroscope theory in classical mechanics is one of the most complex and intricate in terms of analytical solutions. The known mathematical models in the gyroscope theory are mainly based on the actions of the load torque and the rate change in the angular momentum of the spinning rotor. The known gyroscope theory does not give correct answer on numerous practical problems. This is the reason that the gyroscope still attracts researchers that trying to find true theory. The known gyroscope theory and mathematical models for gyroscope effects do not consider the action of the pseudo centrifugal, inertial and Coriolis forces of the rotating mass of spinning rotor, which play a critical role. The new analytical approach demonstrates that the centrifugal and Coriolis forces of the spinning rotor resist any inclination of the rotor's axis and generate the resistance torques. The axial inertial forces of the spinning rotor generate the precession torque. The torque generated by the rate change in the angular momentum vector acts in resistance and precession directions in the gyroscope that depends on the motion of consideration.

New mathematical models for the gyroscope effects lead to new properties and will thus be useful for modeling the behavior of the gyroscopic devices. Formulated analytical equations of the gyroscopic motions, based on the principles of the centrifugal, inertial and Coriolis forces and the rate change in the angular momentum of the spinning rotor that are act simultaneously. The experimental tests and mathematical models of the gyroscope effects for the gyroscope that suspended one side support demonstrate that resistance torque resistance torque does not generate the reactive force of the support. The magnitude of the resistance torque does not exceed the magnitude of the load torque and inertial torque. Blocking motions of the gyroscope about one axis deactivate the resistance torques. The new analytical approach for the gyroscopic effects describes the gyroscope properties and demonstrates new properties

## Acknowledgment

The author would like to thank the University Malaysia Perlis for granting this research and express respect to governing body for their excellent strategic policy in scientific research.

## References

- [1] R. C. Hibbeler., *Engineering Mechanics - Statics and Dynamics*, 12th ed. Prentice Hall, Pearson, Singapore, 2010.
- [2] D. R. Gregory, *Classical Mechanics*, Cambridge University Press, New York, 2006.
- [3] C. Ciminelli, et al., *Advances in Gyroscope Technologies*, Springer-Verlag Berlin and Heidelberg GmbH & Co. KG, Berlin, 2010.
- [4] R.F. Deimel *Mechanics of the Gyroscope*, Dover Publications Inc. New York, 2003.
- [5] G. Greenhill, *Report on Gyroscopic Theory*, General Books LLC, London, 2010.
- [6] M. D. Ardema, *Analytical Dynamics. Theory and Application*. Academic/Plenum Publishers, New York, 2005.
- [7] E. Butikov, "Inertial rotation of a rigid body", *Eur. J. Phys.* 27, pp. 913 – 922, 2006.
- [8] Armenise M.N., et al, *Advances in Gyroscope technologies*, 1st ed., Springer- Verlag New York, LLC, 2010.
- [9] T. J. Quinn and A. Picard, "The mass of spinning rotors: no dependence on speed or sense of rotation", *Nature* 343, France, 1990, pp. 732 – 735, doi: 10.1038/343732a0.
- [10] H. Weinberg, "Gyro Mechanical Performance: the most important parameter", Technical Article MS-2158, Analog Devices, Norwood, MA, 2011, pp. 1-5.
- [11] Y. Ohshima, *Analysis and testing of gyroscope performance for the gravity probe B relativity mission*, Stanford University, CA, 2000.
- [12] L. Zyga, "Gyroscope's unexplained acceleration may be due to modified inertia", *PhysOrg.com*, July 26, 2011.
- [13] J.E. Faller, et al., "Gyroscope-weighing experiment with a null result". *Physical Review Letters* 64, *APS Journal*, Vol. 64. No. 8, 1990, pp. 825–826.
- [14] Z.Zhang, J.Sun, and K.Wu *Error analysis and test study of fiber optic gyroscope north-finder*. *Advanced Sensor Systems and Applications II*, Proceedings of SPIE, Vol. 5634, Cardiff, , 2005, pp. 611-618.
- [15] R. Usubamatov, et al, "Centrifugal forces are the cause of gyroscope stabilization", *Research Bulletin of the Australian Institute of High Energetic Materials*, April, 2011, pp. 226-233.
- [16] R. Usubamatov, "Analysis of inertial forces acting on gyroscope precession", *International Review of Aerospace Engineering*, Vol. 4, N2, 2011, pp. 103-108.
- [17] R. Usubamatov, "Mathematical Model for Gyroscope Effects", *Proceedings of ICMAAE 2013, International Conference on Mechanical, Automotive and Aerospace Engineering*, Kuala Lumpur, Malaysia, Paper 30040, 2013.
- [18] R. Usubamatov, A. B. Harun, M. Fidzwan, "Gyroscope mystery is solved", *Int. Conference on Advances in Engineering Science and Applied Mathematics (ICAESAM 2013)* Cape Town, South Africa, 2013, pp. 38-43.
- [19] R. Usubamatov, "Mathematical Model for Gyroscope Effects", *International Conference on Mathematics, Engineering and Industrial Applications (ICOMEIA 2014)*, 28-30 May Penang, Malaysia, Paper 71, 2014.